Everything* you need to know about

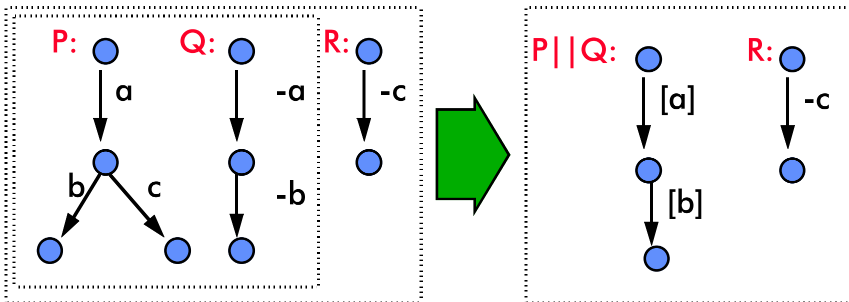
Process Algebra in 20 minutes**

**(more or less)
*(definitely less)

Motivation: Compositionality

- Compositionality ≅ Modularity
  - Ability to “compose” verifications of modules to verify a larger system
  - Logic example: Verify a program using pre- and post-conditions of verified procedures
  - Practical requirement: Verification or analysis results must be summarizations
- Compositionality in finite-state verification
  - Hierarchical analysis, summarizing results at each level
  - Potentially control state-space explosion

Non-Compositional Analysis



- We cannot find all behaviors of P||Q||R by finding behaviors of P||Q, then composing with R

Adding Compositionality ...

- We want algebraic structure
  - Commutativity, associativity, and a congruence
    - e.g., $A+B = C \Rightarrow A+D+B = A+B+D = (A+B)+D = C+D$
- Needed:
  - Account for "potential" behaviors of a subsystem
    - in (P||Q)||R, the partial result P||Q should include action b
  - ... but limit to interface actions
    - record "potential" behaviors only if they are visible outside a module (e.g., actions a and b don't matter to process R)
  - ... and simplify subsystem analyses
    - the difference between [a] and [b] should not matter outside the subsystem P||Q

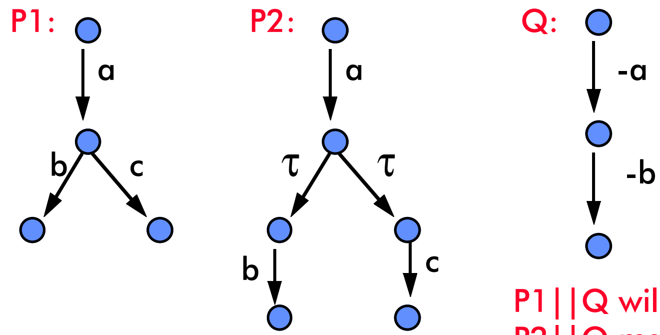
Processes as Terms

- Description of cooperating processes
  - Terms: similar to regular expressions
    - Context free processes are describable but too hairy
  - Process graphs: state machines denoted by terms
    - Regular processes denote finite-state process graphs
- Algebraic laws
  - Associative, commutative laws and substitution of equals for equals (and "less for equals") for incremental reasoning:
    X = A||B implies X||C = A||B||C (equivalence)
    X ≤ A||B implies X||C ≤ A||B||C (preorder)

Process Expressions

- Constants
  - δ (deadlock, or no action)
  - τ (internal, unobservable action, similar to ε)
  - a,b,c, ... Observable actions
- Expressions formed from
  - ; (sequence, with a;b abbreviated as ab)
  - \+ (choice)
  - | (synchronization of 2 events)
  - aP||bQ = (a|b)(P||Q) + a(P||bQ) + b(aP||Q)

Why $\tau \neq \varepsilon$



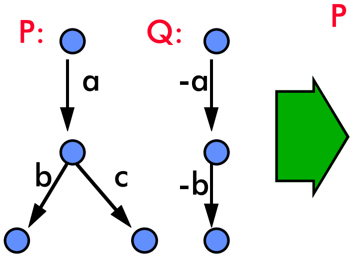
P1||Q will not deadlock
P2||Q may deadlock

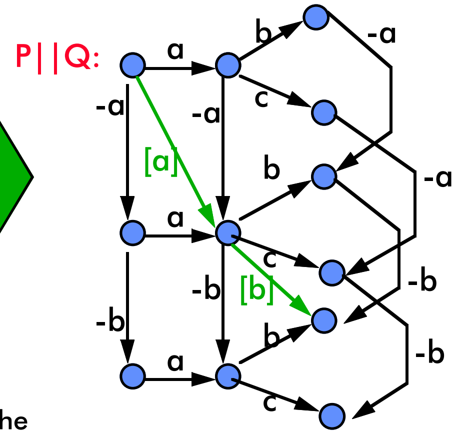
The other axioms of regular expressions come across without change, but note ab+ac = a(τb + τc).

Synchronization

- aP||bQ = (a|b)(P||Q) + a(P||bQ) + b(aP||Q)
  i.e., one moves first or else they move together
- In general, a|b is some action c
- In CCS, a|-a is τ, other pairs are δ
  - synchronization is rendezvous between action and co-action, and rendezvous is unobservable by other processes
- In CSP, a|a is a, other pairs are δ
  - synchonization is agreement to do the same thing

Product of Processes





This is progress?

(not unless we can simplify the intermediate product)

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Equivalence and Congruence

- Language equivalence is too coarse:
  - ab + ac = a(b+c), which we have seen is wrong
  - We want something nearly as coarse, but preserving deadlock, cheap to check and compute quotients
- Bisimulation:
  - P=Q iff P -a-> P' implies Q -a->Q' and P'=Q'
    Q -a->Q' implies P -a-> P' and P'=Q'
  - Strong bisim equivalent if we consider t an action
  - Weak bisim equivalent if an action is $a\tau^*$
  - Cheap to compute: similar to DFA minimization

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Abstraction and Restriction

- Abstraction: Substitute τ for a
  - Meaning: I don't care about a in this context
  - Especially: I don't interact with that action
- Restriction: Substitute δ for a
  - Meaning: That can't happen in this context
  - Especially: That interface isn't visible here
- At module boundaries,
  - Abstract actions that can happen "in the box"
  - Restrict actions in internal interfaces

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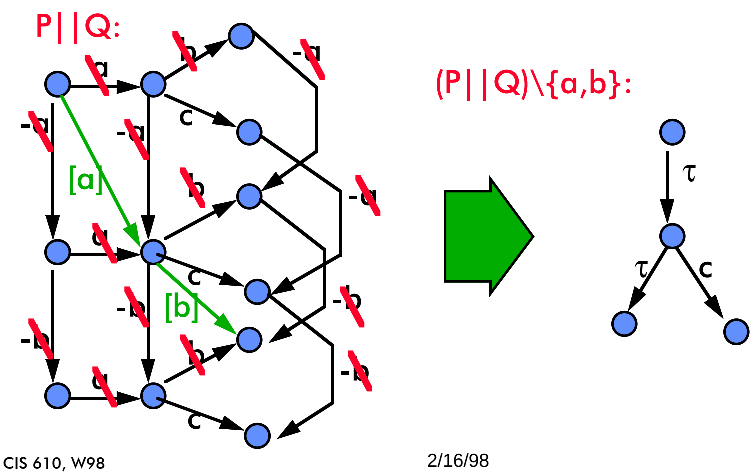
Simplifying P||Q

- Restrict a,b and abstract [a], [b]



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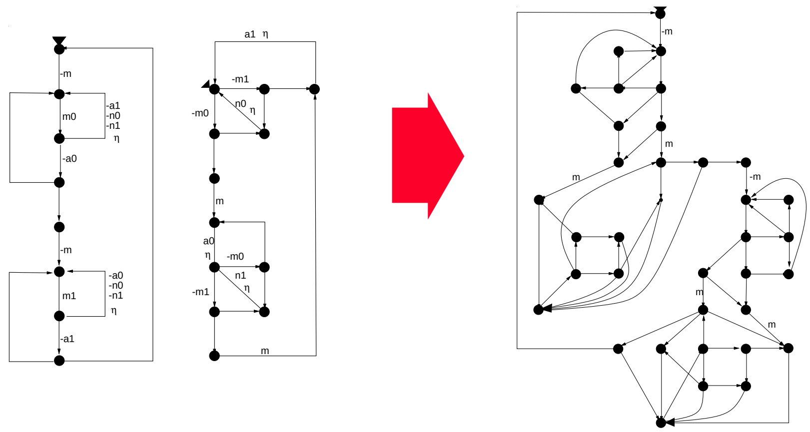
Preorder and Precongruence

- We don't always want equivalence
  - We want to permit looser specs, like a super/sub-type relation among processes
  - Example: Bounded queue of unspecified length
  - A "preorder" relates specification $\leq$ implementation
- The "testing" preorders
  - may: language inclusion
    - if p may pass a test, q may pass that test
  - must: failures inclusion
    - if p must pass a test, q must pass that test

Why should I care?

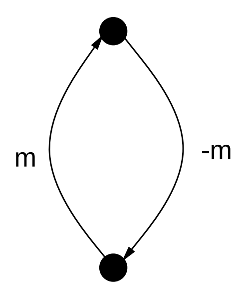
- Congruence (or preferably pre-congruence) is a useful definition of conformance of an implementation to an interface specification
- Process product permits one to say "these processes together meet that spec"
- Abstraction and restriction are the semantic building blocks for modularity
- Algebraic structure is essential (but not sufficient) for reasoning hierarchically about complex systems

State-space exploration example: Alternating Bit Protocol



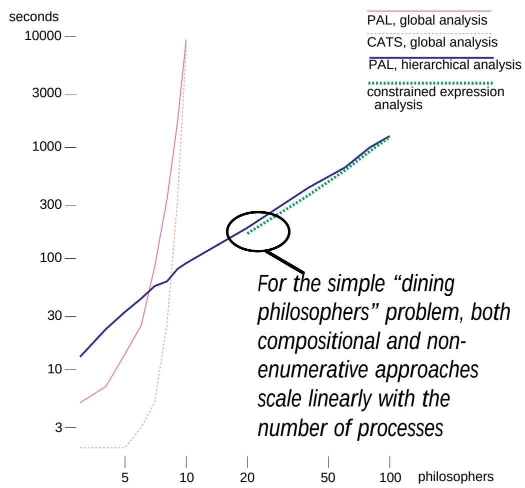
Alternating Bit Protocol: After reduction

- After restriction and abstraction, process graphs can be reduced to equivalent form with respect to a congruence relation



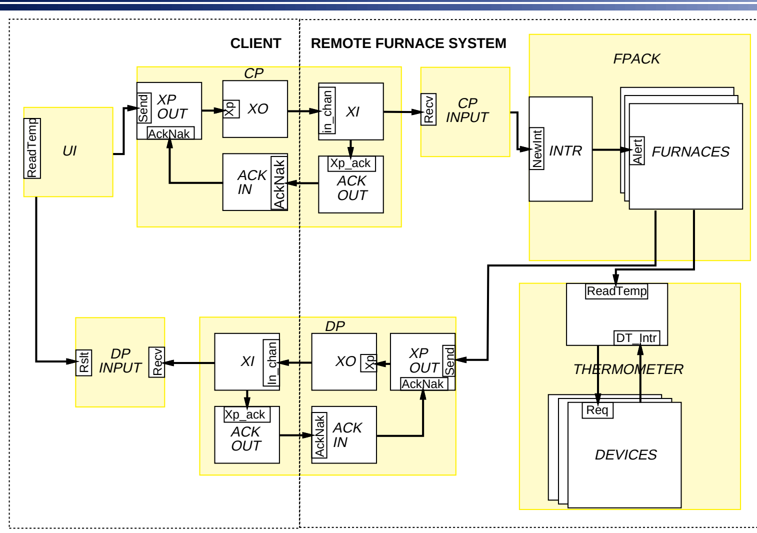
... but radical reductions in process graph size occur only when the system to be analyzed is "well-structured"

Scalable analysis

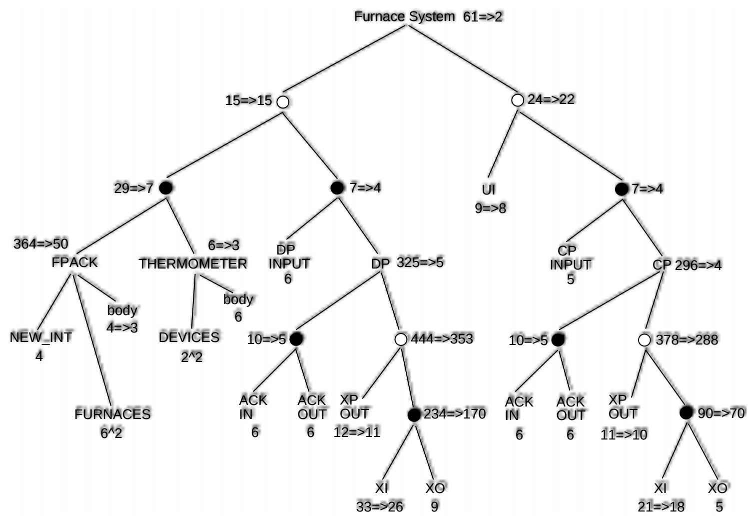


- When compositional analysis “works”, reductions at intermediate steps keep state-space to manageable proportions
- The question is, when does (or can) it work?

An example (redesigned)



Compositional analysis of revised design



Experience with Compositional Analysis using Process Algebra

- Has worked well for well-structured designs, poorly for code and “as built” designs
- (Re-)structuring for analysis is often necessary
  - Analyzable designs are more understandable and modifiable
  - *BUT ...* real designs are seldom structured as we want
  - *AND WORSE ...* there are good reasons for “bad” structure in source code
    - We must accept that the relation between a verified design and the “as built” structure of a system will not be simple