CIS 313

fourth slide

binary search trees

- chapter 12
- we will look at
  - definitions
  - properties
  - operations: insert, delete, search
  - traversals: inorder, postorder, preorder, level order
  - worst case behavior
  - average case behavior
- then move onto self-balancing BSTs: red-black, 2-3, 2-3-4, …

various trees

- free tree
- rooted tree
- ordered tree
- binary tree
- binary search tree
    - (search property) let x be a node in a BST. If y is a node in the left subtree of x, then y.key <=x.key. If y is in the right subtree of x, then y.key >= x.key

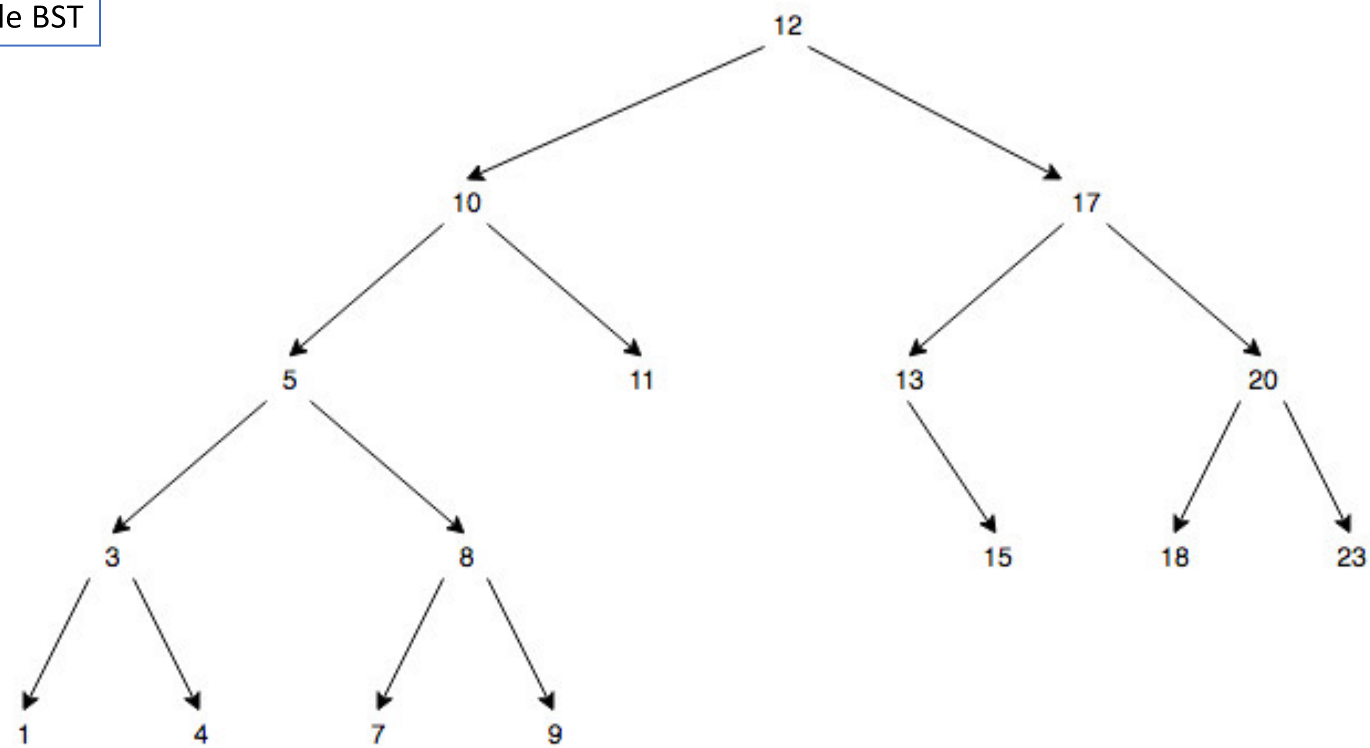
assorted facts and definitions

- any tree with n nodes has n-1 edges
- a binary tree with left/right pointers and n nodes has n+1 null pointers
- a full binary tree with n internal nodes has n+1 external nodes
- full binary tree: all nodes have either 2 children (the internal nodes) or 0 children (external)
- a binary tree of n nodes has height at least *lg* n and at most n-1
- height = distance of node from bottom, depth = distance from top

facts, defs cont’d

- internal path length (I): sum of the depths of all the nodes
- external path length (E): sum of the depths of the nulls (externals)
- fact: E=I+2n (nice exercise)
- I corresponds to successful search in BST, average search time is 1+ I/n
- E corresponds to unsuccessful search, average failed search time is E/(n+1)
- worst case tree: *skew tree* (every node has just one child)

sample BST



BST operations

- find(x)
- insert(x): find a null and put it there
- successor(x)
  - successor(10)=11, successor(15)=17
  - algorithm?
    - if x has right child, go right once, then left until end
    - otherwise, follow parent links until “right” turn
- delete(x): how?
  - if 0 children, remove
  - if 1 child, splice out
  - if 2 children, replace with successor value, then remove successor node

walks

- inorder
  - 1 3 4 5 7 8 9 10 11 12 13 15 17 18 20 23
- preorder
  - 12 10 5 3 1 4 8 7 9 11 17 13 15 20 18 23
- postorder
  - 1 4 3 7 9 8 5 11 10 15 13 18 23 20 17 12

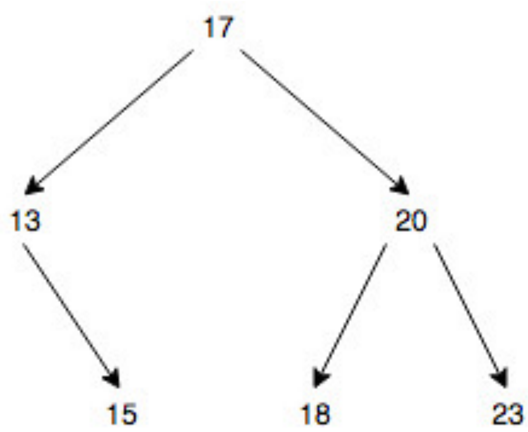
randomly built BST

- we have n values and will insert them one-by-one into a BST
- what will that BST look like?
- there are n! permutations of the input
  - we assume each one equally likely
- how many BST shapes can there be?
  - Catalan number, which is $\frac{1}{n+1}\binom{2n}{n} = \Omega(\frac{4^n}{n^{\frac{3}{2}}})$
  - (hard!)

counting permutations for a tree

- given a tree shape T we can determine the number of permutations which, if inserted into empty BST, would end up with that tree
- build up number bottom up
- at node x, suppose left subtree of x has n nodes and is generated by r permutations, and
- right subtree has m nodes and is generated by s permutations
- the the subtree rooted at x
  - has n+m+1 nodes
  - is generated by $\binom{n+m}{n} \cdot r \cdot s$ permutations

example



intuition: balanced trees more "likely"

- left side generated by 1 permutation: 13 15
- right side by two
  - 20 18 23
  - 20 23 18
- for full tree, pick one permutation each for the left and right sides
- permutation for the whole tree must start with 17 followed by n+m = 2+3 = 5 spaces
  - 17 __ __ __ __ __
- choose two for them for the left tree, which can be done in $\binom{5}{2} = 10$ ways
- example: 2nd and 5th positions
- 17 __ 13 __ __ 15
- either of the two remaining perms can go in remaining three slots
  - 17 <u>20</u> 13 <u>18</u> <u>23</u> 15
  - 17 <u>20</u> 13 <u>23</u> <u>18</u> 15
- total number of permutations for whole tree:

$$1 \cdot 2 \cdot \binom{5}{2} = 20$$

back to sorting theme

- we can build an abstract sort method based on BST
- given unsorted list, insert all values into empty BST
- perform inorder walk

this part is O(n)

```
BST SORT
** input list a=(a₁,a₂,…,aₙ)
create BST T

for i=1 to n
     T.insert(aᵢ)

perform T.inorder
     when visiting a node, store value in list b

return b
```

expected behavior

- if list a is chosen randomly from among all n! permutations
- how long does "`for i=1 to n T.insert(a`$_i$`)`" take?
- worst case: O(n^2)
- want to argue: on average O(n *lg* n)

- main fact: expected search time (1+I/n) in BST built from randomly chosen permutation is $2 \cdot \ln(n+1) + O(1) \approx 1.38 \ \log_2 n + O(1)$

text: exercise 12.4-2, p 303

describe a binary search tree on n nodes such that the average depth of a node in the tree is $\Theta(\lg n)$ but the height of the tree is $\omega(\lg n)$