Assignment 7

due Friday, December 3, 2015

1. exercise 5.9 (3rd ed): Let $T = \{ \langle M \rangle \mid M$ is a TM that accepts $w^R$ whenever it accepts $w \}$. Show that $T$ is undecidable.

2. exercise 6.4 (3rd ed): Let $A_{TM'} = \{ \langle M, w \rangle \mid M$ is an oracle TM and $M^{A_{TM}}$ accepts $w \}$. Show that $A_{TM'}$ is undecidable relative to $A_{TM}$.

Extra credit: (from RF lecture Nov 23)

note: This extra credit has been updated, and can be turned in later (early finals week).

- We looked at the following mapping (encoding) $f$ from natural numbers to binary strings:
  $0 \rightarrow \epsilon, 1 \rightarrow 0, 2 \rightarrow 1, 3 \rightarrow 00, 4 \rightarrow 01, 5 \rightarrow 10, 6 \rightarrow 11, 7 \rightarrow 000, \ldots$ and saw that $l(f(x)) = \log_2 x + O(1)$ ($l$ is the length function, which denotes the number of characters in a string). Give a closed formula for this equation (with no $O(1)$ term).

- (a) Give an algorithm to compute the Kolmogorov complexity of a string given an oracle for the halting problem.

  (b) Recall Chaitin’s incompleteness theorem: Determining whether strings of length $n$ are random (or incompressible; never the output of a shorter input to our UTM than the number of bits in the original string) for arbitrary $n$ is undecidable. Use this to provide an alternative proof of the halting problem. (Hint: use part a)

(Notice that I didn’t ask the symmetric question to (a), which is to solve the halting problem given an oracle for Kolmogorov complexity. This is also do-able, but much harder)