The Implementation of

PARALLEL FUNCTIONAL ARRAY PROGRAMMING

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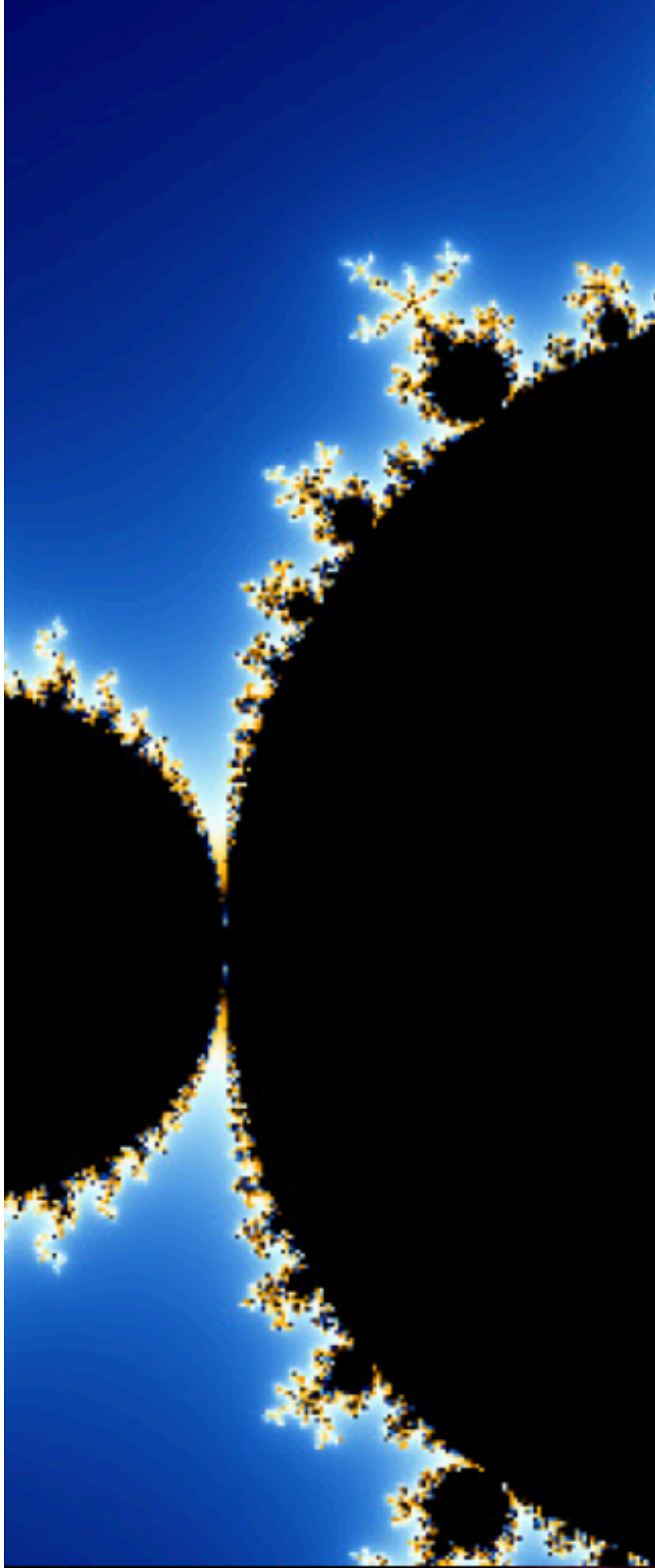


PARALLEL FUNCTIONAL PROGRAMMING

Parallel Programming

performance!

?

Functional Languages

abstraction

higher order functions

controlled side effects

...

MAKING FP (AND TYPES) WORK FOR US

- Abstraction also means the compiler has more information
  - controlled side effects, no user-level pointers,…
- Collection oriented versus explicit loops/recursion
- Expressive type systems help to
  - guide the user
  - guide the compiler write

Composite
data structures

Immutable
structures

Expressive type
system & inference

Haskell

Strong static typing

Higher-order functions
& closures

Principled, pure,
functional programming

Boxed values

Polymorphism
& generics

Strictly isolating
side-effects

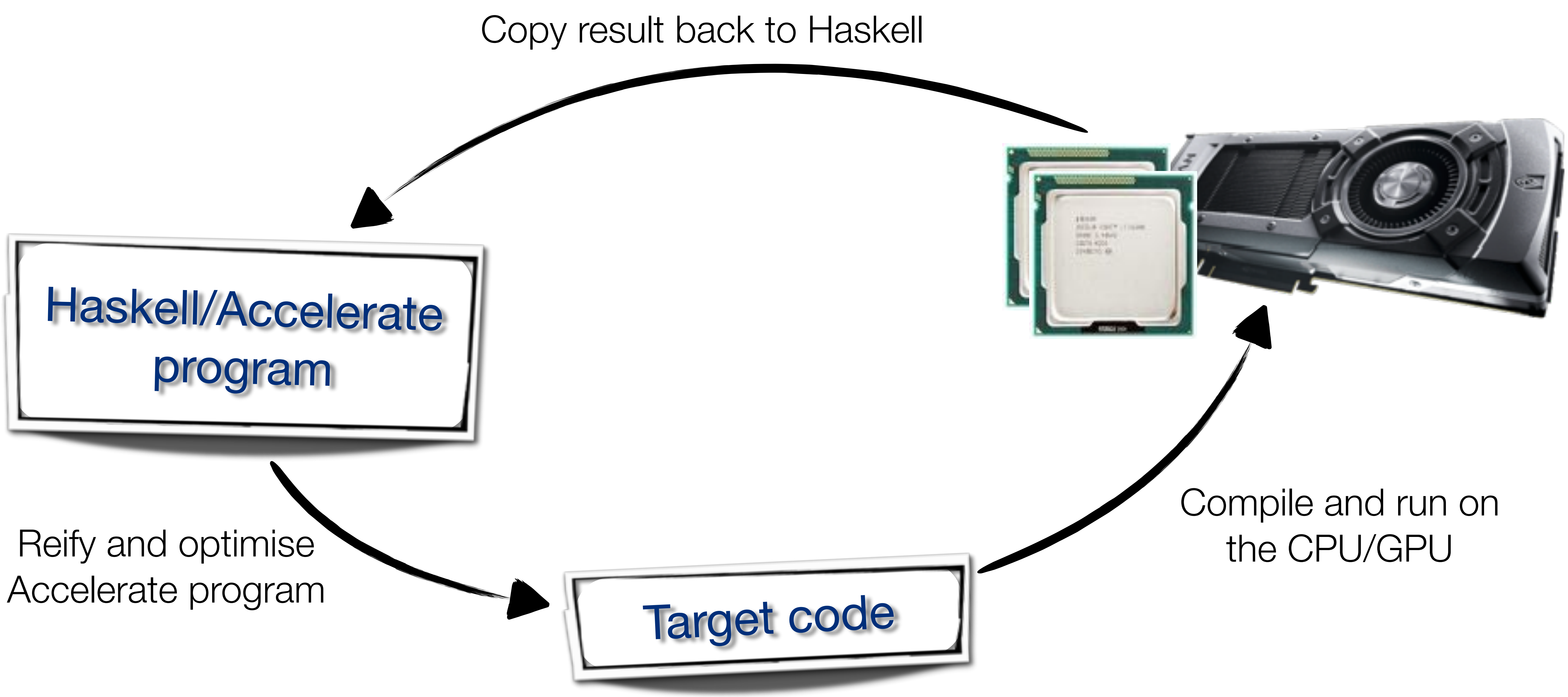
How about
domain specific languages
with
specialised code generation?

DOMAIN SPECIFIC LANGUAGES

- Are restricted languages
  - Generally have specialised features to a particular application domain
  - HTML, Matlab, SQL, postscript, LaTeX …
- Embedded domain specific languages
  - Implemented as libraries in the host language, so can integrate with the host language
  - Reuse the syntax of the host language (as well as parser, type checker…)
  - The host language can generate embedded code
  - Functional languages are great as host languages

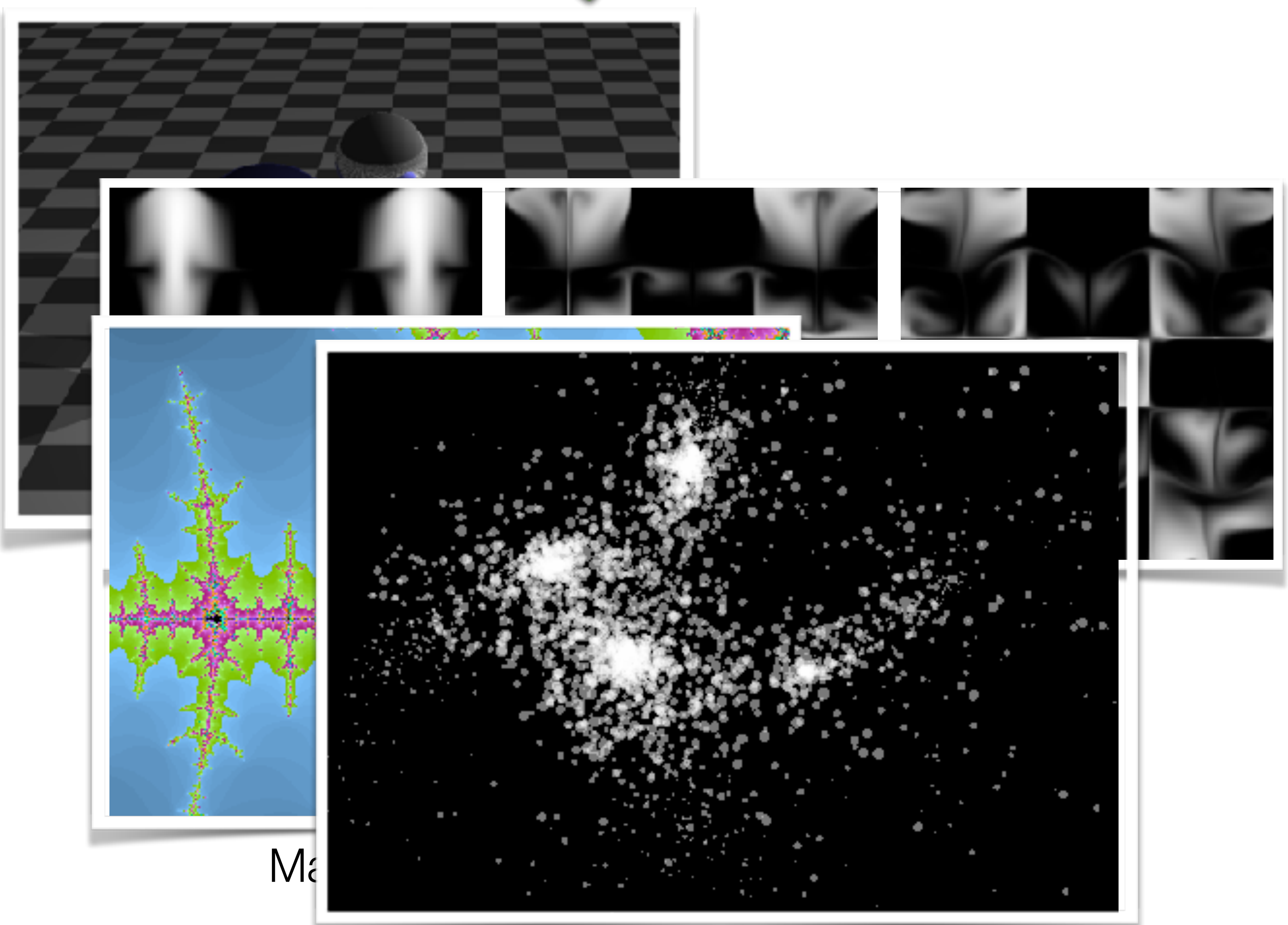
ACCELERATE

- An embedded domain-specific language for high-performance computing in Haskell



ACCELERATE

- Array computations 
- Everything else 



n-body gravitational simulation

DOMAIN SPECIFIC EMBEDDED LANGUAGES

- There are two ways to embed a language
  - shallow embedding
  - deep embedding

SHALLOW EMBEDDING

- Shallow embedding provides fixed interpretation
- Semantics captured in the type
- **Example:** arithmetic expression language

```
type Expr = Float
```

DEEP EMBEDDING

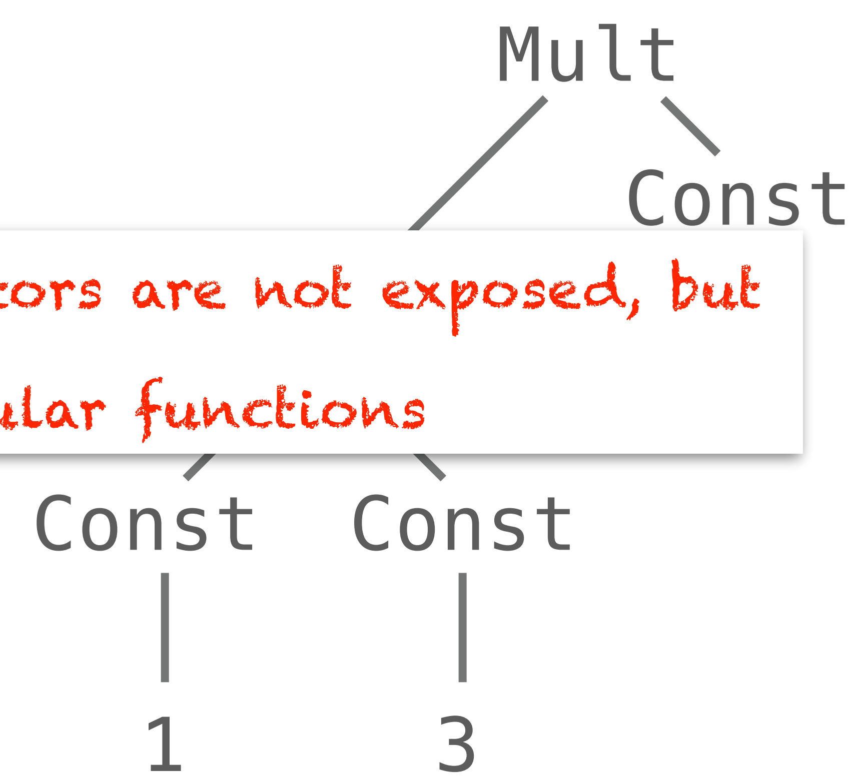
- Captures DSL expression as abstract syntax tree (AST), allowing multiple interpretations

```
data Expr
  = Add   Expr Expr
  | Mult  Expr Expr
  | Neg   Expr
  | Const Float
```

```
data Expr where
  Add   :: Expr  -> Expr -> Expr
  Mult  :: Expr  -> Expr -> Expr
  Neg   :: Expr  -> Expr -> Expr
  Const :: Float -> Expr
```

```
sampleExpr
  = Mult (Add (Const 1) (Const 3)) (Const 5)
```

```
add :: Expr -> Expr -> Expr
add = Add
```



often, constructors are not exposed, but wrapped in regular functions

DEEP EMBEDDING

- Captures DSL expression as AST, allowing multiple interpretations

```
data Expr where
  Add   :: Expr  -> Expr -> Expr
  Mult  :: Expr  -> Expr -> Expr
  Neg   :: Expr  -> Expr -> Expr
  Const :: Float -> Expr
```

```
eval :: Expr -> Float
eval (Const x)     =
eval (Add e1 e2)   =
eval (Mult e1 e2)  =
eval (Neg e)       =
```

DEEP EMBEDDING

```
data Expr where
  Add   :: Expr  -> Expr -> Expr
  Mult  :: Expr  -> Expr -> Expr
  Neg   :: Expr  -> Expr -> Expr
  Const :: Float -> Expr
```

```
simplify :: Expr -> Expr
```

execute :: Expr -> IO Float

DEEP EMBEDDING

➤ The expression representation is untyped:

```
data Expr where
  Add    ::          Expr -> Expr  -> Expr
  Mult   ::          Expr -> Expr  -> Expr
  Neg    ::          Expr -> Expr
```

```
eval :: Expr -> Float
```

could be Float or Bool!

DEEP EMBEDDING

- The expression representation is untyped:

```haskell
data Expr where
  Add    ::               Expr -> Expr  -> Expr
  Mult   ::               Expr -> Expr  -> Expr
  Neg    ::               Expr -> Expr
  If     :: Expr -> Expr -> Expr  -> Expr
  Less   ::               Expr -> Expr  -> Expr
  NConst ::                       Float -> Expr
  BConst ::                       Bool  -> Expr
```

```haskell
data Result where
  FRes :: Float -> Result
  BRes :: Bool  -> Result
```

```haskell
eval :: Expr -> Result
```

ASIDE: PARAMETRISED ALGEBRAIC DATA TYPES

```haskell
data Tree a
  = Leaf
  | Node a (Tree a) (Tree a)
```

```haskell
data Tree a where
  Leaf ::                               Tree a
  Node :: a -> Tree a -> Tree a -> Tree a
```

ASIDE: PARAMETRISED ALGEBRAIC DATA TYPES

```
data Tree a
  = Leaf
  | Node a (Tree a) (Tree a)
```

```
data Tree a where
  Leaf ::                               Tree b
  Node :: c -> Tree c -> Tree c -> Tree c
```

DEEP EMBEDDING

- Generalised Algebraic Data Types (GADTs)

```
data Expr    where
  Add     ::          Expr         -> Expr         -> Expr
```

```
eval :: Expr a -> a
eval (Const c) = c
eval (If cond e1 e2) =
   if  (eval cond)
      then eval e1
      else eval e2
```

DEEP EMBEDDING

➤ Generalised Algebraic Data Types (GADTs)

```
data Expr a where
  Add    ::           Expr Float -> Expr Float -> Expr Float
  Mult   ::           Expr Float -> Expr Float -> Expr Float
  Neg    ::                         Expr Float -> Expr Float
  Less   ::           Expr Float -> Expr Float -> Expr Bool
  Const  ::                               a    -> Expr a
  If     :: Expr Bool -> Expr a -> Expr a      -> Expr a
```

```
simplify :: Expr a  -> Expr a
simplify (Const n) = Const n
simplify (Neg (Neg e))
  = simplify e
simplify (Add e1 e2)
  = Add (simplify e1) (simplify e2)
simplify (Mult e1 e2)
  = Mult (simplify e1) (simplify e2)
```

LET'S LOOK AT ACCELERATE NOW!

ACCELERATE

- Computations take place on dense, multidimensional arrays
- Parallelism is introduced in the form of collective operations on arrays

- The usual suspects: maps, zipWiths, folds, generators, permutes and backpermutes, stencil operations

FIRST EXAMPLE

- dot-product in Haskell (on lists)

```haskell
class Num a where
    (+), (-), (*)       :: a -> a -> a
    negate              :: a -> a
    abs                 :: a -> a
    signum              :: a -> a
    fromInteger         :: Integer -> a
```

```haskell
import Prelude

dotp :: Num a => [a] -> [a] -> a
dotp xs ys = foldl (+) 0 (zipWith (*) xs ys)
```

```haskell
zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]

foldl :: (b -> a -> b) -> b -> [a] -> b
```

FIRST EXAMPLE

- dot-product in Haskell (on vectors):

```haskell
import Data.Vector.Unboxed

dotp :: (Num a, Unbox a)
     => Vector a
     -> Vector a
     -> a
dotp xs ys = foldl (+) 0 (zipWith (*) xs ys)
```

```haskell
zipWith :: (a -> b -> c) -> Vector a -> Vector b -> Vector c

foldl :: (b -> a -> b) -> b -> Vector a -> b
```

FIRST EXAMPLE

- dot-product in Haskell (using Accelerate):

```
import Data.Array.Accelerate

 dotp :: (Num a, Elt a)
      => Acc (Vector a)
      -> Acc (Vector a)
      -> Acc (Scalar a)
 dotp xs ys = fold (+) 0 (zipWith (*) xs ys)
```

```
zipWith : (Elt a, Elt b, Elt c)
  => (Exp a -> Exp b -> Exp c) -> Acc (Vector a) -> Acc (Vector b)
                                                 -> Acc (Vector c)
```

```
fold :: Elt a =>
  (Exp a -> Exp a -> Exp a) -> Exp a -> Acc (Vector a) -> Acc (Scalar a)
```

DETOUR: TYPE CLASSES IN HASKELL

```
instance Num Int where
  (+) = ...
  (*) = ...
  ...
```

```
foo :: Num a => a -> a
foo x = x * x + x
```

```
foo' :: NumDict a -> a -> a
foo' dict x = (getAdd dict)((getMult dict) x x) x
```

type annotations may become necessary

DIFFERENT RUN FUNCTIONS

- Running an accelerate program:

```
dotp :: (Num a, Elt a) =>
   Acc (Vector a) -> Acc (Vector a) -> Acc (Scalar a)

vec1, vec2 :: Acc (Vector Float)
vec1 = …
vec2 = …

accPrg:: Acc (Scalar Float)
accPrg = dotp vec1 vec2
```

```
run :: Arrays a => Acc a -> a
```

```
putStrLn $ show $ run (dotp vec1 vec2)
```

RUNNING AN ACCELERATE PROGRAM

- Plugging it all together:

```
dotp :: (Num a, Elt a) =>
    Acc (Vector a) -> Acc (Vector a) -> Acc (Scalar a)

dotp vec1 vec2 = …

vec1, vec2 :: Vector Float
vec1 = …
vec2 = …

main = P.putStrLn $ P.show $
         run1 (uncurry dotp) (vec1, vec2)
```

```
run1 :: Arrays a => (Acc a -> Acc b) -> a -> b
```

DIFFERENT RUN FUNCTIONS

- Compiling Accelerate programs at Haskell compile time:

```
runQ
```

ACCELERATE EXPRESSIONS

- Accelerate expressions can be of two distinct types:
  - Embedded sequential, scalar expression:

```
Exp a
```

  - Embedded array computations:

```
Acc a
```

  - What is the difference between these two?

```
Exp Int

Acc (Scalar Int)
```

ACCELERATE EXPRESSIONS

- Nested parallel computations ~~almost~~ can't be expressed:

```
map :: (Elt a, Elt b) =>
  (Exp a -> Exp b) -> Acc (Vector a) -> Acc (Vector b)
```

DEFINITION OF EXP

- Exp is a GADT whose constructors represent scalar operations

```haskell
data Exp a where
  Const   :: Elt c
          => c
          -> Exp c

  PrimApp :: (Elt a, Elt r)
          => PrimFun (a -> r)
          -> Exp a
          -> Exp r
  …
```

`Apply primitive scalar function: (+), (*) …`

AD-HOC POLYMORPHISM FOR EXP

- Overloaded the standard type classes to reflect arithmetic expressions
- The `Num` instance for `Exp` terms allows us to reuse standard operators like `(+)` and `(*)`

```
instance Num (Exp Int) where
  x + y = PrimAdd numType `PrimApp` tup2 (x, y)
  ...
```

AD-HOC POLYMORPHISM FOR EXP

- Use explicit dictionary passing to support ad-hoc polymorphism
    - Type checker chooses the correct instance when creating the dictionary
    - Pattern matching on the dictionary constructor makes the class constraints available

AD-HOC POLYMORPHISM FOR EXP

- How does the dictionary trick work?
- With a standard algebraic data type the following are equivalent:

```
foo :: Foo a -> a -> a
foo _        x = x+1

bar :: Foo a -> a -> a
bar (Foo _) x = x+1
```

- But, with GADTs this is not the case

```
data Foo a where
  Foo :: Num a => a -> Foo a
```

ACCELERATE TYPES

- We encountered two different Accelerate array types:

```
dotp :: (Num a, Elt a) =>
    Acc (Vector a)-> Acc (Vector a) -> Acc (Scalar a)
```

- These are just two special cases of Accelerate's Array types
  - parametrised with the shape type `sh`
  - element type `a`

```
Array sh a
```

ARRAY SHAPES

- The shape of an array determines its dimensionality and extent

```
data Z              = Z
data head :. tail = head :. tail


type DIM0 = Z
type DIM1 = DIM0 :. Int
type DIM2 = DIM1 :. Int

type Scalar a = Array DIM0 a
type Vector a = Array DIM1 a
```

ARRAY SHAPES

- Operations are shape polymorphic:

```
map :: (Shape sh, Elt a, Elt b) =>
 (Exp a -> Exp b) -> Acc (Array sh a) -> Acc (Array sh b)


zipWith :: (Shape sh, Elt a, Elt b, Elt c) =>
(Exp a -> Exp b -> Exp c) ->
 Acc (Array sh a) -> Acc (Array sh b) -> Acc (Array sh c)


fold :: (Shape sh, Elt a) =>
  (Exp a -> Exp a -> Exp a) -> Exp a ->
  Acc (Array (sh :. Int) a) -> Acc (Array sh a)


generate :: (Shape sh, Elt a) =>
   Exp sh -> (Exp sh -> Exp a) -> Acc (Array sh a)
```

ARRAY SHAPES

- This means that out dot-product has actually a more general type:

```
dotp :: (Num a, Elt a)
    => Acc (Vector a)
    -> Acc (Vector a)
    -> Acc (Scalar a)
dotp xs ys = fold (+) 0 (zipWith (*) xs ys)
```

```
dotp :: (Num a, Elt a, Shape sh) =>
    Acc (Array (sh :. Int) a) ->
    Acc (Array (sh :. Int) a) ->
    Acc (Array     sh      a)
```

SUMMARY SO FAR

- Looked at deep and shallow embedding
  - GADTs to maintain types in the AST
- Programming model of Accelerate
  - writing simple Accelerate programs
  - `fromList ::(Elt e, Shape t) => t -> [e] -> Array t e`
  - `use :: Arrays arrays => arrays -> Acc arrays`

THE ELT CLASS

- Members of the `Elt` class contain admissible surface types for array elements:
  - `()`
  - `Int,`
  - `Float`
  - `Char`
  - `Bool`
  - Array indices formed from `Z` and `(:.)`
  - Tuples of all of these, e.g. `(Bool, Int, (Float, Float))`
  - To meet hardware restrictions, there are no nested arrays in Accelerate

```
dotp :: (Num a, Elt a, Shape sh) =>
     Acc (Array (sh :. Int) a) ->
     Acc (Array (sh :. Int) a) ->
     Acc (Array      sh      a)
```

ELT CLASS

- GPUs are efficient processing arrays of elementary type
- not so much for aggregate types, pointers
- similarly CPU when using SIMD vector instructions
- set of types LLVM supports is fixed
- We map the user-friendly surface types to efficient representations

ELT CLASS IS USER EXTENSIBLE

- Using type families (i.e., functions from type to type)

```
type family EltRepr t
type instance EltRepr Int   = Int
type instance EltRepr Float = Float
type instance EltRepr (a,b) =
       ProdRepr ( EltRepr a, EltRepr b )

type family ProdRepr t
type instance ProdRepr (a,b)   = (((), a), b)
type instance ProdRepr (a,b,c) = ((((), a), b), c)
```

- To extend the class, define

```
fromElt  :: a -> EltRepr a
toElt    :: EltRepr a -> a
```

LIFTING

- How can we construct values of `Exp` type?
  - Accelerate supplies `Exp` versions of Haskell Prelude ops, some constant values via overloading
- `lift`/`unlift` to switch to and (sometimes) back

```
trueExp :: Exp Bool
trueExp = lift True

true :: Bool
true = unlift trueExp


swap :: Exp (Int, Int) -> Exp (Int, Int)
swap pairExp =
  let (x, y) = unlift pairExp :: (Exp Int, Exp Int)
  in lift (y, x)
```

LIFTING

c is a type constructor
(e.g., Exp, Acc)

Plain is an **associated type** (function from type e to some other type)
Strips away the surface type constructors

```
class Lift c e where
  type Plain e
  lift :: e -> c (Plain e)



class Lift c e => Unlift c e where
  unlift :: c (Plain e) -> e
```

```haskell
instance Lift Exp Int where
  type Plain Int = Int
  lift = Exp . Const
```

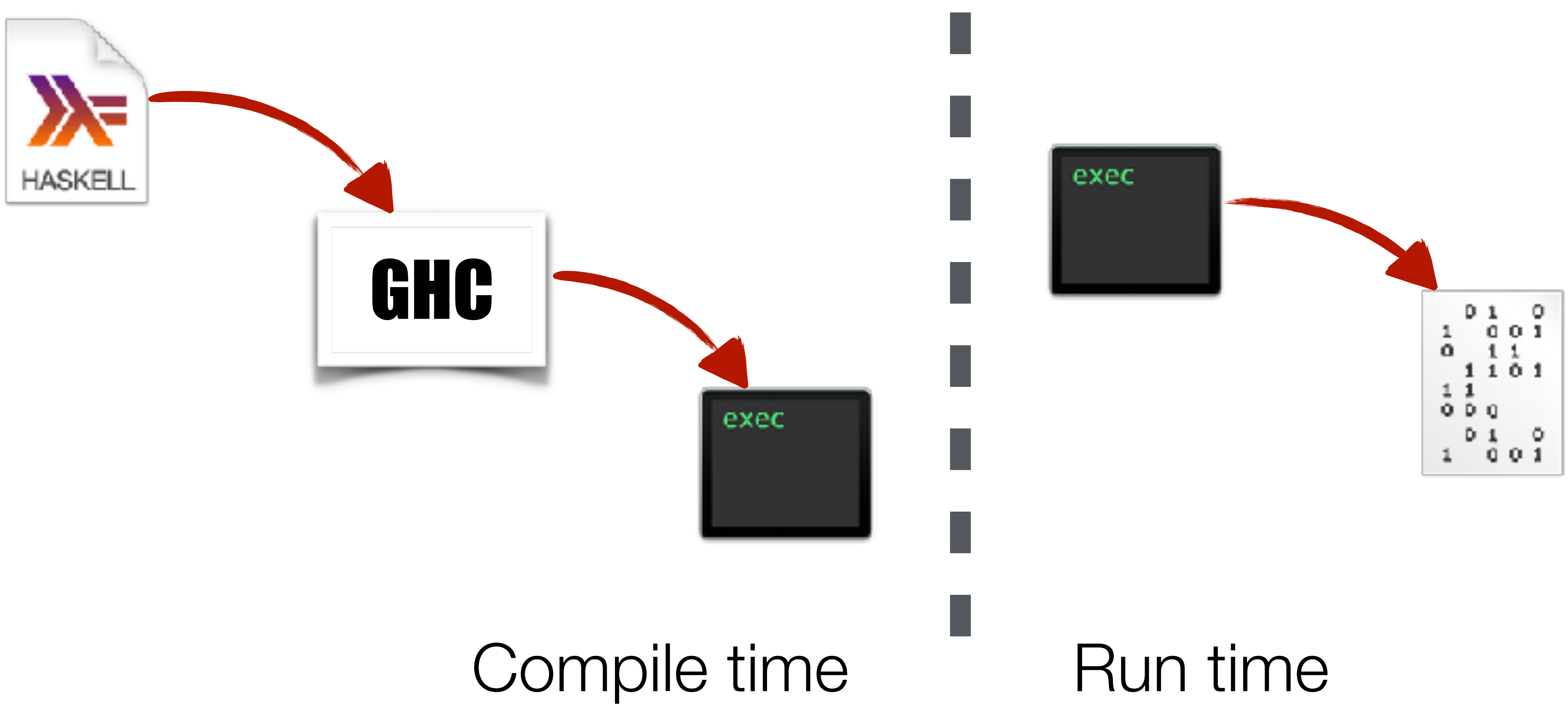
```haskell
instance Lift Exp (Exp e) where
  type Plain (Exp e) = e
  lift = id
```

```haskell
Plain (Exp Int, Int) ~ (Int,Int) ~ Plain (Int, Exp Int)
```

BACK TO THE IMPLEMENTATION OF EDSLS

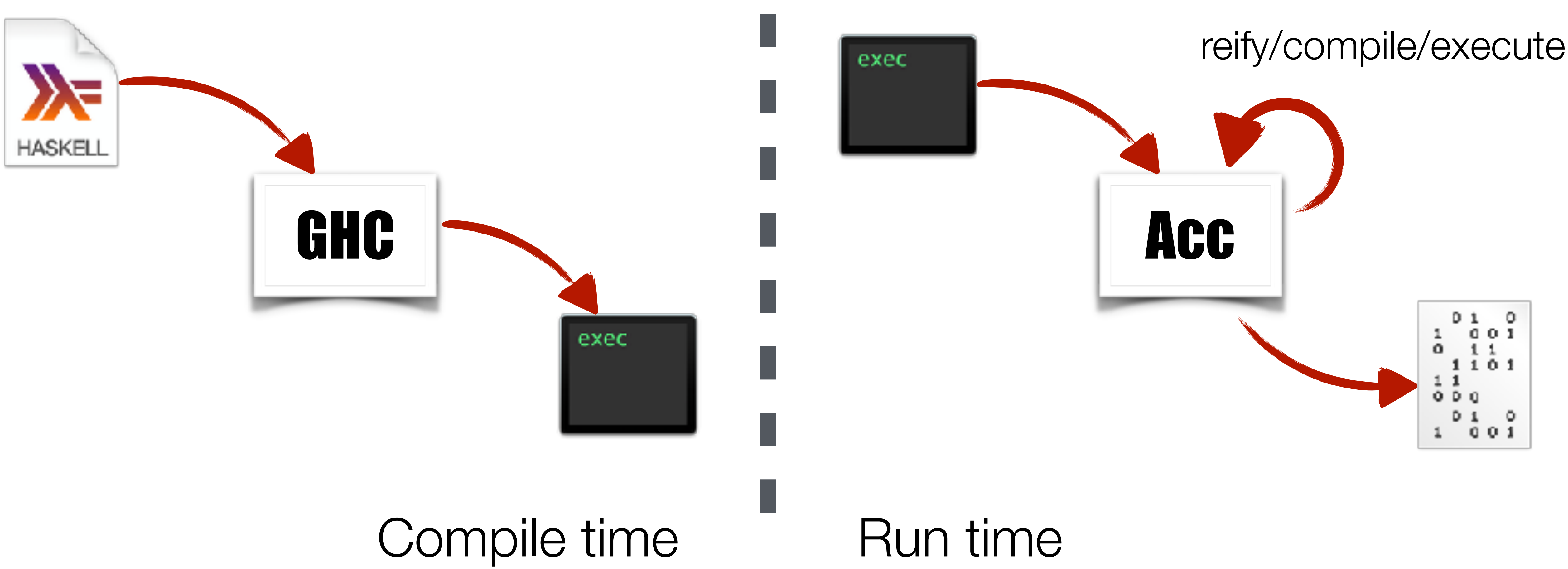
EXECUTION OF AN ACCELERATE PROGRAM

- What happens when we compile & run a regular Haskell program?



EXECUTION OF AN ACCELERATE PROGRAM

- What happens when we compile & run an Accelerate program?



WHY A TYPED AST?

- We compile the AST during application runtime
  - embedded compile time errors become application runtime errors
- Source of the type error can be
  - Accelerate user error
  - bugs in the Accelerate compiler
- Type checking the intermediate representation during Accelerate compilation
  - only shows this particular program is correct
  - transformation

WHY A TYPED AST?

- Applies to all runtime compiled EDSLs
- but particularly important for high-performance DSL
  - compilation is more challenging
- We learned the hard way
  - original CUDA backend was untyped
  - many bugs we found could have been avoided with a typed backend

FIRST ORDER AND HIGHER ORDER AST

- Let us look at a slightly more interesting EDSL:
  - values of the source language can be lifted into the DSL

    like the `Const` constructor in the arithmetic DSL
  - function application
  - lambda-abstraction

How do we model variables?

HIGHER ORDER AST

- We can use the variables and abstraction mechanism of the host language:

```
data HOExpr a where
  HConst   ::                          a           -> HOExpr a
  HApp     :: HOExpr (a -> b) -> HOExpr a -> HOExpr b
  HLambda :: (HOExpr a -> HOExpr b)       -> HOExpr (a -> b)
```

```
let f = \ea -> (HApp (HApp (HConst (+)) ea) ea)

eval (HApp (HLambda f) (HConst 5))
```

```
evalHO :: HOExpr a -> a
evalHO (HConst c)    = c
evalHO (HApp e1 e2) = (evalHO e1) (evalHO e2)
evalHO (HLambda f)  = \a -> (evalHO (f (HConst a))
```

HIGHER-ORDER AST

- Convenient to write and evaluate:
  - abstraction
  - application of the host language
- Not suitable for transformation & analysis of the AST
  - can't see inside functions
- Summary: good for surface syntax, not great for internal representation

FIRST ORDER AST

- Variables as regular terms of the language

```
type VarId = String

data FOExpr a where
  FConst  ::                          a      -> FOExpr a
  FApp    :: FOExpr (a -> b) -> FOExpr a -> FOExpr b
  FVar    :: VarId -> FOExpr a
  FLambda :: VarId -> FOExpr b  -> FOExpr (a -> b)
```

```
evalFO :: FOExpr a -> a
evalFO (FVar varId)  = ???
```

we need an environment of some sort

but what is its type??

DE BRUIJN INDEX

- Alternative representation of lambda-terms eliminating names:

$$\lambda x.\ \lambda y.\ (x + y)$$

$$\lambda x.\ \ (x + (\lambda f.\ f\ x)(+1))$$

- Names are replaced with indices encoding the nesting depth of the binder

$$\lambda\ \lambda\ (i_1 + i_0)$$

$$\lambda\ (\ i_0 + (\lambda\ i_0\ i_1)(+1))$$

FIRST ORDER SYNTAX WITH DE BRUIJN

- **Idea:**
  - a environment is either empty, or a tuple of value and rest environment
  - the type of the environment describes the type of all the values it contains
  - *i*th entry is the value of variable bound at nesting level n

```
data Val env where
  Empty :: Val ()
  Push  :: Val env -> t -> Val (env, t)
```

```
Push (Push Empty 5) True :: Val (((), Int), Bool)
```

FIRST ORDER SYNTAX WITH DE BRUIJN

- **Idea:**
  - a variable is a typed index
  - the type encodes the type of the value the variable it represents, as well as the type of the environment is needs

```
data Idx env t where
  ZeroIdx ::                Idx (env, t) t
  SuccIdx :: Idx env t -> Idx (env, s) t
```

```
prj :: Idx env t -> Val env -> t
prj ZeroIdx       (Push val v) = v
prj (SuccIdx idx) (Push val _) = prj idx val
```

DEMO

- A term type in our language is parametrised with two types:
  - the result type `t`
  - the environment type `env`

```
data Term env t where
  Var :: Idx env t                               -> Term env t
  Con :: t                                       -> Term env t
  Lam :: Term (env, s) t                         -> Term env (s -> t)
  App :: Term env (s -> t) -> Term env s -> Term env t
```

demo

FIRST ORDER SYNTAX WITH DE BRUIJN

- The type-safe evaluator is now pretty straight forward:

```
eval :: Term env t -> Val env -> t
eval (Var ix)        val = prj ix val
eval (Con v)         val = v
eval (Lam body)      val = eval body . (val `Push`)
eval (App fun arg)  val = (eval fun val) (eval arg val)
```

FIRST ORDER SYNTAX WITH DE BRUIJN

- Typed higher-order abstract syntax is
  - convenient as surface syntax
  - not suitable for program analysis, program transformations
- Typed De Bruijn first order abstract syntax
  - impractical to use as surface syntax
  - well suited as internal representation
- Solution:
  - user writes program in HO-syntax
  - we convert it to De Bruijn representation

PROBLEM SOLVED NOW, RIGHT?

UHM, NO...

SUMMARY

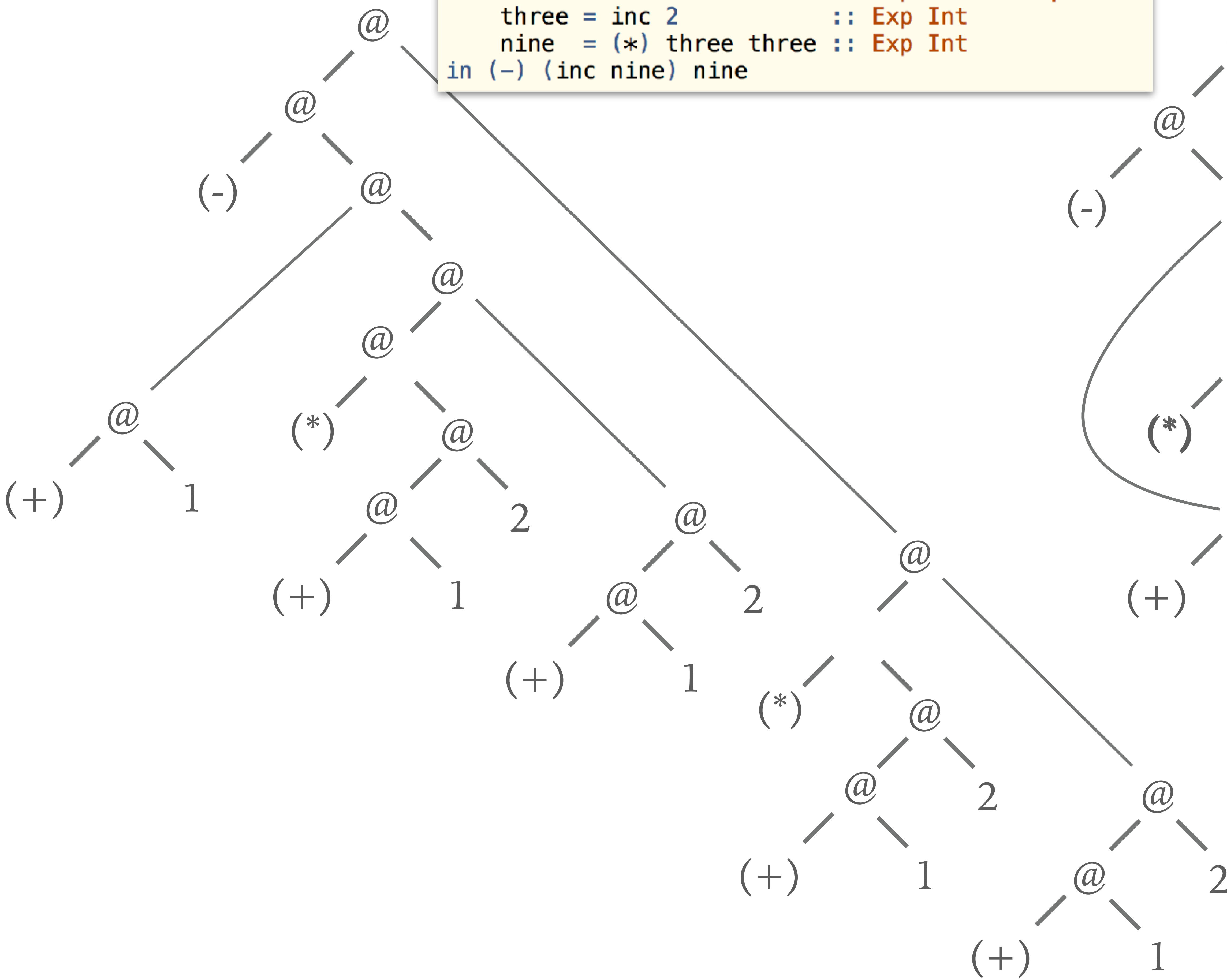
- Surface types and representation types
- First-order and Higher-order abstract syntax
- Typed De Bruijn representation

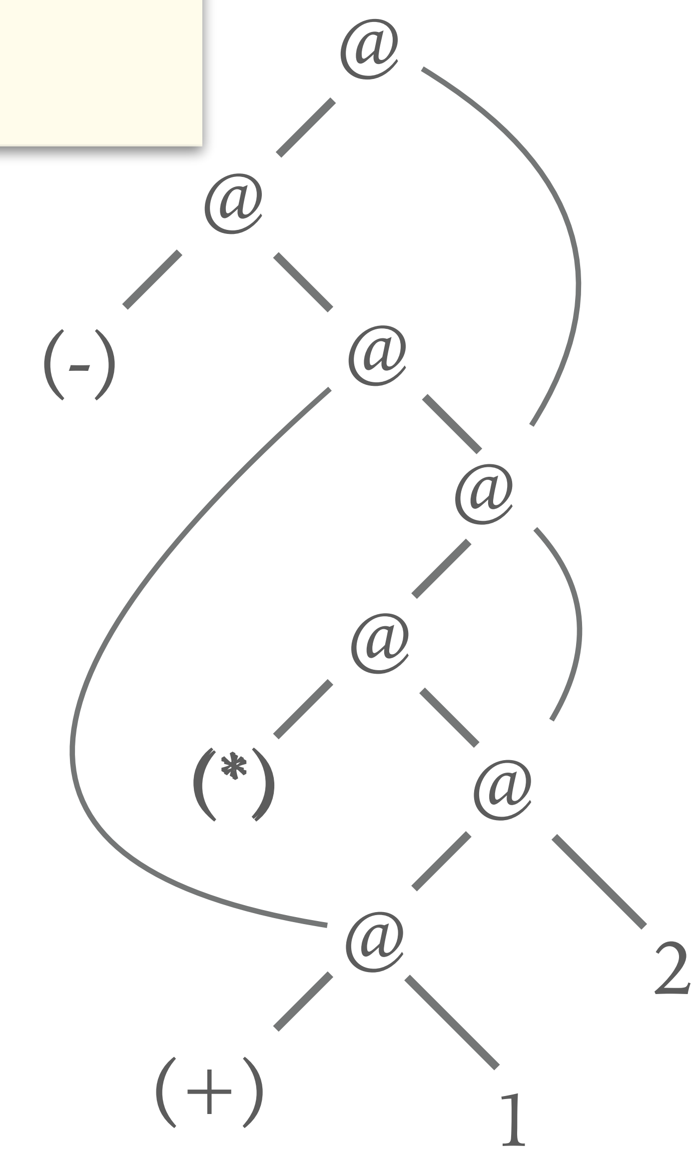
SHARING

```
let
    inc   = (+) 1             :: Exp Int -> Exp Int
    three = inc 2             :: Exp Int
    nine  = (*) three three   :: Exp Int
in (-) (inc nine) nine
```

- What does the AST look like for this expression?

```
let
    inc   = (+) 1               :: Exp Int -> Exp Int
    three = inc 2               :: Exp Int
    nine  = (*) three three :: Exp Int
in (-) (inc nine) nine
```





PROBLEM

- Sharing in the host language internal representation
- Not readily observable
- Processing the tree means we are loosing the sharing information
  - often results in large ASTs
  - expressions are evaluated multiple times
  - really, really inefficient

PROBLEM

- Black-Scholes option pricing
  - Accelerate without sharing 20 times slower than CUDA implementation on GPU

```
blackscholes :: Vector (Float, Float, Float)
             -> Acc (Vector (Float, Float))
blackscholes = map callput . use
  where
  callput x =
    let (price, strike, years) = unlift x
        r       = constant riskfree
        v       = constant volatility
        v_sqrtT = v * sqrt years
        d1      = (log (price / strike) +
                  (r + 0.5 * v * v) * years) / v_sqrtT
        d2      = d1 - v_sqrtT
        cnd d   = let c = cnd' d in d >* 0 ? (1.0 - c, c)
        cndD1   = cnd d1
        cndD2   = cnd d2
        x_expRT = strike * exp (-r * years)
    in
    lift ( price * cndD1 - x_expRT * cndD2
         , x_expRT * (1.0 - cndD2) - price * (1.0 - cndD1))
```

```
riskfree, volatility :: Float
riskfree   = 0.02
volatility = 0.30

horner :: Num a => [a] -> a -> a
horner coeff x = x * foldr1 madd coeff
  where
    madd a b = a + x*b

cnd' :: Floating a => a -> a
cnd' d =
  let poly     = horner coeff
      coeff    = [0.31938153, -0.356563782,
                  1.781477937, -1.821255978,
                  1.330274429]
      rsqrt2pi = 0.39894228040143267793994605993438
      k        = 1.0 / (1.0 + 0.2316419 * abs d)
  in
  rsqrt2pi * exp (-0.5*d*d) * poly k
```

PROBLEM

- Including ‘let’ in the surface language would make it extremely awkward to use
- Can we have ‘let’ in the internal representation, and convert without loosing sharing?

PROBLEM

- We need to be able to observe an implementation detail pure functional languages abstract over
  - referential equality:
    - not enough to know two values are the same, we need to check if they share a location
    - language level reference equality clashes with referential transparency, garbage collection, compiler optimisations
- Luckily, the need for referential equality pops up in other contexts as well
  - memoization, O(1) comparison of large objects,…

STABLE NAMES

- Idea:
  - associate values with an address-like **stable** name

```haskell
data StableName a

mkStableName :: a -> IO (StableName a)
hashStableName :: StableName a -> Int

instance Eq  (StableName a)
instance Ord (StableName a)
```

STABLE NAMES

```
mkStableName x = mkStableName y

⇒

x = y
```

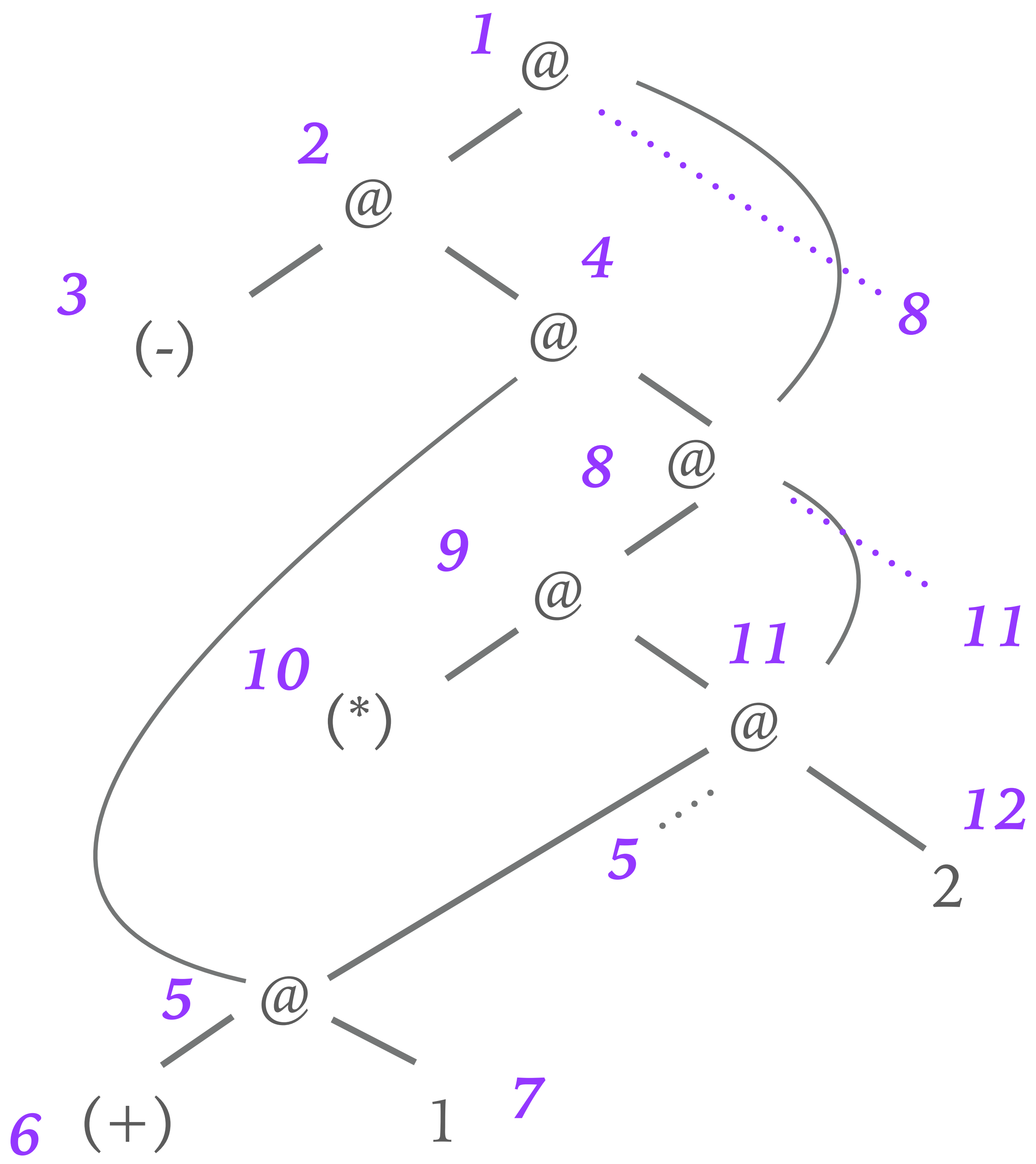
```
x ≠ y

⇒

mkStableName x ≠ mkStableName y
```

```
x = y

⇏

mkStableName x = mkStableName y
```

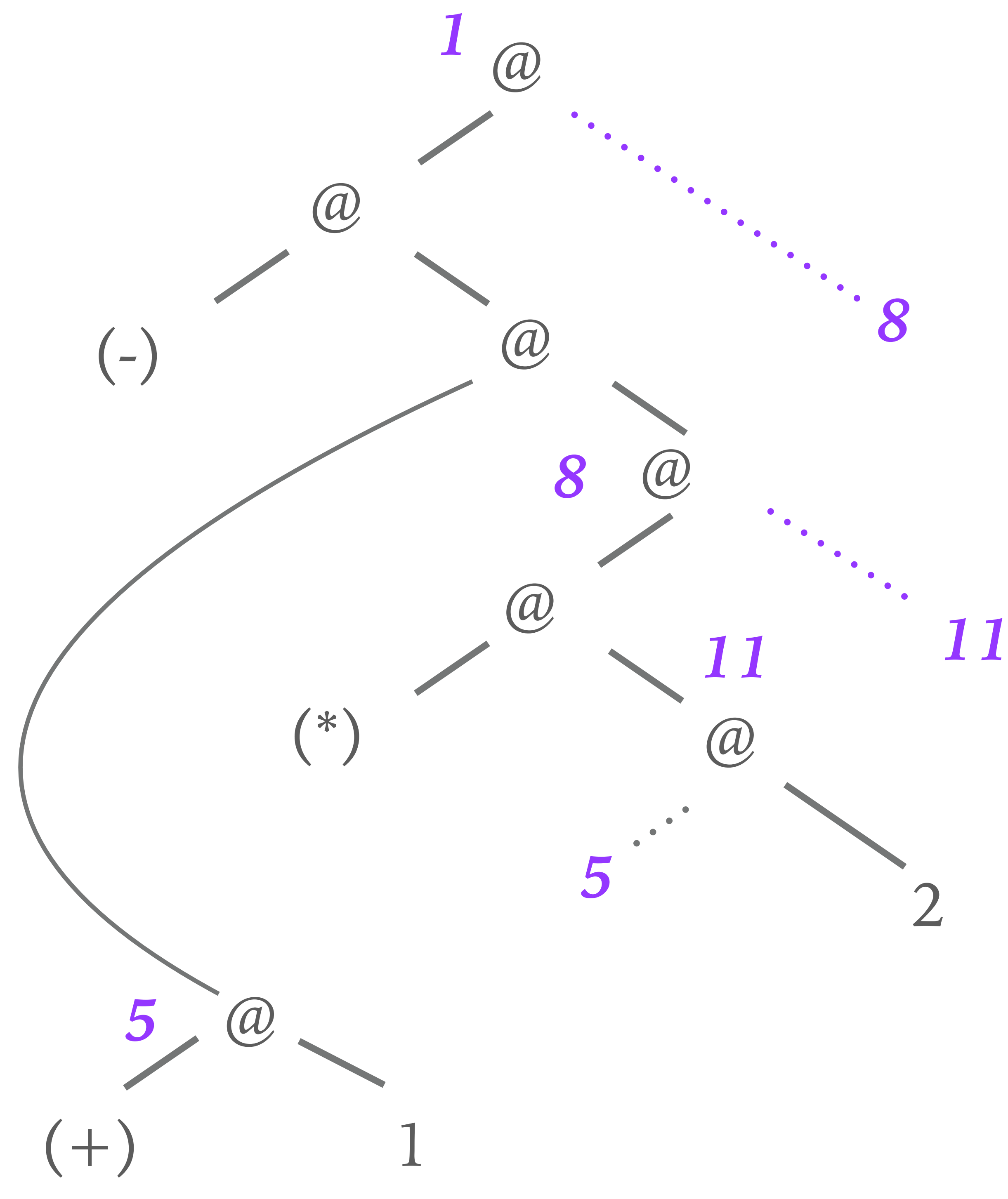
SHARING RECOVERY

- Traverse the ADT structure and identify the shared nodes with the help of stable names
- insert let-bindings in the de Bruijn internal representation at the right positions

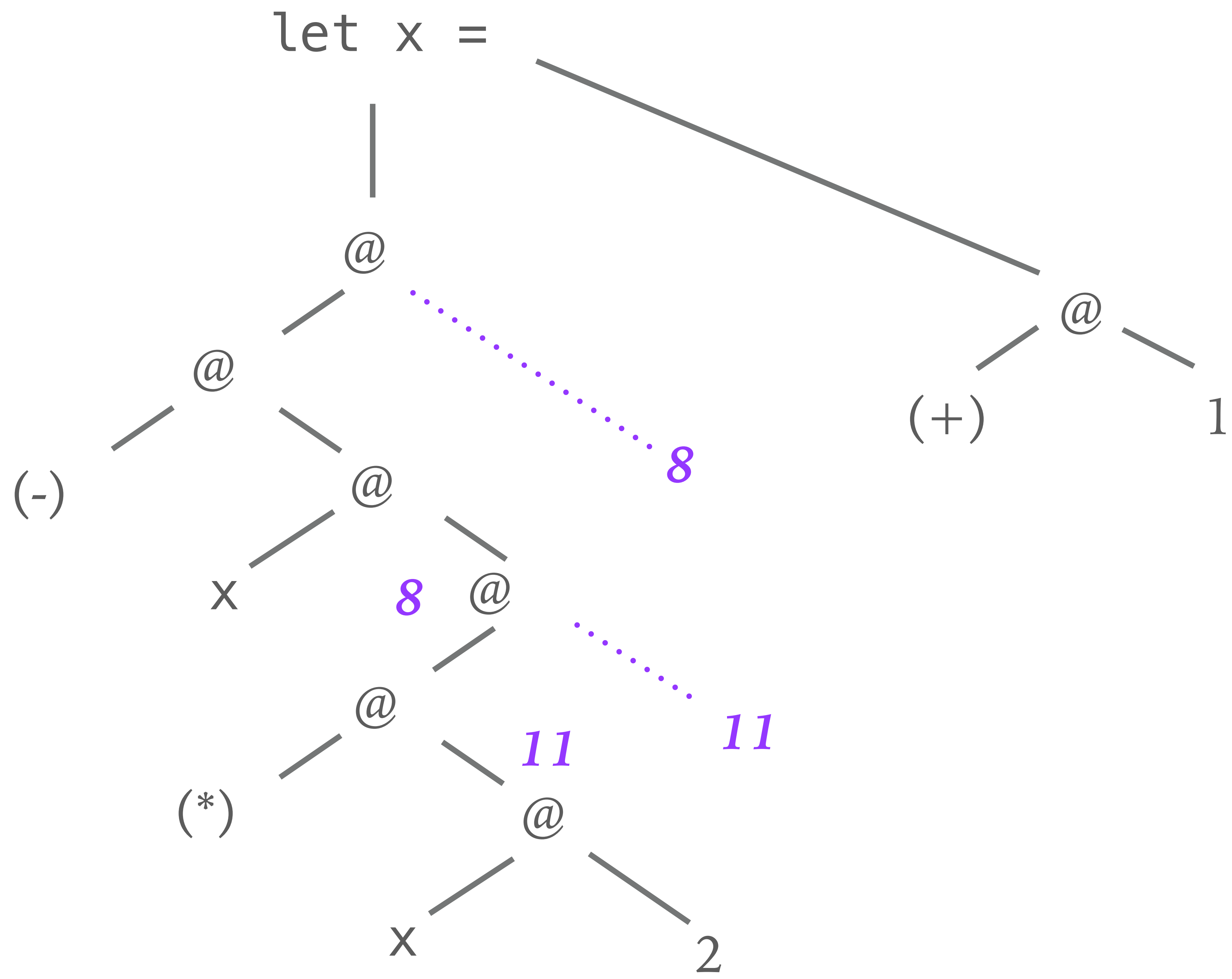
SHARING RECOVERY



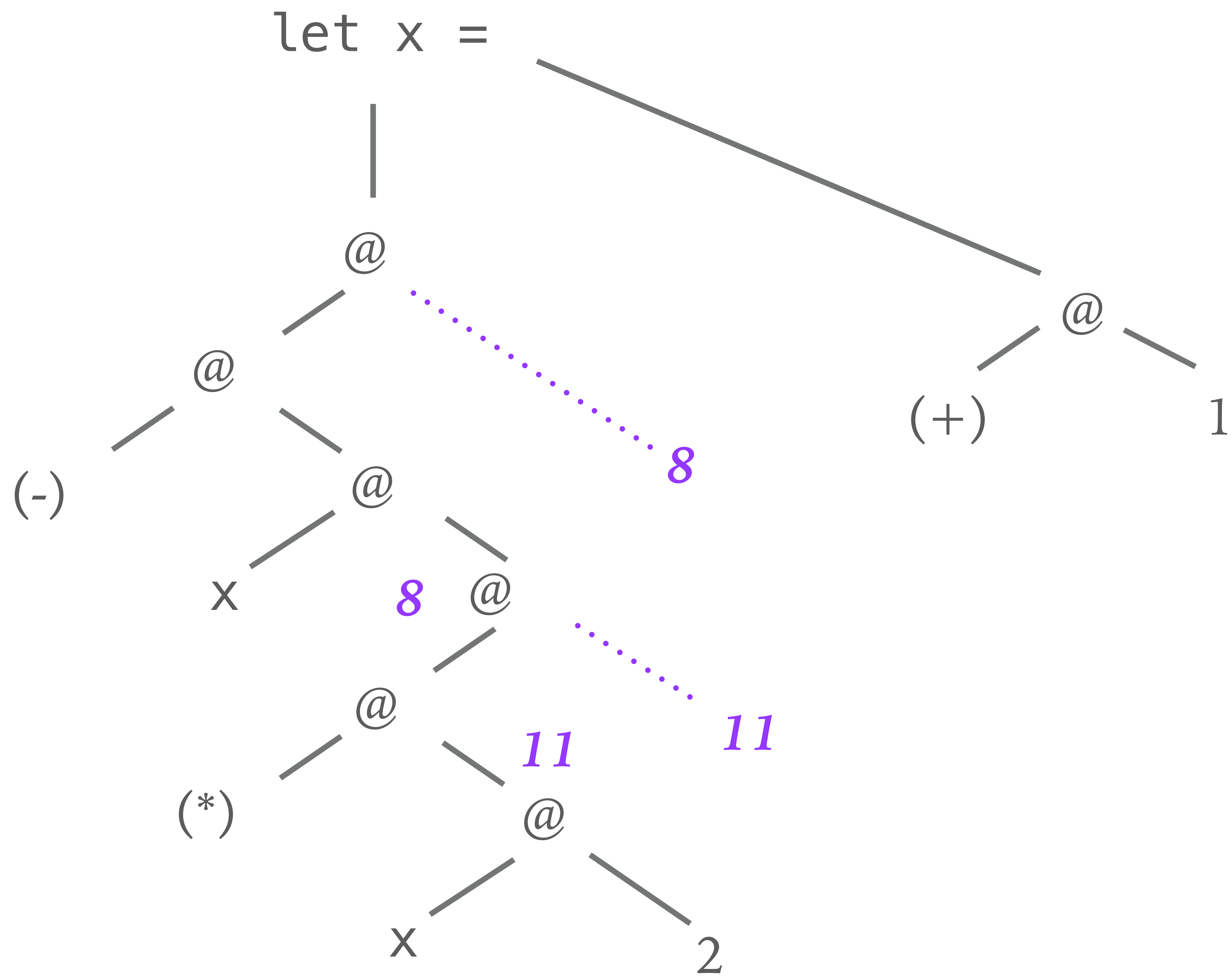
SHARING RECOVERY



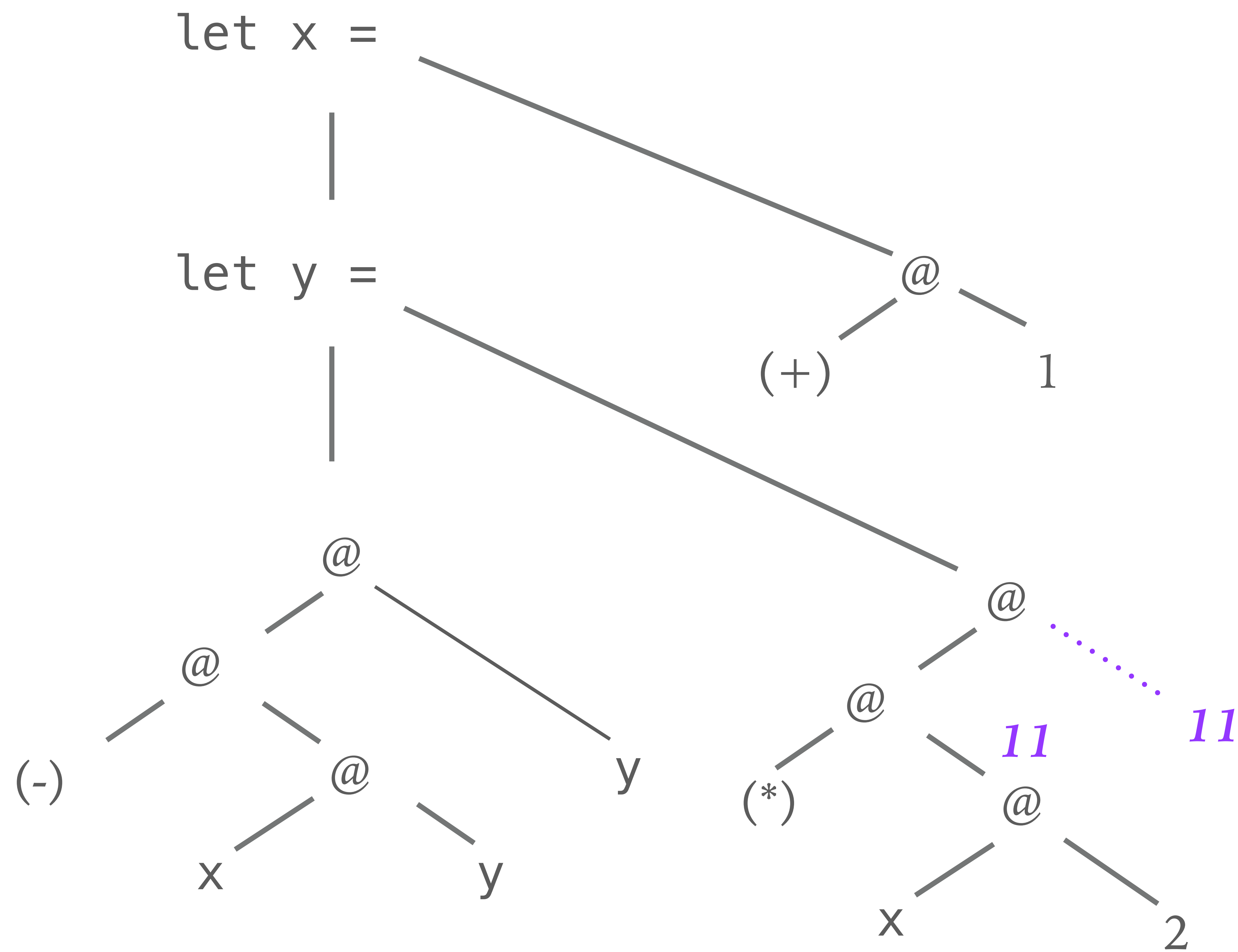
SHARING RECOVERY



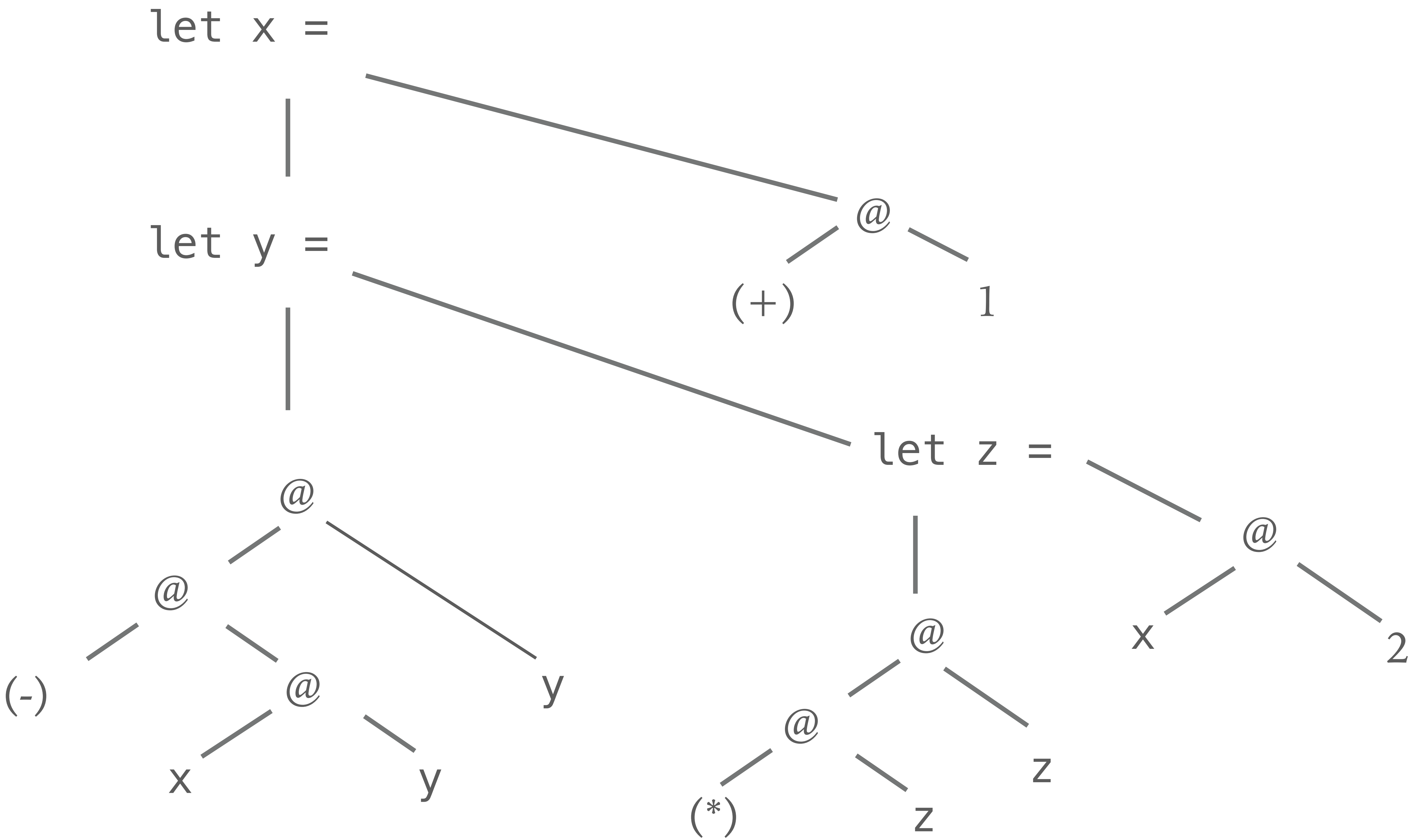
SHARING RECOVERY



SHARING RECOVERY



SHARING RECOVERY



COMBINING SHARING RECOVERY AND DE BRUIJN CONVERSION

- The two transformation are fused into a single one
  - applying De Bruijn conversion first would destroy sharing

- We loose type information during conversion from HOAS to De Bruijn
  - type checked dynamically

PERFORMANCE

- **Impact of sharing recovery*:**
  - Black Scholes, 20M elements:
    - CUDA, handwritten: 6.7ms
    - Accelerate, w/o sharing: 116ms
    - Accelerate w. sharing: 6.12ms
  - Canny edge detection, 16M pixel:
    - OpenCV: 50.6ms
    - Accelerate, w/o sharing: 82.7ms
    - Accelerate w. sharing: 78.4ms
  - Fluid-flow simulation, 2M particles
    - Accelerate, w/o sharing: 107ms
    - Accelerate w. sharing: 119ms

**Tesla T10 processor (compute capability 1.3, 30 multiprocessors = 240 cores at 1.3GHz, 4GB RAM) backed by two quad-core Xenon E5405 CPUs (64-bit, 2GHz, 8GB RAM), running GNU/Linux (Ubuntu 12.04 LTS). The reported GPU runtimes are averages of 100 runs.*

FUSION

- Well-known problem of collection-oriented programming:

```
map f $ map g xs

fold (+) $ enumFromThenTo 0 1 1000000000

let
  xs = map sqrt $ enumFromThenTo 0 1 1000000000
in fold (+) $ zipWith (*) $ map (+ (fold (+) xs)) xs
```

- Unnecessary intermediate structures, traversals

FUSION

- Like sharing recovery, fusion is essential if we care about performance
  - Mandelbrot: speed up of 1000%
  - typically, at least 50% faster

FUSION

- Many of the classical techniques don't work in this context:
  - e.g., `build/fold like` fusion approaches destroy the parallel pattern

```
build ::
  (forall b. (a -> b -> b) -> b -> b) -> [a]
build g = g (:) []


foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f z []     =  z
foldr f z (x:xs) =  f x (foldr f z xs)
```

```
foldr k z (build g) = g k z
```

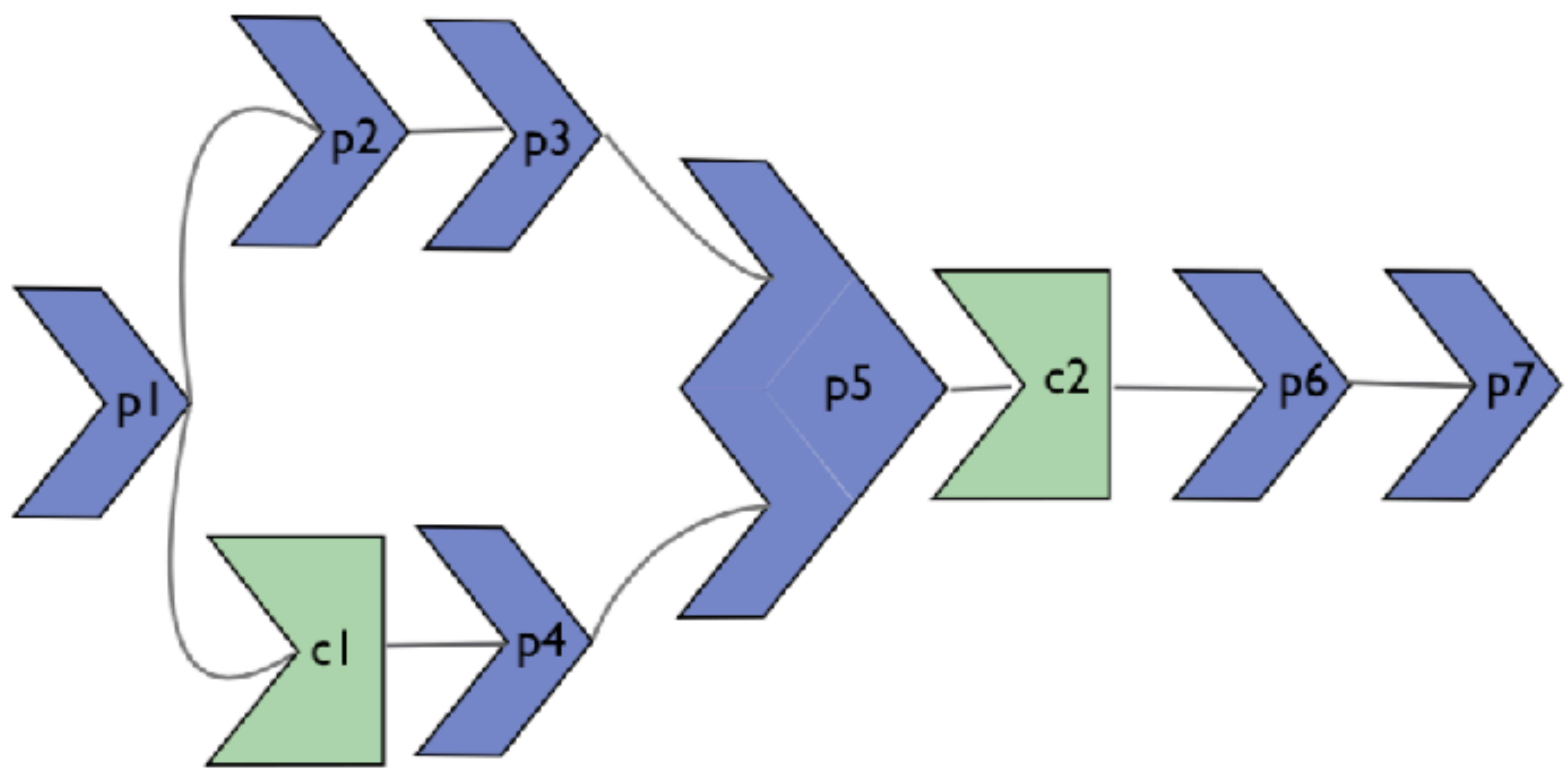
```
map f xs = build (\c n -> foldr
            (\a b -> c (f a) b) n xs)
…
```

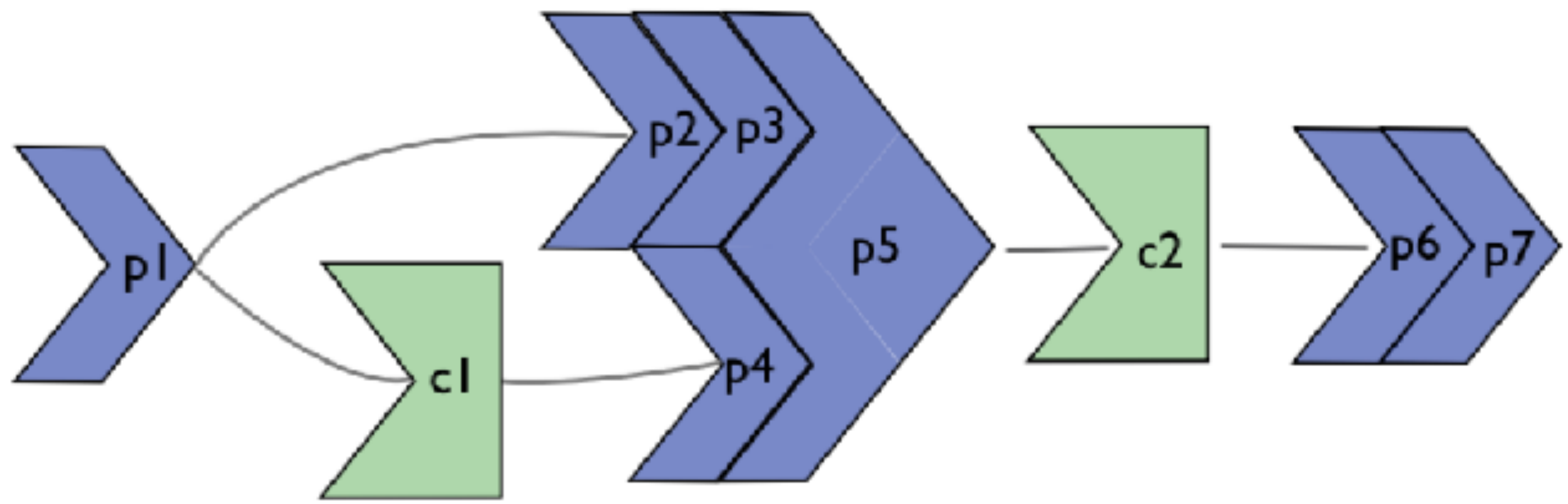
FUSION

- Fusion often relies on inlining for producer/consumer pairs to be detected, only done conservatively
- Accelerate fusion happens at run time, need to be aware of the costs (but also: more information available)
- Result of fusion transformation needs to fit in to our code generation templates

FUSION

- Producers:
  - each element of the result depends on at most one element of input array (e.g, map, backpermute, generate)
- Consumers:
  - each element of result depends on multiple elements of input array (e.g., folds, scans, stencil operations)
- We treat them separately
  - Producer/Producer fused via program transformation
  - Producer/Consumer during code generation


p1
p2
p3
c1
p4
p5
c2
p6
p7


p1
p2
p3
p4
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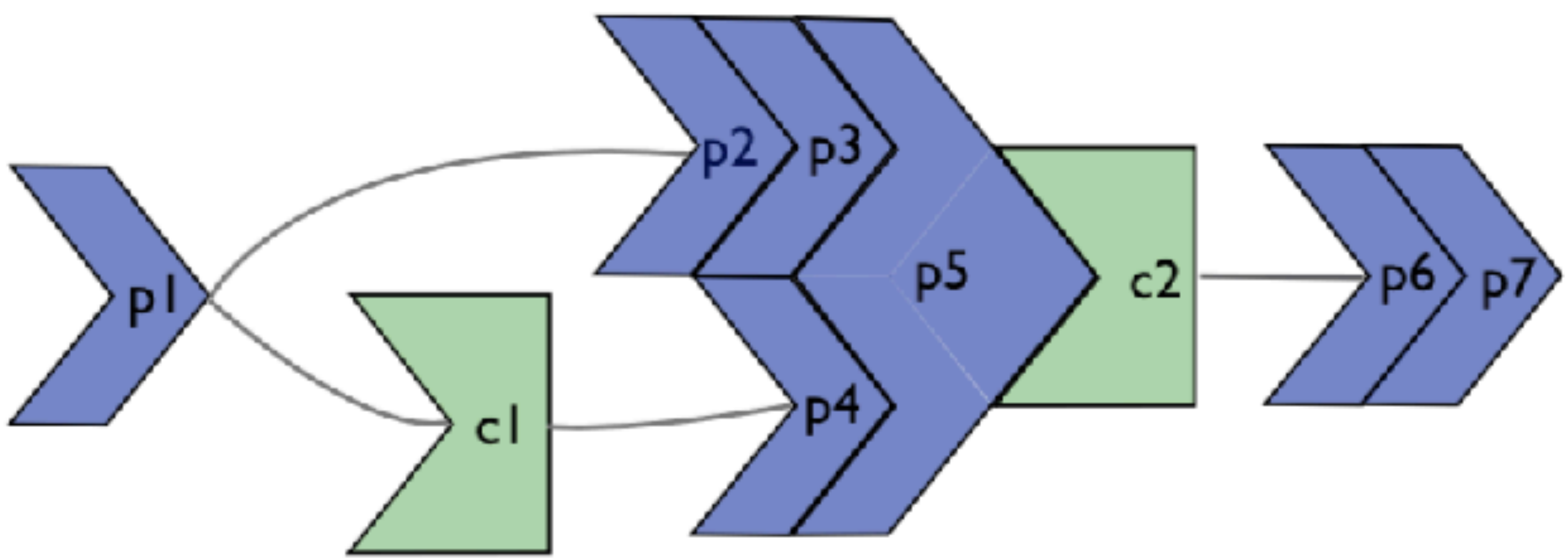
p5
c2
p6
p7


p1
c1
p2
p3
p4
p5
c2
p6
p7

PRODUCER/PRODUCER FUSION

- Arrays represented as delayed computations:

```
data DelayedAcc a where
   Done  :: Acc a
         -> DelayedAcc a
   Yield :: (Shape sh, Elt e)
         => Exp sh
         -> Fun (sh -> e)
         -> DelayedAcc (Array sh e)
   Step  :: (Shape sh, Shape sh', Elt e, Elt e')
         => Exp sh'
         -> Fun (sh' -> sh)
         -> Fun (e -> e')
         -> Idx (Array sh e)
         -> DelayedAcc (Array sh' e')
```

FUSION

- Example: map

```
mapD :: (Shape sh, Elt a, Elt b)
       => Fun (a -> b)
       -> DelayedAcc (Array sh a)
       -> DelayedAcc (Array sh b)
mapDf(Step shpgv)
  = Step shp (f.g) v
mapD f (Yield sh g)
  = Yield sh (f . g)
```

- To prevent fusion, arrays can be made manifest

```
compute :: Arrays a => Acc a -> Acc a
```

PRODUCER/CONSUMER FUSION

- Producer/consumer fusion is done during code generation
- Producer operations are inserted in the consumer code templates

- No support for consumer/consumer fusion yet

TYPE SAFE CODE GENERATION

- LLVM IR represents types as value-level data structure
- We track them as Haskell types in the LLVM binding
- Guarantees we only generate type correct LLVM programs
- GADT to define LLVM instruction set:

```haskell
data Instruction a  where
  Add :: NumType a
      -> Operand a
      -> Operand a
      -> Instruction a
  ...
```

- We translate the well-typed Accelerate AST into a well-typed LLVM AST

LLVM BACKEND FRAMEWORK

- LLVM is a reusable framework, portable across diverse architectures
- Accelerate LLVM backend framework
  - a set of re-usable components
  - reduces the cost of implementing future backends
- Existing backends:
  - vectorising multicore CPU
  - GPU backend

Benchmark	Input Size	Contender (ms)		Accelerate full (ms)		Accelerate no fusion (ms)		Accelerate no sharing (ms)	
Black Scholes	20M	6.70	(CUDA)	6.19	(92%)	(not needed)		116	(1731%)
Canny	16M	50.6	(OpenCV)	78.4	(155%)	(not needed)		82.7	(164%)
Dot Product	20M	1.88	(CUBLAS)	2.35	(125%)	3.90	(207%)	(not needed)	
Fluid Flow	2M	5461	(Repa -N7)	107	(1.96%)	(not needed)		119	(2.18%)
Mandelbrot (limit)	2M	14.0	(CUDA)	24.0	(171%)	245	(1750%)	245	(1750%)
N-Body	32k	54.4	(CUDA)	607	(1116%)	(out of memory)		(out of memory)	
Radix sort	4M	780	(Nikola)	442	(56%)	657	(84%)	657	(84%)
SMVM (protein)	4M	0.641	(CUSP)	0.637	(99%)	32.8	(5115%)	(not needed)	

Name	Non-zeros (nnz/row)	CUSP	Accelerate	Accelerate no fusion
Dense	4M (2K)	14.48	14.62	3.41
Protein	4.3M (119)	13.55	13.65	0.26
FEM/Spheres	6M (72)	12.63	9.03	4.70
FEM/Cantilever	4M (65)	11.98	7.96	4.41
Wind Tunnel	11.6M (53)	11.98	7.33	4.62
FEM/Harbour	2.37M (50)	9.42	6.14	0.13
QCD	1.9M (39)	7.79	4.66	0.13
FEM/Ship	3.98 (28)	12.28	6.60	4.47
Economics	1.27M (6)	4.59	0.90	1.06
Epidemiology	1.27M (4)	6.42	0.59	0.91
FEM/Accelerator	2.62M (22)	5.41	3.08	2.92
Circuit	959k (6)	3.56	0.82	1.08
Webbase	3.1M (3)	2.11	0.47	0.74
LP	11.3M (2825)	5.22	5.04	2.41

GFLOPS/s (higher is better)

THE ACCELERATE PROJECT

- Open source project
  - https://github.com/AccelerateHS/
- Current project members
  - Trevor McDonell
  - Rob Everest
  - Josh Meredith
  - Manuel Chakravarty