

## Extra Credit

The following are some slightly more challenging problems you might enjoy trying. We will assign some extra credit for these. This extra credit will be added to your score after the grades curve is determined - in this way there is no penalty for not doing any.

You may turn in a problem whenever you choose (note the first Note, however). You do not need to turn them in together. They will be graded strictly. Do not guess at a solution - you will receive no partial credit unless you are very close to being correct. It is possible that it will be returned to you ungraded for further elaboration, which you can then give for reconsideration (at no penalty).

These will be worth a varying number of points, depending on how hard the instructor feels they are and how much work you do. For example, the second one would be usually worth two points but if you generalize the problem you could get even more. In general, an extra credit point is worth more than a point on a homework.

**Note 1:** Only two extra credit problems per person will be accepted during dead week. None will be accepted during finals week. However, resubmissions of previously submitted problems are not subject to these constraints.

**Note 2:** This page may be updated throughout the term, so check occasionally for further problems.

- Here are two strange looking recurrence relations which can be massaged into a form that you will have seen before. Your goal is to solve them.
  - Solve  $T(n) = 8T(\sqrt{n}) + \lg n$ . Hint: Let  $S(n) = T(2^n)$ , derive a recurrence relation for  $S$ , solve it, and back-substitute to get a solution for  $T$ .
  - Solve  $T(n) = 3T(n - 1) + n2^n$ . Hint: let  $S(n) = T(\lg n)$ .
- On one of the homeworks you were asked to simulate a stack with two queues. More interesting is to simulate a queue with two stacks. This can be done more efficiently - any sequence of  $n$  *enqueue* and *dequeue* operations can be performed in a total of  $O(n)$  time. Explain how.
- Explain how to find the *second* largest of a list of  $n$  elements using at most  $n + \lg n$  comparisons. (*Hint:* run a tournament as a complete binary tree.)
- Let  $T$  be a binary tree. It is not necessarily full - there may be nodes with 0, 1, and 2 children - and in this context we consider a *leaf* to be a node with no children. Let nodes  $v_1, v_2, \dots, v_k$  be all the leaves, and for each  $i$  let  $l_i = \text{depth}(v_i)$ . Prove that

$$\sum_{i=1}^k 2^{-l_i} \leq 1.$$

This is known as *Kraft's Inequality*.

- Actually, Kraft's Inequality is more general and refers to codes. The previous question is an application to binary trees, and is only one direction (of an "if and only if"). For this question, show the following converse to the previous problem: If we have integers  $l_1, l_2, \dots, l_k$  such that

$$\sum_{i=1}^k 2^{-l_i} \leq 1,$$

then there is a binary tree  $T$  with leaves  $v_1, v_2, \dots, v_k$  such that

$$\forall i, l_i = \text{depth}(v_i).$$

- You saw on a homework that there is a tree that can be colored as a red-black tree which is not an AVL tree. Now prove that *any* AVL tree can be colored as a red-black tree.
- **(2-3-4 heaps)** exercise 19-1, pp 473-474
- Recall the voting problems from assignment 6: you are given a list of  $n$  votes, each vote is represented by the student number of the candidate receiving the vote. Since these student numbers are very large integers, they cannot easily be used as an index into an array. The problem here is to determine if someone received the *majority* (that is,  $> \frac{n}{2}$ ) of the votes. Surprisingly, this is easier. Show how determine if someone received the majority of the votes and, if so, who, in  $O(n)$  time. (*Hint:* Look at last year's final. Be very careful how you deal with an odd number of votes.)