Scheduling to Minimize Lateness

algorithms

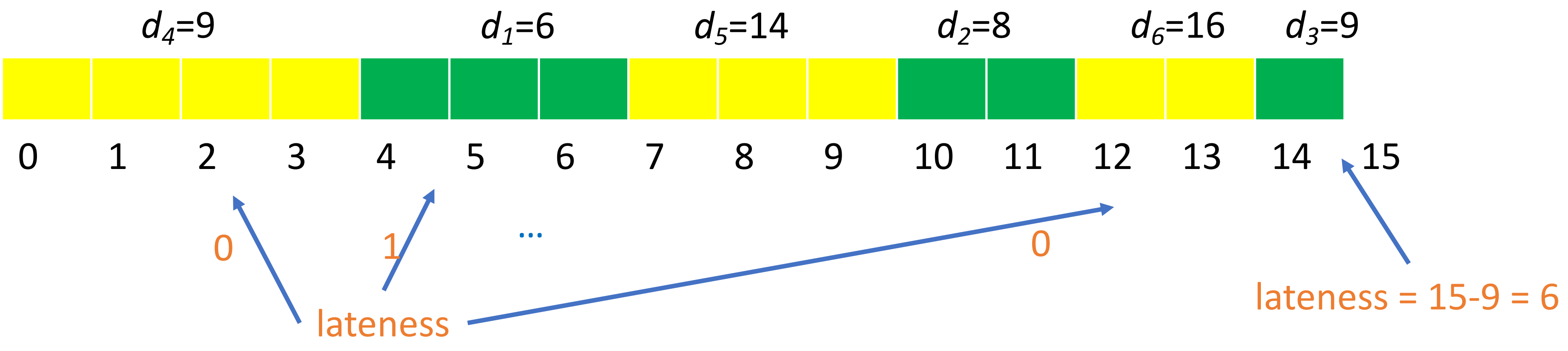
minimize the maximum lateness

- need to schedule a series of n jobs on a single processor
- the i^{th} job requires t_i units of processing time
- … and has a deadline of d_i
- if job i is scheduled to start at time s, it finishes at $f_i=s+ t_i$
- the lateness of the i^{th} job is l_i = max $\{0, f_i – d_i\}$
- goal is to minimize max $\{ l_i \mid 1 \le i \le n \}$
- input: $t_1, t_2, …, t_n$ and $d_1, d_2, …, d_n$

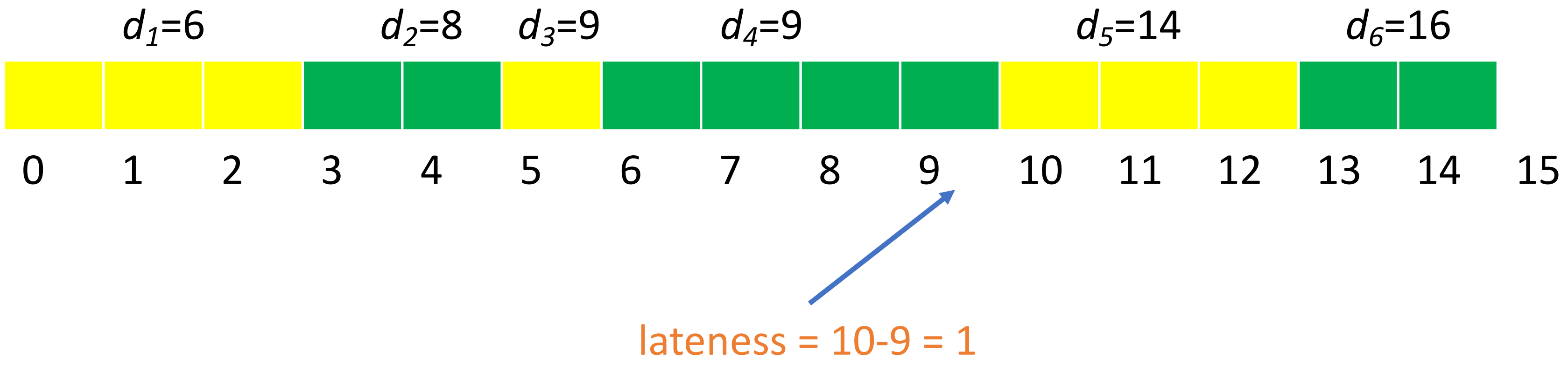
example

	1	2	3	4	5	6
t_i	3	2	1	4	3	2
d_i	6	8	9	9	14	15

schedule by decreasing computation time:



optimal: schedule by increasing deadline



proof of optimality

- suppose there is some other “optimal” schedule that does not satisfy earliest deadline first
- it does not satisfy $d_1 \leq d_2 \leq \ldots \leq d_n$
- look for an inversion
- then there must be an i such that $d_i > d_{i+1}$ (why does it exist?)
- specifically look for two neighbors that are out of order
- makes proof a bit simpler
- swap jobs i and $i+1$ and show that lateness does not get worse

proof of optimality - continued

- we swap jobs i and $i+1$ where $d_i > d_{i+1}$
- before finish and lateness values: $f_1, f_2, \ldots, f_n$ and $l_1, l_2, \ldots, l_n$
- after finish and lateness values: $f'_1, f'_2, \ldots, f'_n$ and $l'_1, l'_2, \ldots, l'_n$
- note: by swapping adjacent jobs, other finish times don't change
- … so $f_j = f'_j$ for all $j \neq i$ or $i+1$
- small note: we assume that in original schedule, one job starts as soon as another finishes
- GOAL: show that $\max\{ l'_i , l'_{i+1} \} \leq \max\{ l_i , l_{i+1} \}$
- this is enough since other lateness values don't change

making the swap of jobs i and i+1

- before: job i starts at time (say) s
- before: $f_i = s + t_i$ and $f_{i+1} = s + t_i + t_{i+1}$
- before: and $l_{i+1} = s + t_i + t_{i+1} – d_{i+1}$

- after: $l'_i = f'_i - d_{i+1} = s + t_{i+1} - d_{i+1} \leq s + t_{i+1} + t_i - d_{i+1} = l_{i+1}$
- after: $l'_{i+1} = f'_{i+1} - d_i = f_{i+1} - d_i \leq f_{i+1} – d_{i+1} = l_{i+1}$
- … (since $f_{i+1} = f'_{i+1}$ and $d_i > d_{i+1}$)

- Therefore: $\max\{ l'_i , l'_{i+1} \} \leq l_{i+1} \leq \max\{ l_i , l_{i+1} \}$ (done!)

i^{th}
l_i
d_i
f_i
t_i
$\leq$
$\neq$