Main topics of the week:

- **Countable and uncountable sets**
- **Diagonalization proof of uncountable**
- **Undecidable halting problem A_{TM}**
- **Complement of A_{TM} is not even Turing recognizable**
- **Review for final**

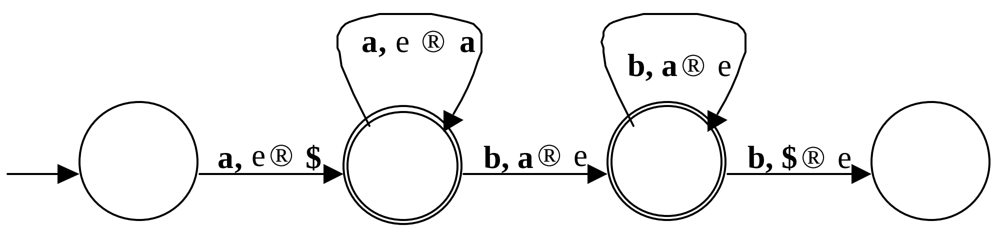
Review problems for final.

Let G be the grammar $\quad S \rightarrow aS \mid aA \mid a$

$\qquad\qquad\qquad\qquad\qquad\quad A \rightarrow aAb \mid ab$

Give a concise description of L(G) and a PDA to accept L(G).

We observe that the symbol A generates $a^n b^n$ for all $n \geq 1$. The rules for S using aA and a simply add another 'a' in front, so that gives us strings of the form $aa^n b^n$ for all $n \geq 0$. The rule aS for S allows an arbitrary number of a's to be added at the beginning, so L(G) = $\{ a^m b^n \mid m > n \geq 0 \}$.



Show that the C programming language is not a context free language.

Proof: Suppose that C is a CFL. By the pumping lemma for CFLs, there must be a pumping length p. Consider the legal C program:

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f(){int aa…a;aa…a;aa…a;}
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In this program, the variable is a^p, i.e., the symbol 'a' repeated p times. Suppose we realize this string as uvxyz, where $|vxy| \leq p$. If vy contains the blank or any of the symbols preceding it, then we certainly have a syntax error in uxz or at least an undeclared variable. If vy contains the last semi-colon or closing brace, then again uxz will have a syntax error. So vxy must be between the space and the last semi-colon. If vy contains a semi-colon and possibly some characters before and after, again uxz will have an undeclared variable since one set of the a's will combine to be at least p+1 a's while the other stays the same. If vxy contains no semi-colon, then uvvxyyz will have a second identifier, so again the program will not be legal since something will be undeclared. This exhausts all the possibilities; so legal C programs are not a context free language.

Let A = {<R,S> | R and S are regular expressions and L(R) Í L(S) } Show that A is decidable.

Proof: We describe a TM that decides A:

"On input <R,S> where R and S are regular expressions,

1) Convert the regular expression R to an equivalent DFA A, using the procedure for converting a regular expression to an NFA, and the procedure for converting an NFA to a DFA.
2) Likewise convert the regular expression S to a DFA B.
3) Using the procedure from an exercise, convert B to a DFA C that recognizes the complement of L(S).
4) Using the procedure from the problem on the midterm (just like the one for the union of regular languages), construct a DFA D from A and C to recognize $L(R) \cap L(\overline{S})$.
5) Run the TM for E_{DFA} on D to determine if $L(R) \cap L(\overline{S}) = \emptyset$.
6) If it is empty, *accept*, otherwise *reject*."

Note that each stage of our TM uses procedures that we know to halt, so this is a decider. By determining if the intersection of L(R) with the complement of L(S) is empty, we determine whether $L(R) \subseteq L(S)$ or not.

Let S ={<M> | M is a DFA that accepts w^R whenever M accepts w}. Show that S is decidable.

Proof: We describe a TM that decides S. Basically we test to see whether the language recognized by M is the same as the reverse of that language.

"On input <M>, where M is a DFA,

1) Construct a DFA N to recognize $\{w^R \mid w \in L(M)\}$. (Recall that we saw in a problem how to construct an NFA to do this by adding a single accept state and reversing all the arrows, and we know how to convert an NFA to a DFA).
2) Run the TM for EQ_{DFA} with input <M,N>.

If it accepts, *accept*, otherwise *reject*."