Main topics of the week:

- Definition of regular language and regular operations
- Closure of regular languages under union
- Examples and definition of nondeterminism


## Example of construction in proof of Theorem 1.12:

Here's an example of the construction used in the proof that the union of regular languages is regular for two simple DFAs. Let A be the language over the alphabet $\{\mathrm{a}, \mathrm{b}\}$ with even occurrences of $a$ and $B$ be the language with even occurrences of $b$. The union is pretty clearly the strings with an even number of a's or b's. The diagram shows the construction of the DFA to recognize this language. Note that we have labeled the states in the constructed DFA as the pair of states, one from each of the DFAs for A and B.


Construction recognizing $\mathbf{A} \cup \mathbf{B}$

## Example of an NFA.



The following diagram is a DFA that recognizes the same language. You should convince yourself that these two automata do recognize the same language. Notice that the $\varepsilon$ transitions and multiple and missing transitions of the NFA make it much easier to understand.


Another example. The following DFA has 5 states and recognizes the language that we can describe as any sequence of ab and aba. Even with just 5 states, it is not immediately obvious what the DFA does. Between states $\mathrm{q}_{4}$ and $\mathrm{q}_{5}$, we can see that any number of ab occurrences keeps the string accepted, but beyond that, it takes a lot of careful analysis to see what is going on. We have a dead state $\mathrm{q}_{3}$, that is apparently reached by seeing two a's in a row.


Let's contrast this with recognizing the same language with an NFA.


Note that here we have only three states and it is pretty easy to see what is going on, since the state progression is in only one direction, caused by a followed by $\mathbf{b}$, and then possibly another a or not. This clearly recognizes strings composed of ab or aba.

