Assignment 5

due Wednesday, February 17, 2016

For the problems below, just give a description of the subproblem and a recurrence relation for the optimal solution value. There is no need to write any code (and none is desired).

1. chapter 6, exercise 10, pp 321-322

2. Consider the following version of the sequence alignment problem. In converting $X = X_1X_2 \ldots X_n$ to $Y = Y_1Y_2 \ldots Y_m$ there are various fixed costs:
   - $d$ is the cost to delete a character from $X$
   - $s$ is the cost to insert a character of $Y$
   - $m$ is the cost to match two identical characters of $X$ and $Y$
   - $r$ is the cost to replace a character of $X$ with a different one from $Y$

   (a) Give a subproblem and recurrence to find the least cost to convert $X$ to $Y$
   (b) What values assigned to $(d, s, m, r)$ describe the Edit Distance problem (from the Dasgupta et al text, section 6.3, p 174)?
   (c) What values assigned to $(d, s, m, r)$ describe the Longest Common Subsequence problem - look online for problem definition? (Hint: use some negative numbers.)

3. (exercise 6.23, p 196, from DPV) A mission-critical production system has n stages that have to be performed sequentially; stage $i$ is performed by machine $M_i$. Each machine $M_i$ has a probability $r_i$ of functioning reliably and a probability $1 - r_i$ of failing (and the failures are independent). Therefore, if we implement each stage with a single machine, the probability that the whole system works is $r_1 \cdot r_2 \cdots r_n$. To improve this probability we add redundancy by having $m_i$ copies of the machine $M_i$ that performs stage $i$. The probability that all $m_i$ copies fail simultaneously is only $(1 - r_i)^{m_i}$, so the probability that stage $i$ is completed correctly is $1 - (1 - r_i)^{m_i}$ and the probability that the whole system works is $\Pi_{i=1}^{n}[1 - (1 - r_i)^{m_i}]$. Each machine $M_i$ has a cost $c_i$, and there is a total budget $B$ to buy machines. (Assume that $B$ and the $c_i$ are positive integers.)

   Given the probabilities $r_1, r_2, \ldots, r_n$, the costs $c_1, c_2, \ldots, c_n$, and the budget $B$, find the maximum reliability that can be achieved within budget $B$.

4. The “Bone’s Battery” problem, linked to from class page. Remember - just subproblem, recurrence, no code.