CIS 313:
Intermediate Data Structure

fifth slide

hash tables

- chapter 11
- we want to manage a dynamic set K ($|K|=n$) where each element has a key in universe $U = \{0,1,\ldots,u-1\}$
- support efficient operations SEARCH, INSERT and DELETE (i.e, in O(1))
- if u is small, an array $T[0,\ldots,u-1]$ would suffice
- each slot in T corresponds to a key in the universe
- if the set doesn't contain key k, then $T[k]$ = NIL.

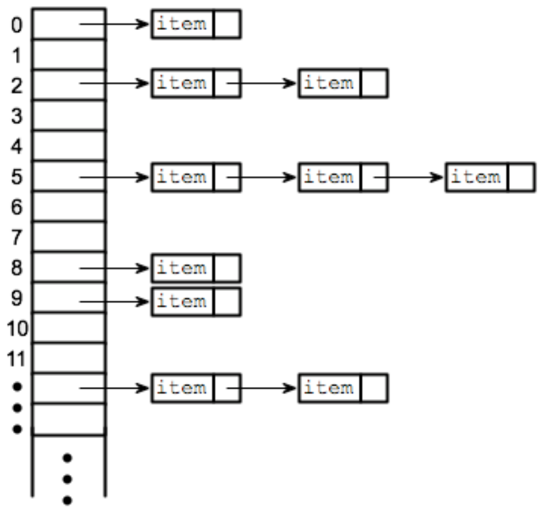
hash tables

- if $u = |U|$ is large, an array of size $|U|$ might be impractical/impossible
- idea: the number of keys actually used n might be much smaller than $|U|$
- we can thus reduce the storage requirement while still achieving the efficiency
- hash table: store n items of K in a table T of size m ($m \ll |U|$)
- hash function h determines where to put an item ($h: U \rightarrow \{0,1,..,m-1\}$)
- issues
  - what to do when two items hash to same location (collision)
  - how to choose good hash function h (minimize collisions)
  - how to choose table size m
  - dynamically increase table size
    - important in databases but not addressed here

collision resolution

- what to do with two items x and y that hash to same location?
- h(x.key) = h(y.key)
- open addressing
  - look at other locations in the table
  - table might overflow
  - more complicated
- closed addressing
  - all items that hash to location t stay there in some structure
  - bucket, linked list, …

chaining



- first: simple version of chaining
- table T with m slots, each containing a linked list
- hash function h maps keys to {0, 1, …, m-1}
- INSERT(T, x): put x in a node at the head of T[h(x.key)]
- SEARCH(T,k): search for an item with key k in the list T[h(k)]
- DELETE(T,x): delete x from the list T[h(x.key)] (done in O(1) with doubly linked list)
- load factor: $\alpha = n/m$, where *n* is the number of items in the set.
- simple uniform hashing (ideal): search time is $1 + \Theta(\alpha)$ (average-case)
- also called *closed addressing* (since item stored at that location)

choosing a hash function

- let k be the key and T a table of size m
- want h(k) to distribute keys uniformly across locations {0,1,…,m-1} (i.e, approximate the simple uniform hashing)
- division method: h(k) = k *mod* m
  - choice of table size m important
  - if $m=2^P$, then only low order bits of k matter (poor choice)
  - if k not distributed well, then h(k) prone to be biased
  - best if m a prime

multiplication method

- pick constant A with 0<A<1
- $h(k) = \lfloor m \cdot ((k \cdot A) \ mod\ 1) \rfloor$ (here “mod 1” means fractional part of real number)
- Knuth suggests $A = \frac{\sqrt{5} - 1}{2} \cong 0.6180339 \ldots$
- nice example on p 264 of text

universal hashing

- problem with fixed hash function: all keys might hash to same slot
- universal hashing: family of hash functions $\mathcal{H}$, maps key universe U onto {0, 1, …, m-1}
- remark: no single input will always exhibit worst-case behavior (good average-case performance)
- want for any $k, l \in U$ that the number of $h \in \mathcal{H}$ such that $h(k) = h(l)$ is at most $\frac{|\mathcal{H}|}{m}$ (universal hashing)
- idea is to pick an $h \in \mathcal{H}$ randomly if possible
- intuitively if keys $\mathrm{k} \in U$ not distributed well a random $h \in \mathcal{H}$ will still distribute the locations well and excess avoid collision
- example family: $\mathcal{H}$ will depend on fixed p, m
    - m is table size, $p>m$ is a prime so that all keys $k<p$
    - choose a,b with $0<a<p$, $b<p$ (randomly)
    - h(k) = ((ak+b) mod p) mod m
    - proof that $\mathcal{H}$ is universal in text, depending on basic number theory (nice proof)

back to collision resolution: open addressing

- instead of using lists in chaining, all elements are stored in the hash table, so no storage requirement for points, saving spaces to reduce collisions
- for key k=x.key, if location T[h(k)] is full (via collision), need to put x in a different location
- look in a sequence of locations depending on *k.* This is called the *probe sequence*
- using the hash function h<k,i> to determine the slot to probe at time i on key k
- look in locations h<k,0>, h<k,1>, h<k,2>, … until find empty slot in which to place x
- requirement: for every key k, (h<k,0>, h<k,1>, …, h<k,m-1>) be a permutation of (0,1,…,m-1) so every position of the hash table is considered eventually

strategies for probe sequences

- simplest (and worst): *linear probing*
  - $h<k,i> = (h(k)+i) \bmod m$
  - that is, if h(k) is full, look in locations h(k)+1, h(k)+2, h(k)+3, …
  - problem: primary clustering (slots are clustered in long lines)
- quadratic probing
  - pick constants c, d
  - $h<k,i> = (h(k) + c*i + d*i^2) \bmod m$
  - c, d, m need to be chosen carefully so that h<k,i> can probe entire table
  - problem: secondary clustering (milder than primary clustering)
- double hashing (the current best one)
  - use two hash functions h_1, h_2
  - $h<k,i> = (h_1(k) + i*h_2(k)) \bmod m$
  - need m and $h_2(k)$ to be relatively prime

other uses of hash functions

- database indexing
  - need extendible hash tables as many insertions happen
  - not good for *range queries* (“find all values between a and b”)
  - B-tree indexes more popular
- cryptographically secure hashing
  - password files
  - multi-party communication
  - hash functions very different looking
- Bloom filters, count-min sketch