Model Checking

Static analysis techniques for finite-state models and design representations

A note on terminology

- "Model checking" often means "temporal logic model checking"
  - And recently, often just "Symbolic model checking with OBDD models"
- Related terms:
  - Finite-state verification (of concurrent programs)
  - Reachability analysis, concurrency analysis
- Closely related to flow analysis of sequential and concurrent programs

Models and Formulae

- An object may *be a model of* a formula
  - i.e., the models of a specification are objects that satisfy it; an inconsistent specification has no models
- Model checking: Given an object and a formula (specification), determine whether the object is a model of the formula
- Models derived from programs or designs, formulas express desired properties

Models & Formulae: Examples

- Models
  - Control flow graphs, data flow graphs
  - Reachability graphs (of Petri nets, process graphs, etc.)
- Formulae & other specs
  - Logics: Propositional or first-order, ordinary or temporal, real-time, authentication, . . .
  - Languages: Regular expressions, context-free languages
  - Particular properties of interest, e.g., freedom from deadlock

Temporal Logic

- Like a standard (first order or propositional) logic with additional connectives
  - first-order: with quantifiers; propositional: without

◊ “eventually” (“future,” “somtime”) Abbrev: F

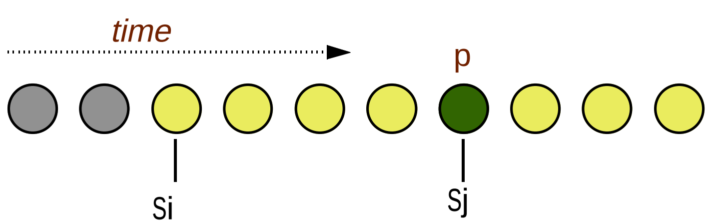
❑ “always” (“henceforth,” “globally”) Abbrev: G

U “until” Abbrev: U

❍ “next” (seldom desirable at spec level) X

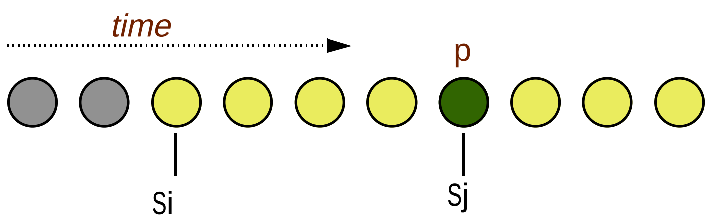
Meaning of “Eventually”

- Interpret propositional temporal logic as first-order statements about a sequence of program states S_0, S_1, ...
- Si |– p iff p is true in Si
- Si |– F p iff Sj |– p, for some j ≥ i



Alternate definition of "eventually"

- S |– p iff S_0 |– p
- S |– F p iff S |– p or X S |– E p
  - This latter definition is the basis of model-checking algorithms



Other temporal connectives

- Eventually q: q in this state, or
  eventually q in the next state
- Always p: p in this state, and
  always p in the next state
- p Until q: q in this state, or
  p in this state and p Until q in the next
- Next p: p in the next state

Why temporal logic?

- To say:

  "Eventually the call gets through"

  "Race conditions never occur"

  "N/S green does not come on until E/W light is red"

  "If scheduler is fair, all processes eventually run"
- Properties of progress, but not of *metric* time
- Especially for eventuality; safety (never, always) can be specified in other ways

Why use logic at all?
vs. operational spec or model

- Twin dangers of *over* and *under*-specification
  - Logic specs often say too little
  - Operational models often say too much
- Combination appears to be attractive
  - Say a few simple things with an appropriate logic
  - If the logic gets messy, move part of it into another kind of spec
- Example: Lamport's transition axiom method
  - State machine with invariants for safety properties, temporal logic for liveness properties

Temporal logic model checking

- Given a graph model of a program
  - State machine in which the propositional variables can be evaluated
- Given a propositional temporal logic formula
- Determine whether the model satisfies ("is a model of") the formula

CTL: Restricted branching-time logic

- Branching time: Quantification over paths
  - A graph of possible execution histories, not a single path through the program
  - A: All paths (from here)
  - E: Some path (from here)
- Restriction: Require quantifier with each temporal connective (for efficient checking)
  - AF, EF (inevitably, potentially)
  - AG, EG (always)
  - AU, EU (until)

Checking AFp

- Evaluate p in every state
- Initialize AFp to false in every state
- Apply inductive definition in each state until no values change
  – actual algorithm is a depth-first search, 1 pass over the graph

Model checking algorithm

- Decompose specification formula into a tree
- Each node => one pass over the graph
- Example: a and b:
  – Evaluate a at each node
  – Evaluate b at each node
  – Combine a and b at each node
- For temporal connectives,
  node values propogate along edges; order of evaluation is important for 1-pass evaluation

Fixed points

- A *fixed point* of a function f is a value x such that   f(x) = x
- A set of equations (constraints) may have a set of solutions (fixed points), among them a *least fixed point*
- Inductive definitions of temporal connectives can be formulated as finding a least fixed point solution

Temporal logic & fixed points

- AF p == p or AX AF p
- EF p == p or EX EF p
- AG p == p and AX AG p
- p AU q == q or (p and AX (p AU q))

"AX" and "EX" mean: Look at (all, any) of the edges from this node to its successors. The inductive defintions become a set of constraints, and a fixed point solution gives the value of the temporal formulae at each node.

Expressiveness of CTL

- There is no CTL equivalent for

  GF p => GF q

  - And this does come up in practice!

  – Example: If at least some packets get through, the protocol will eventually deliver a message
- Solution: Hack the algorithm
  - Hard-wire the fairness property into the model checking algorithm
  - See Clarke, Emerson, Sistla 85 (Toplas) for details

Complexity and Expressiveness

- Restricted branching time logics: CTL, LTAC
  – linear time checking procedures: |f| * |M|
- Linear time logic: PTL
  – 2^|f| * |M|
  - Why? Because formula is evaluated (in the worst case) on all paths.
- Cheap extensions:
  – arbitrary state machines as temporal connectives
  – PTL to CTL* (linear time to unrestricted branching time)

Symbolic Model Checking

- The model (graph) could be very large.
- Q: Can we do better than explicitly evaluating formulae in every state?
- A: Not always, but sometimes symbolic representations and lazy evaluation help
- Represent graph as next-state function (symbolically), represent formula as evaluation