CIS 427/527 Assignment #3 - Due Thursday, January 31, 2008

1. Understand and be able to complete the proof of soundness of propositional logic.
2. A set Γ of propositions is consistent if $\Gamma \nvdash \bot$.
   Prove that the following conditions are equivalent:
   (a) Γ is consistent
   (b) For no ϕ, $\gamma \vdash \phi$ and $\Gamma \vdash \neg\phi$
   (c) There is at least one ϕ such that $\gamma \nvdash \phi$.
3. Prove the following theorems in the system of Natural Deduction:
   (a) $\vdash (A \wedge B) \wedge C \rightarrow A \wedge (B \wedge C)$
   (b) $\vdash A \wedge B \rightarrow A \vee B$
   (c) $\vdash \neg(A \wedge B) \rightarrow \neg A \vee \neg B$
   (d) $\vdash \neg\neg A \rightarrow A$
   (e) $\vdash A \rightarrow \neg\neg A$
   (f) $\vdash \neg A \vee \neg\neg A$
   (g) $\vdash \neg\neg(A \vee \neg A)$
   (h) $\vdash (A \rightarrow B) \vee (B \rightarrow A)$
   (i) $\vdash \neg A \vee B \rightarrow (\neg\neg B) \vee \neg A$
   (j) $\vdash \bot \rightarrow A$
4. Which of the above theorems are provable in intuitionistic logic
5. Assume that α, β and γ are formulas of propositional logic. Prove or disprove each of the following:
   (1) $\alpha, \beta \models \gamma$ if and only if $\alpha \models \beta \rightarrow \gamma$;
   (2) $\alpha \models \beta$ and $\beta \models \alpha$ if and only if $\alpha = \beta$;
   (3) if $\alpha \models \beta$ or $\alpha \models \gamma$ then $\alpha \models \beta \vee \gamma$;
   (4) if $\alpha \models \beta$ or $\alpha \models \gamma$ then $\alpha \models \beta \wedge \gamma$;
   (5) if $\alpha \models \beta$ and $\alpha \models \gamma$ then $\alpha \models \beta \wedge \gamma$;
6. Assume that α and β are formulas of propositional logic.
   (1) Show that if α is unsatisfiable then $\alpha \models \beta$ for any formula β