CS 420/520 Automata Theory
Fall 2023

Assignment 3

due Thursday, October 26, 2023

1. Use the pumping lemma to show that the following are not regular:
   (a) $\{ 0^n 1^m 0^n \mid n, m \geq 0 \}$
   (b) $\{ w \in \{0,1\}^* \mid w$ has at most 4 more 0s than 1s $\}$
   (c) $\{ w0^{|w|} \mid w \in \{0,1\}^* \}$

2. Let $\Sigma = \{0, 1, +, =\}$ and

   $$\text{ADD} = \{x = y + z \mid x, y, z \text{ are binary integers and } x \text{ is the sum of } y \text{ and } z \}$$

   Show that ADD is not a regular language.

3. (a) Let $B = \{ 1^k y \mid y \in \{0,1\}^*$ and y contains at least k 1's, for $k \geq 1 \}$. Show that B is a regular language.
   (b) Let $C = \{ 1^k y \mid y \in \{0,1\}^*$ and y contains at most k 1's, for $k \geq 1 \}$. Show that C is not a regular language.

4. Give a context-free grammar accepting the languages below. For these, the alphabet is $\Sigma = \{a, b\}$.
   (a) $\{ w \mid w = w^R,$ that is, w is a palindrome $\}$
   (b) $\{ wa^{|w|} \mid w \in \Sigma^* \}$
   (c) $\{ a^i b^j a^{i+j} \mid i, j \geq 0 \}$