CSE 413/513: Advanced Data Structures

Homework #7
Due in class on Thursday, June 3, 2010

Guidelines: You may brainstorm with others, but please write up the answers by yourself. Acknowledge all collaborations and external resources used.

1. Suppose you start with a “degenerate” skip list consisting of $n$ level-1 nodes and no higher level nodes, and then insert $n/2$ new nodes using the standard insert algorithm for skip lists, with $p = 1/2$.
   
   (a) Assume that the new nodes and old nodes are intermixed randomly (i.e., the probability that a new node directly precedes an old node is 1/3). What is the expected time to find a node in the final skip list with $3n/2$ nodes? Show your analysis.
   
   (b) Explain what could happen if the new nodes are not intermixed randomly. (For example, the new nodes could all have larger keys than the $n$ original nodes.)
   
   (c) (Grads only) Note that the expected time to insert the first new node is $n/2$. What is the total expected insertion time of all $n/2$ new nodes? Assume that new nodes are intermixed randomly.

2. In this question, you will consider skip lists where the maximum level is set artificially low.
   
   (a) Suppose that the maximum level of a skip list is two. In other words, when inserting a node, it is added to both lists with probability $p$ and to just the bottom list with probability $1 - p$. With $p = 1$, the expected worst-case number of comparisons is $n$. With $p = 1/2$, the expected number of comparisons is $n/2 + 1$. With $p = 1/3$, the expected number of comparisons is $n/3 + 2$. Find the optimal $p$ that minimizes the expected number of comparisons.
   
   (b) Generalize your answer to the case where there are exactly $k$ lists. (You may assume that $k < \log n$.)