Guidelines: You may brainstorm with others, but please write up the answers by yourself. Acknowledge all collaborations and external resources used.

1. CLRS, Exercise 26.1-7

2. CLRS, Exercise 26.2-1

3. Prove the following: For any graph $G$ with edge capacities $\text{cap}(u, v)$ and any flow $f$, $\text{res}(u, v) + \text{res}(v, u) = \text{cap}(u, v) + \text{cap}(v, u)$. (Based on CLRS, Exercise 26.2-4)

4. (Grads only) Consider a modified network flow in which vertices as well as edges have capacities. Specifically, $\text{cap}(v)$ is a function from vertices $v \in V$ to non-negative numbers. The vertex capacities impose the following constraints on legal flows:
   For $v \neq s, t$, $\sum_w \max(f(v, w), 0) \leq \text{cap}(v)$

   (a) Prove that the following constraint is equivalent to the one stated above:
   For $v \neq s, t$, $\sum_w \max(f(w, v), 0) \leq \text{cap}(v)$

   (b) Describe how to translate a network flow with vertex capacities into one with only edge capacities, such that the maximum flow in the translated graph can be used to determine the maximum flow in the original graph.