Ensure you understand the course policies for assignments.

This homework is organized similarly to HW4 in that it contains two separate parts. You can select to do either Part I or Part II (you do not have to do both, but can elect to do so). If you do both parts, you can elect to replace the score for a past homework with the score for the optional part in this homework.

Part I: Continuation Passing Style

The problems below are all trivial or straightforward recursive problems. You must first write each of the functions in this part of the assignment in direct style (according to the problem specification), then transform the function definition into continuation-passing style. Several examples of direct and CPS style were given in lecture (e.g., Factorial). In the first section, you will familiarize yourself with the basics of CPS transformations: returning values, creating continuations, and linearizing computations.

1. Write the following low-level functions in continuation-passing style. A description of what each function should do follows:

- **addk** adds two integers
  
  ```
  val addk : int -> int -> (int -> a) -> a = <fun>
  ```

- **subk** subtracts one integer from another
  
  ```
  val subk : int -> int -> (int -> a) -> a = <fun>
  ```

- **timesk** multiplies two integers
  
  ```
  val timesk : int -> int -> (int -> a) -> a = <fun>
  ```

- **plusk** adds two floats
  
  ```
  val plusk : float -> float -> (float -> a) -> a = <fun>
  ```

- **take_awayk** subtracts one float from another
  
  ```
  val take_awayk : float -> float -> (float -> a) -> a = <fun>
  ```

- **multk** multiplies two floats
  
  ```
  val multk : float -> float -> (float -> a) -> a = <fun>
  ```

- **catk** concatenates two strings
  
  ```
  val catk : string -> string -> (string -> a) -> a = <fun>
  ```

- **consk** creates a new list by adding an element onto the front of a list
  
  ```
  val consk : a -> a list -> (a list -> b) -> b = <fun>
  ```

- **lessk** determines if one argument is less than another
  
  ```
  val lessk : a -> a -> (bool -> b) -> b = <fun>
  ```

- **eqk** tests if two arguments are equal
  
  ```
  val eqk : a -> a -> (bool -> b) -> b = <fun>
  ```

You can use the following report function to test your functions, e.g., `addk 3 4 report`.

```ocaml
let report x =
  print_string "Result: ";
  print_int x;
  print_newline();;
val report : int -> unit = <fun>
```
2. Nesting continuations We wish to add three numbers, but addk itself only adds two numbers. We could define add3k as follows:

```ocaml
et add3k a b c k =
    addk a b (fun ab -> addk ab c k);
val add3k : int -> int -> int -> (int -> a) -> a = <fun>
# add3k 1 2 3 report;;
Result: 6
- : unit = ()
```

On line 2 we give the first call to addk a function that saves the sum of a and b in the variable ab. Then this function adds ab to c and passes its result to the continuation k.

Using multk and plusk as helper functions, write a function abcdk, which takes four float arguments \(a, b, c, d\) and “returns” \((a + (b \times c)) + (d \times a)\). You may only use the \texttt{multk} and \texttt{plusk} operators to do the arithmetic. The order of evaluation of operations must be as indicated by the parentheses in the given formula. Where there are ambiguities, evaluate the expression on the right first.

```ocaml
# let abcdk a b c d k = ...
val abcdk : float -> float -> float -> float -> (float -> a) -> a = <fun>
# abcdk 2.0 3.0 4.0 5.0 (fun y -> report (int_of_float y));;
Result: 24
- : unit = ()
```

3. Transforming recursive functions. Consider the factorial example from class.

```ocaml
# let rec factorial n =
    if n = 0 then 1 else n * factorial (n - 1);;
val factorial : int -> int = <fun>
# factorial 5;;
- : int = 120
```

We can rewrite this making each step of the computation explicit:

```ocaml
# let rec factoriale n =
    let b = n = 0 in
    if b then 1
    else let s = n - 1 in
      let m = factoriale s in
      n * m;;
val factoriale : int -> int = <fun>
# factoriale 5;;
- : int = 120
```

To put the function into full CPS, we must make factorial take an additional argument, a continuation, to which the result of the factorial function should be passed. When the recursive call is made to factorial, instead of it returning a result to build the next higher factorial, it needs to take a continuation for building that next value from its result. In addition, each intermediate computation must be converted so that it also takes a continuation. We use the functions defined in Problem 1 and the code becomes:
# let rec factorialk n k =
  eqk n 0
  (fun b -> if b then k 1
   else subk n 1
     (fun s -> factorialk s
      (fun m -> timesk n m k)));
# factorialk 5 report;;
Result: 120
- : unit = ()

Note that to make a recursive call, we needed to build an intermediate continuation capturing all the work that must be done after the recursive call returns and before we can return the final result. If \( m \) is the result of the recursive call in direct style (without continuations), then we need to build a continuation to:

- take the recursive value: \( m \)
- build to the final result: \( n \times m \)
- pass it to the final continuation \( k \)

This is an extension of the “nested continuation” method.

In this problem, you are asked to first write a function in direct style and then manually transform the code into continuation-passing style. When writing functions in continuation-passing style, all uses of functions need to take a continuation as an argument. All uses of primitive operations (e.g. +, -, *, \( \times \), =) should use the corresponding functions defined in Problem 1. If you need to make use of primitive operations not covered in Problem 1, you should include a definition of the corresponding version that takes a continuation as an additional argument, as in Problem 1.

For each problem in this section, **first write the function as described in direct style.** Then, write the function again in continuation-passing style; append \( k \) to the name of the second definition. For example, if a problem asks you to write a function \( \text{splat} \), then you should define \( \text{splat} \) in direct style and \( \text{splatk} \) in continuation-passing style.

(a) Factorial range

i. Write the function \( \text{fact_range} \), which takes an integer \( n \), multiplies it to all integers less than \( n \) down to \( m \) (inclusive), and returns the result. If \( n < m \), then return 1.

```ocaml
# let rec fact_range n m = ...;;
val fact_range : int -> int -> int = <fun>
# fact_range 5 1;;
- : int = 120
```

ii. Write the function \( \text{fact_rangek} : \text{int} \to \text{int} \to \text{int} \to \text{a} \to \text{a} \) that is the CPS transformation of \( \text{fact_range} \) defined in part a.i.

```ocaml
# let rec fact_rangek n m k = ...;;
val fact_rangek : int -> int -> (int -> a) -> a = <fun>
# fact_rangek 7 5 report;;
Result: 210
- : unit = ()
```

4. Apply function to list.

(a) Write the function \( \text{app_all} : (\text{a} \to \text{b}) \text{ list} \to \text{a} \to \text{b} \text{ list} \) that takes a list of functions \( \text{flst} \) and a value \( \text{x} \) and creates the list made by applying each function in \( \text{flst} \) to \( \text{x} \). The functions should be applied in the order in which they occur in the list, and the results list should be the order corresponding to the function list. You should assume OCaml order of evaluation.
let rec app_all flst x = ...;;
val app_all : (a -> b) list -> a -> b list = <fun>
# app_all [((+) 1); (fun x -> x * x); (fun x -> 13)] 5;;
- : int list = [6; 25; 13]

(b) Write the function app_allk : (a -> (b -> c) -> c) list -> a -> (b list -> c) -> c
that is the CPS transformation of the code you wrote in part i. Your definition of app_allk must assume the functions in the input list will also be in continuation-passing style; that is, the type of the list is not (a -> b) list, but (a -> (b -> c) -> c) list.

let rec app_allk flstk x k = ...;
val app_allk : (a -> (b -> c) -> c) list -> a -> (b list -> c) -> c = <fun>
# app_allk [(addk 1); (fun x -> timesk x x); (fun x -> (fun k -> k 13))]
5 (fun x -> x) ;;
- : int list = [6; 25; 13]

5. Using continuations to alter control flow. As we have seen in the previous sections, continuations allow us a way of explicitly stating the order of events, and in particular, what happens next. We can use this ability to increase our flexibility over the control of the flow of execution (referred to as control flow). If we build and keep at our access several different continuations, then we have the ability to choose among them which one to use in moving forward, thereby altering our flow of execution. You are all familiar with using an if-then-else as a control flow construct to enable the program to dynamically choose between two different execution paths going forward.

Another useful control flow construct is that of raising and handling exceptions. In class, we gave an example of how we can use continuations to abandon the current execution path and roll back to an earlier point to continue with a different path of execution from that point. This method involves keeping track of two continuations at the same time: a primary one that handles “normal control flow, and one that remembers the point to roll back to when an exceptional case turns up. As in regular continuation-passing style, the primary continuation should be continuously updated; however, the exception continuation remains the same. The exception continuation is then passed the control flow (by being called) when an exceptional state comes up, and the primary continuation is used otherwise.

Write the function sum wholesk that takes a list of integers, a regular continuation, and an exception continuation, and sums all the numbers in the list. If any of the integers is negative, call the exception continuation and pass the offending value to it. Your definition must be in continuation-passing style, and must follow the same restrictions about calling primitives as the previous sections problems. (For this problem, you will receive no points for the direct style definition of sum wholes, though writing it will be helpful for you to convert it to continuation-passing style.)

let rec sum_wholesk l k xk = ...;;
val sum_wholesk : int list -> (int -> a) -> (int -> a) -> a

[0; -1; 2; 3]
report
(fun i ->
    print_string ("Error: " (string_of_int i) " is not a whole number");
    print_newline());;
Error: -1 is not a whole number
- : unit = ()

**Turn in:** OCaml code in file hw5.ml.
Part II: System F, Concurrent ML

hw5code.tar, available on the course website, contains several Caml files you need.

6. (System F and parametricity)
   
   (a) Give 4 values $v$ in System F such that:
   
   - $\vdash v : \forall \alpha.(\alpha \times \alpha) \rightarrow (\alpha \times \alpha)$
   - Each $v$ is not equivalent to the other three (i.e., given the same arguments it may have a different behavior).
   
   For one of your 4 values, give a full typing derivation.

   (b) Give 6 values $v$ in Caml such that:
   
   - $v$ is a closed term of type $\forall \alpha.\forall \beta.(\alpha \times \alpha) \rightarrow (\alpha \times \alpha)$ or a more general type. For example, $\forall \alpha.\forall \beta.(\alpha \times \beta) \rightarrow (\alpha \times \alpha)$ is more general than $\forall \alpha.\forall \beta.(\alpha \times \alpha) \rightarrow (\alpha \times \alpha)$ because there is a type substitution that produces the latter from the former (namely $\forall \alpha$ for $\forall \beta$).
   - Each $v$ is not totally equivalent to the other five.
   - None perform input or output.

For problems 7 and 8, do not use mutable references, mutexes, condition variables, fork-join, etc. All you need is thread-creation and the Concurrent ML primitives provided in the Event module. Remember to compile hw5.ml via ocamlc -vmthread -o hw5cml threads.cma cml.ml or similar. Also note Caml terminates when the main thread terminates, so it is sometimes convenient when testing to block the main thread (e.g., by calling Thread.yield in an infinite loop).

7. (Concurrent ML) Implement the first commented-out interface in cml.mli. Define type barrier and functions new_barrier and wait. If a barrier is created by new_barrier $i$ and a thread makes one of the first $i - 1$ calls to wait with that barrier, then the thread should block until the $i$th call, at which point all $i$ threads should proceed. A thread that calls wait with a barrier after there have been $i$ calls with that barrier should block forever.

   How to do it: new_barrier should return a channel that a newly created thread receives on. wait should send on this channel another channel and then receive on the channel it sends. (It does not matter what is sent on this channel; () is a fine choice.) The newly created thread should “remember” how many waiters there are and what channels to send on after the last arrives. The thread can then terminate. You do not need choose or wrap.

8. (Concurrent ML) Implement the second commented-out interface in cml.mli. This interface is for shared/exclusive locks (better known as readers/writer locks). Function newSelock creates a new lock. Functions shared_do and exclusive_do take a lock and a thunk and run the thunk. An implementation is correct if for each lock $lk$:

   - If there are any thunks that have not completed, then at least one of these thunks gets to run.
   - While a thunk passed to exclusive_do $lk$ runs, no other thunk passed to exclusive_do $lk$ or shared_do $lk$ runs.
   - If there are no uncompleted exclusive thunks, then the uncompleted shared thunks get to run in parallel.
(a) A correct solution to this problem has been given to you, but it is much longer and more complicated than necessary. In it, the “server” maintains explicit lists of waiting thunks. Reimplement the interface without these lists. Instead just use a simpler protocol that relies on the fact that CML allows multiple threads to block on the same channel, effectively forming an implicit queue. Hint: You still need \texttt{choose} and \texttt{wrap}, but you need fewer code cases and fewer total messages.

(b) Suppose a call to \texttt{shared\_do lk} blocks before a call to \texttt{exclusive\_do lk}. For the two implementations of the interface (the one given to you and your simpler one), describe the states under which we are sure the call to \texttt{shared\_do lk} will unblock before the call to \texttt{exclusive\_do lk}. Explain your answer, which can be different for the two implementations. Note \texttt{choose} is nondeterministic.

\textbf{Turn in:} OCaml code in file \texttt{cml.ml}. Written answers for Part II either electronically via turnin script or hard copies in class or Deschutes dropbox.