CIS 313 Introduction to Data Structures
Winter 2007

MIDTERM SAMPLE SOLUTION

1. Provide solutions (using big-Oh or big-Theta) for the following recurrence relations.

   (a) $T(n) = 7\ T(\frac{n}{3}) + n \lg n$

   (b) $T(n) = 314\ T(\frac{n}{313}) + 2^{315} \cdot n$

   (c) $T(n) = 25\ T(\frac{n}{5}) + n^2$

   {*sol'n*} The answers are $\Theta(n^{\log_3 7})$, $\Theta(n^{\log_{313} 314})$, and $\Theta(n^2 \lg n)$, respectively.

2. Into an initially empty AVL tree, insert the following values:

   $$62,\ 47,\ 32,\ 15,\ 26,\ 30,\ 27,\ 28,\ 29,\ 10,\ 5.$$

   {*sol'n*} See the attached graphics below.

3. Insert the values above into an initially empty 2-3-4 tree.

   {*sol'n*} See the attached graphics below.

4. What are the run-times of the following pieces of code?

   (a)
   ```
   for i = n downto 1 {
         j = i
         while (j>=1) {
            sum++
            j=j/313
            }
         }
   ```

   (b)
   ```
   for i = 1 to n*n*n
         for j = 1 to i*n
               sum++
   ```

   {*sol'n*} The first piece of code is $\Theta(n \lg n)$ - note that the inner loop runs for $\log_{313} i = \Theta(\lg i)$ steps.

   In part (b), the maximum value of i is n^3, and so the inner loop runs for at most n^4 steps. With the outer loop running n^3 times, the maximum total is $O(n^3 \cdot n^4) = O(n^7)$.

   A more accurate accounting would be

   $$\sum_{i=1}^{n^3} i \cdot n = n \cdot \sum_{i=1}^{n^3} i = n \cdot \frac{n^3(n^3-1)}{2} = \frac{n^7 - n^4}{2} = \Theta(n^7).$$

5. Write a recursive routine which, given an integer `k`, prints the keys of all nodes at *height* `k`. They can be printed in any order. The fields of each node are called `key`, `lchild`, and `rchild`.

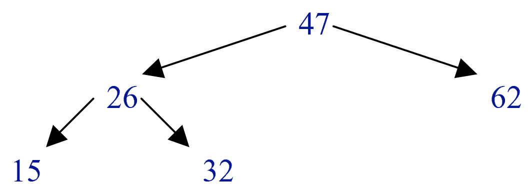
   {*sol'n*} This is similar to the balance factor problem of HW3 - in this case rather than calculating the balance factor, we compare our height to k. The initial call should be `getHeights(T.root, k)`.

```
procedure getHeights(node p, int k) returns int

   if (p==null) return -1

   lHeight = getHeights(p.lchild, k)
   rHeight = getHeights(p.rchild, k)
   thisHeight = max(lHeight,rHeight) + 1

   if (thisHeight==k)
      print p.key

   return thisHeight
```

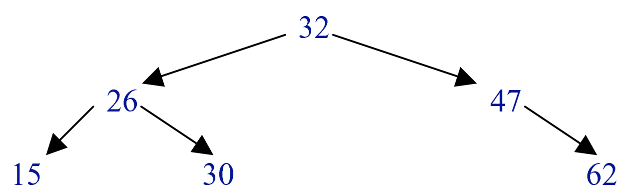
   The idea is to calculate the height of each node - get the heights of the left and right children, add one to the maximum of those two values. At this point, we compare our height to `k`, and print the key if equal. We will necessarily be doing a postorder traversal since we can't know the height of the current node before we know the heights of the children.

Question 2: *build AVL tree*

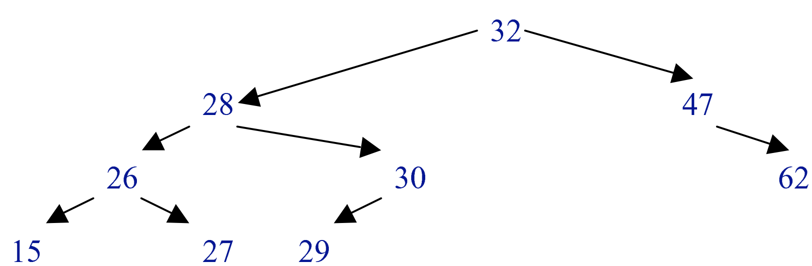
After the insertion of 26:



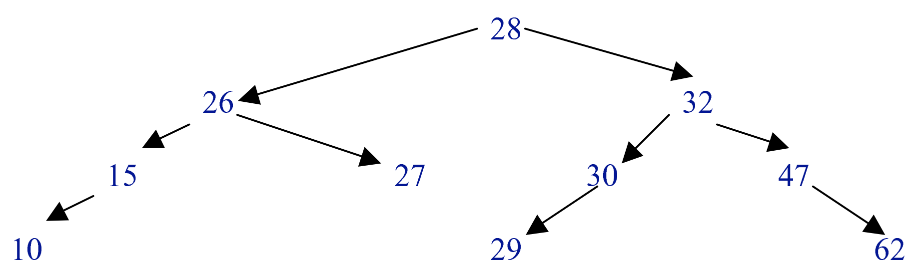
After 30:



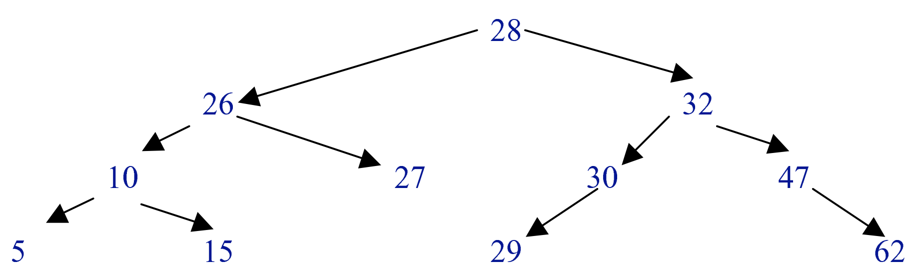
After 29



After 10



And the final tree



Question 3: *build 2-3-4 tree*

The final tree, using the bottom-up technique described in class, is