Part 3: Safety and liveness

Safety vs. liveness

Safety: something “bad” will never happen
Liveness: something “good” will happen (but we don’t know when)

Safety vs. liveness for sequential programs

Safety: the program will never produce a wrong result ("partial correctness")
Liveness: the program will produce a result ("termination")

Safety vs. liveness for state-transition graphs

Safety: those properties whose violation always has a finite witness ("if something bad happens on an infinite run, then it happens already on some finite prefix")
Liveness: those properties whose violation never has a finite witness ("no matter what happens along a finite run, something good could still happen later")

This is much easier.

Safety: the properties that can be checked on finite executions
Liveness: the properties that cannot be checked on finite executions (they need to be checked on infinite executions)
Example: Mutual exclusion
It cannot happen that both processes are in their critical sections simultaneously.

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Safety

Example: Bounded overtaking
Whenever process P1 wants to enter the critical section, then process P2 gets to enter at most once before process P1 gets to enter.

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Safety

Example: Starvation freedom
Whenever process P1 wants to enter the critical section, provided process P2 never stays in the critical section forever, P1 gets to enter eventually.

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Liveness
Example: Starvation freedom

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Liveness

LTL (Linear Temporal Logic)

- safety & liveness
- linear time

[Prüel 1977; Lichtenstein & Prüel 1982]

LTL Syntax

\[ \varphi ::= a \mid \varphi \land \varphi \mid \neg \varphi \mid \varnothing \varphi \mid \varphi \cup \varphi \]

LTL Model

infinite trace \( t = t_0, t_1, t_2 \ldots \)

(sequence of observations)

Language of deadlock-free state-transition graph \( K \) at state \( q \):

\( L(K, q) = \) set of infinite traces of \( K \) starting at \( q \)

\( (K, q) \models \varphi \iff \) for all \( t \in L(K, q), \ t \models \varphi \)

\( (K, q) \models \exists^\omega \varphi \iff \) exists \( t \in L(K, q), \ t \models \varphi \)
LTL Semantics

\[ t \models a \iff a \in t_0 \]
\[ t \models \varphi \wedge \psi \iff t \models \varphi \text{ and } t \models \psi \]
\[ t \models \neg \varphi \iff \neg t \models \varphi \]
\[ t \models \Diamond \varphi \iff t_1, t_2, \ldots \models \varphi \]
\[ t \models \varphi \U \psi \iff \text{exists } n \geq 0 \text{ s.t.}
  \begin{align*}
  &1. \text{ for all } 0 \leq i < n, \ t_i, t_{i+1} \ldots \models \varphi \\
  &2. \ t_n, t_{n+1} \ldots \models \psi
  \end{align*}
\[ (K,q) \models \Diamond \varphi \iff (K,q) \models \neg \Diamond \neg \varphi \]

Defined modalities

- \( \Diamond \varphi \) : next
- \( U \varphi \) : until
- \( \Diamond \varphi = \text{true} U \varphi \) : eventually
- \( \Box \varphi = \neg \Diamond \neg \varphi \) : always
- \( \varphi W \psi = (\varphi U \psi) \vee \Box \varphi \) : waiting-for (weak-until)

Important properties

Invariance

- \( \Box a \) : safety
- \( \Box \neg (pc1=\text{in} \wedge pc2=\text{in}) \)

Sequencing

- \( a W b W c W d \) : safety
- \( (pc1=\text{req}) \Rightarrow (pc2=\text{in}) W (pc2=\text{in}) W (pc2=\text{in}) W (pc1=\text{in}) \)

Response

- \( \Box (a \Rightarrow \Diamond b) \) : liveness
- \( \Box (pc1=\text{req}) \Rightarrow \Diamond (pc1=\text{in}) \)

Composed modalities

- \( \Diamond \Diamond a \) : infinitely often a
- \( \Diamond \Box a \) : almost always a

Example: Starvation freedom

Whenever process P1 wants to enter the critical section, provided process P2 never stays in the critical section forever, P1 gets to enter eventually.

\[ \Box \Diamond (pc2=\text{in} \Rightarrow \Diamond (pc2=\text{out})) \Rightarrow \Box (pc1=\text{req} \Rightarrow \Diamond (pc1=\text{in})) \]

State-transition graph

- \( Q \) : set of states \( (q_1,q_2,q_3) \)
- \( A \) : set of atomic observations \( (a,b) \)
- \( \rightarrow \subseteq Q \times Q \) : transition relation \( q_1 \rightarrow q_2 \)
- \( [\cdot] : Q \rightarrow 2^A \) : observation function \( [q_i] = \{a\} \)
\( (K, q) \models \forall \varphi \)

Tableau construction
(Vardi-Wolper)

\( (K', q', BA) \) where \( BA \subseteq K' \)

Is there an infinite path starting from \( q' \) that hits \( BA \) infinitely often?

Is there a path from \( q' \) to \( p \in BA \) such that \( p \) is a member of a strongly connected component of \( K' \)?

```
dfs(s) {
    add s to dfsTable
    for each successor t of s
        if (t \notin dfsTable) then dfs(t)
        if (s \in BA) then (seed := s; ndfs(s))
    }

ndfs(s) {
    add s to ndfsTable
    for each successor t of s
        if (t \notin ndfsTable) then ndfs(t)
        else if (t = seed) then report error
    }
```