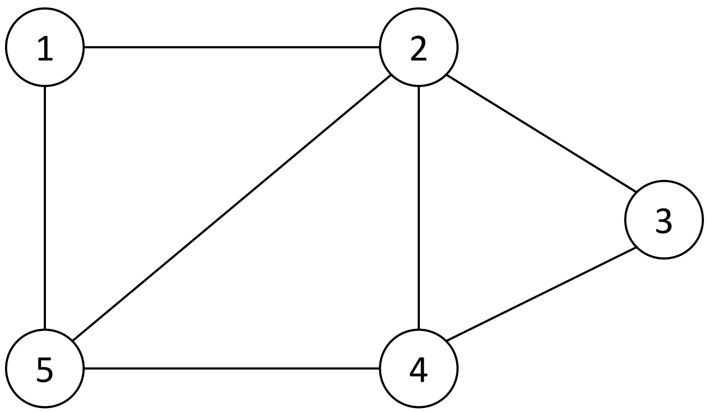
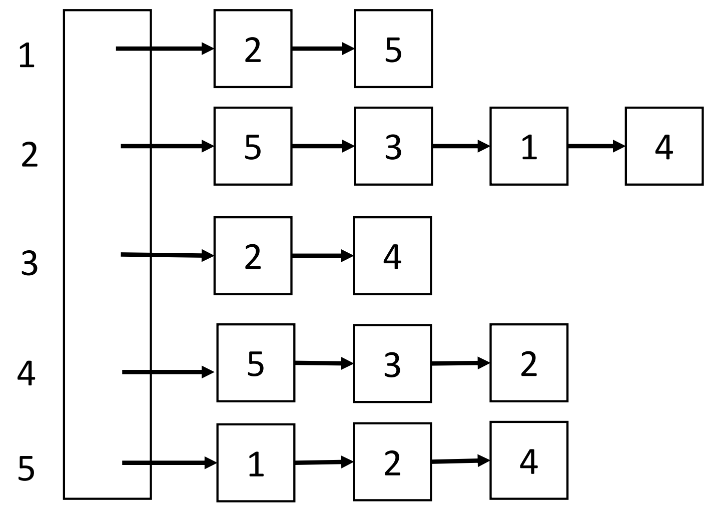
Graph Representation Example

Graph G=(V,E)



adjacency list for G



size parameters n=|V| and m=|E|

size n + 2m

adjacency matrix

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

size n^2

- poor space usage
- especially for sparse graphs
- easy to code
- why does $M = M^T$?

matrix multiplication

$$M^2 = \begin{pmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & 4 & 1 & 2 & 2 \\ 1 & 1 & 2 & 1 & 2 \\ 2 & 2 & 1 & 3 & 1 \\ 1 & 2 & 2 & 1 & 3 \end{pmatrix}$$

$$M^3 = \begin{pmatrix} 2 & 6 & 3 & 3 & 5 \\ 6 & 6 & 6 & 7 & 7 \\ 3 & 6 & 2 & 5 & 3 \\ 3 & 7 & 5 & 4 & 7 \\ 5 & 7 & 3 & 7 & 4 \end{pmatrix}$$

what do these numbers mean?

consider $M^* = I + M + M^2 + M^3 + \ldots + M^{n-1}$

theme: re-use operations, here matrix mult

+ → OR → MIN

* → AND → +

original: number of paths

existence of a path

shortest path?

is there a path (boolean)

$M^2(i,j)$ = “there is a path of length 2 from i to j”
= (edge i to 1) AND (edge 1 to j) --OR—
(edge i to 2) AND (edge 2 to j) –OR–
…
(edge i to n) AND (edge n to j)

= [M(i,1) AND M(1,j)] OR [M(i,2) AND M(2,j)] OR … OR [M(i,n) AND M(n,j)]

= $\sum_{k=1}^{n} M(i,k) * M(k,j)$

shortest path on 2 edges

$$M^2(i,j) = \min_{1 \le k \le n} (M(i,k) + M(k,j))$$

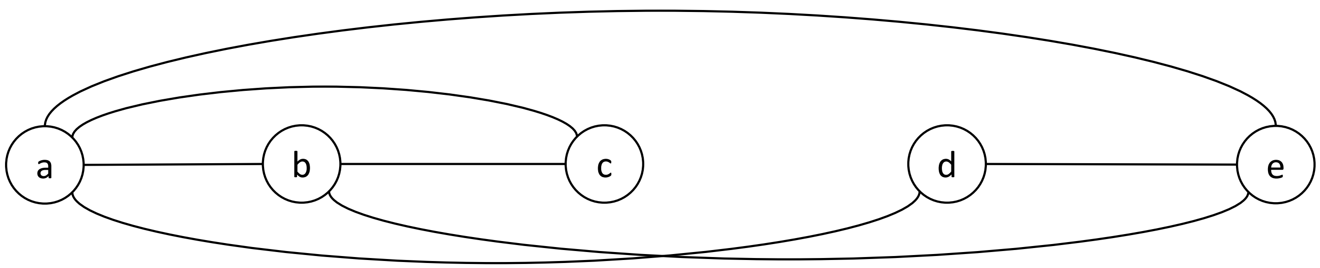
abstract “+”

abstract “*”

linear algebra on the matrix

- what are the eigenvalues?
- what does the rank tell us?
- whole area of *spectral graph theory*
- https://en.wikipedia.org/wiki/Spectral_graph_theory
- used to define *expander graphs*, used in error-correcting codes, de-randomization, etc.
- very much outside the scope of this class

graph isomorphism



is this the same graph as G?

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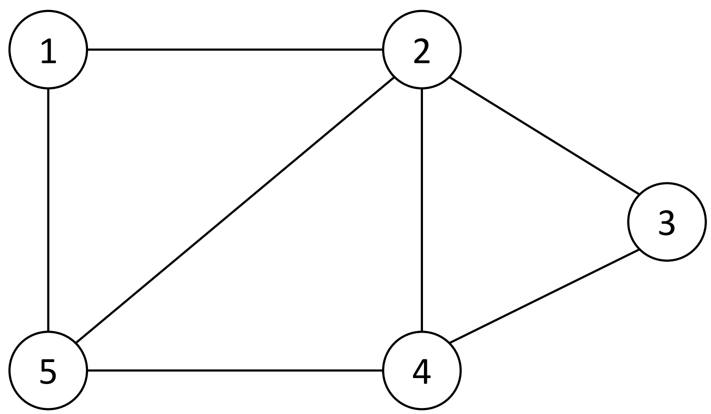

graph isomorphism problem

- best candidate for “not in P” and “not NP-complete”
- first breakthrough: Prof. Eugene Luks, $O(2^{\sqrt{nlgn}})$ time
- recent breakthrough: Laci Babai, $2^{O((\lg n)^c)}$ time (“whispering distance “ of P)
- https://jeremykun.com/2015/11/12/a-quasipolynomial-time-algorithm-for-graph-isomorphism-the-details/
- both results use deep computational group theory