Gale-Shapley

example

favorite ↓ least favorite ↓

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

hospitals' preference lists

favorite ↓ least favorite ↓

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

students' preference lists

- n hospitals, n students
- one student per hospital
- each hospital ranks all students
- each student ranks all hospitals

unstable

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

A–Y is an unstable pair for matching M = { A–Z, B–Y, C–X }

the algorithm:

GALE–SHAPLEY (*preference lists for hospitals and students*)

INITIALIZE M to empty matching.

WHILE (some hospital h is unmatched and hasn't proposed to every student)

  $s \leftarrow$ first student on h's list to whom h has not yet proposed.

  IF (s is unmatched)

    Add h–s to matching M.

  ELSE IF (s prefers h to current partner h')

    Replace h'–s with h–s in matching M.

  ELSE

    s rejects h.

steps

1. s remains matched from the first time they receive a proposal; the sequence of h they are matched to gets better over time
2. the sequence of s that an h proposes to gets worse over time
3. the GS algorithm terminates after at most n^2 iterations
4. if h is unmatched at some point in the execution, there there is an s to which h has not yet proposed
5. the set M returned at termination is a perfect matching
6. the set M returned is a stable matching (pf on next page)

Claim. In Gale–Shapley matching M^*, there are no unstable pairs.

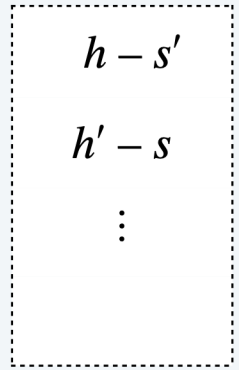
Pf. Consider any pair h–s that is not in M^*.

- Case 1: h never proposed to s.
  - $\Rightarrow$ h prefers its Gale–Shapley partner s' to s. ← hospitals propose in decreasing order of preference
  - $\Rightarrow$ h–s is not unstable.

- Case 2: h proposed to s.
  - $\Rightarrow$ s rejected h (either right away or later)
  - $\Rightarrow$ s prefers Gale–Shapley partner h' to h.
  - $\Rightarrow$ h–s is not unstable.

students only trade up

- In either case, the pair h–s is not unstable. ▪



Gale–Shapley matching M*