Smaller, Faster, Cheaper: Architectural Designs for Efficient Machine Learning

by

Steven Walton

A dissertation accepted and approved in partial fulfillment of the

requirements for the degree of

Doctor of Philosophy

in Computer Science

Dissertation Committee: Hank Childs, Chair Humphrey Shi, Core Member Daniel Lowd, Core Member Thien Nguyen, Core Member Edward Rubin, Institutional Representative

> University of Oregon Summer 2025

 $\bigodot$  2025 Steven Walton

This work, including text and images of this document but not including supplemental files (for example, not including software code and data), is licensed under a Creative Commons Attribution 4.0 International License.



### DISSERTATION ABSTRACT

### Steven Walton

### Doctor of Philosophy in Computer Science

Title: Smaller, Faster, Cheaper: Architectural Designs for Efficient Machine Learning

Major advancements in the capabilities of computer vision models have been primarily fueled by rapid expansion of datasets, model parameters, and computational budgets, leading to ever-increasing demands on computational infrastructure. However, as these models are deployed in increasingly diverse and resource-constrained environments, there is a pressing need for architectures that can deliver high performance while requiring fewer computational resources.

This dissertation focuses on architectural principles through which models can achieve increased performance while reducing their computational demands. We discuss strides towards this goal through three directions. First, we focus on data ingress and egress, investigating how information may be passed into and retrieved from our core neural processing units. This ensures that our models make the most of available data, allowing smaller architectures to become more performant. Second, we investigate modifications to the core neural architecture, applied to restricted attention in vision transformers. This section explores how removing uniform context windows in restricted attention increases the expressivity of the underlying neural architecture. Third, we explore the natural structures of Normalizing Flows and how we can leverage these properties to better distill model knowledge.

 $\mathbf{3}$ 

These contributions demonstrate that careful design of neural architectures can increase the efficiency of machine learning algorithms, allowing them to become smaller, faster, and cheaper.

This dissertation includes previously published and unpublished co-authored material.

### CURRICULUM VITAE

### NAME OF AUTHOR: Steven Walton

### GRADUATE AND UNDERGRADUATE SCHOOLS ATTENDED:

University of Oregon, Eugene, OR, USA Embry-Riddle Aeronautical University, Prescott, AZ, USA

### DEGREES AWARDED:

Doctor of Philosophy in Computer Science, 2025, University of Oregon Master of Science in Computer Science, 2023, University of Oregon Bachelor of Science in Space Physics, 2014, Embry-Riddle Aeronautical University

### AREAS OF SPECIAL INTEREST:

Computer Vision Machine Learning Artificial Intelligence Generative Modeling

### PROFESSIONAL EXPERIENCE:

Graduate Researcher, University of Oregon, Eugene, OR, Aug. 2018 - Jun. 2025
Metropolis Intern, Nvidia, Sep. 2023 - Mar. 2024
Ph.D. Research Intern, Picsart AI Research, Eugene, OR, Jun. 2021 - Nov. 2022
Computer Science Intern, Lawrence Livermore National Labratory, Livermore, CA, Jun. - Sept. 2020
Computer Science Intern, Lawrence Livermore National Labratory, Livermore, CA, Jun. - Sept. 2019
ASTRO Intern, Oak Ridge National Labratory, Oak Ridge, TN, Jun. - Aug. 2018

### GRANTS, AWARDS AND HONORS:

Outstanding Reviewer, CVPR 2025

### PUBLICATIONS:

- Steven Walton, Ali Hassani, Xingqian Xu, Zhangyang Wang, and Humphrey Shi. Efficient image generation with variadic attention heads. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR) Workshops, 2025
- Steven Walton, Valeriy Klyukin, Maksim Artemev, Denis Derkach, Nikita Orlov, and Humphrey Shi. Distilling normalizing flows. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR) Workshops, 2025.
- Ali Hassani, Fengzhe Zhou, Aditya Kane, Jiannan Huang, Chieh-Yun Chen, Min Shi, Steven Walton, Markus Hoehnerbach, Vijay Thakkar, Michael Isaev, Qinsheng Zhang, Bing Xu, Haicheng Wu, Wen mei Hwu, Ming-Yu Liu, and Humphrey Shi. Generalized neighborhood attention: Multi-dimensional sparse attention at the speed of light, 2025. arXiv:2504.16922
- Jonathan Roberts, Mohammad Reza Taesiri, Ansh Sharma, Akash Gupta, Samuel Roberts, Ioana Croitoru, Simion-Vlad Bogolin, Jialu Tang, Florian Langer, Vyas Raina, Vatsal Raina, Hanyi Xiong, Vishaal Udandarao, Jingyi Lu, Shiyang Chen, Sam Purkis, Tianshuo Yan, Wenye Lin, Gyungin Shin, Qiaochu Yang, Anh Totti Nguyen, David I. Atkinson, Aaditya Baranwal, Alexandru Coca, Mikah Dang, Sebastian Dziadzio, Jakob D. Kunz, Kaiqu Liang, Alexander Lo, Brian Pulfer, Steven Walton, Charig Yang, Kai Han, and Samuel Albanie. Zerobench: An impossible visual benchmark for contemporary large multimodal models, 2025. arXiv:2502.09696
- Noble Kennamer, Steven Walton, and Alexander Ihler. Design amortization for bayesian optimal experimental design. Proceedings of the AAAI Conference on Artificial Intelligence, 37(7):8220–8227, 2023.
- Ali Hassani, Steven Walton, Jiachen Li, Shen Li, and Humphrey Shi. Neighborhood attention transformer. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), pages 6185–6194, 2023.

- Steven Walton. Isomorphism, normalizing flows, and density estimation: Preserving relationships between data, 2022. https://www.cs.uoregon.edu/Reports/AREA-202307-Walton.pdf
- Jitesh Jain, Anukriti Singh, Nikita Orlov, Zilong Huang, Jiachen Li, Steven Walton, and Humphrey Shi. Semask: Semantically masked transformers for semantic segmentation. In Proceedings of the IEEE/CVF International Conference on Computer Vision (ICCV) Workshops, pages 752–761, 2023.
- Jiachen Li, Ali Hassani, Steven Walton, and Humphrey Shi. Convmlp: Hierarchical convolutional mlps for vision. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR) Workshops, pages 6307–6316, 2023.
- David Pugmire, James Kress, Jieyang Chen, Hank Childs, Jong Choi, Dmitry Ganyushin, Berk Geveci, Mark Kim, Scott Klasky, Xin Liang, Jeremy Logan, Nicole Marsaglia, Kshitij Mehta, Norbert Podhorszki, Caitlin Ross, Eric Suchyta, Nick Thompson, Steven Walton, Lipeng Wan, Matthew Wolf, Jeffrey Nichols, Becky Verastegui, Arthur 'Barney' Maccabe, Oscar Hernandez, Suzanne Parete-Koon, and Theresa Ahearn. "Visualization as a Service for Scientific Data". In "Driving Scientific and Engineering Discoveries Through the Convergence of HPC, Big Data and AI", pages "157–174", "Cham", "2020". "Springer International Publishing".
- Steven Walton, Ali Hassani, Abulikemu Abuduweili, and Humphrey Shi. Training compact transformers from scratch in 30 minutes with pytorch. medium.com/pytorch, 2021. arXiv:2104.05704
- Ali Hassani, Steven Walton, Nikhil Shah, Abulikemu Abuduweili, Jiachen Li, and Humphrey Shi. Escaping the big data paradigm with compact transformers, 2022.
- Steven Walton. Datum: Dotted attention temporal upscaling method. 2020. https://www.cs.uoregon.edu/Reports/DRP-202006-Walton.pdf

### ACKNOWLEDGEMENTS

I'd like to thank my mentors and professors from my Universities for helping get me to where I am today. Thank you Jeff Spear, for being the first to show me how to be creative with math. To Karla Westphal, for helping me find passion and dedication to the subject. To my undergraduate professors: Timothy Callahan, Andri Gretarsson, Edward Poon, Hisaya Tsutsui, and Darrel Smith who taught me my passion for math, physics, and providing me the tools to understand the world around me. To my graduate professors and advisors, who helped get me through these difficult times. I especially want to thank Hank Childs for encouraging me to pursue Machine Learning and to be my acting advisor after Humphrey moved to Georgia Tech. I want to thank Humphrey Shi for being my advisor and helping me make all the connections and pushing me to become a better researcher.

I'd like to thank my friends and family for helping get through this. It was a journey that I could not have made alone. Noble, you've been a close friend for so many years and your insights helped shape my research and encouraged me to go to graduate school. You constantly challenge my ideas, often frustratingly so, but they always end up better and more refined for it. Never change. Ali, I couldn't ask for a better co-author nor friend. Your intelligence and work ethic have always pushed me to better myself, and I look forward to calling you "doctor". I want to thank my cat Hypatia, who has been my best friend for the last decade. She's had to listen to many explinations and I'm sorry you have not received formal recognition for your contributions despite frequent appearances in my works (including this one). Lastly, I want to thank my wonderful girlfriend: Jaichung Lee. We have been through so much and I could not have crossed the finish line without

you. I know it was as much of a challenge for you as it was for me, and this PhD would not have been possible without your many efforts. Thank you.

### DEDICATED TO

My mom, and the many years of watching Star Trek together. My dad, and the many years of reading Asimov together. Jaichung, and the many years to build the future together.

## TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION	18
1.1. Motivation $\ldots$	18
1.2. Research Goals and Approaches	19
1.3. Dissertation Outline	21
1.4. Co-Authored Material	22
II. BACKGROUND	24
2.1. Learned Data Mappings	24
2.2. Scale Is Not All You Need	27
2.2.1. Scaling Data	28
2.2.2. Model Size	30
2.3. The Foundations That Shape Us	31
2.3.1. Transformers	31
2.3.2. Adversarial Generation	33
2.3.3. Normalizing Flows	34
2.4. The Tyranny of Measurements	37
III. ESCAPING THE BIG DATA PARADIGM	39
3.1. Vision Transformers	40
3.2. Data Efficient Vision Transformers	43
3.2.1. Convolutional Tokenizer	43
3.2.2. SeqPool	45
3.3. Experiments	46
3.3.1. Datasets	47

	3.3.2.	Computational Resources	47
	3.3.3.	Hyperparameters	47
	3.3.4.	Transformers On Small Datasets	48
	3.3.5.	Ablations	51
	3.3.6.	Scaling Study	54
	3.3.7.	Natural Language Processing	61
3.4.	Conclu	usion $\ldots$	61
IV. VA	RIADI	IC NEIGHBORHOOD ATTENTION	64
4.1.	Locali	zed Attention	67
4.2.	Neighl	borhood Attention	68
4.3.	Variad	dic Attention Heads	70
4.4.	Genera	ating The Right Experiment	72
	4.4.1.	Datasets	77
	4.4.2.	Hyperparameters	78
4.5.	When	Faced With Sparse Attention	80
4.6.	A Bur	mp While Headed To Church	83
4.7.	Metric	cs Are Not Enough	85
	4.7.1.	The Face Says It All	86
	4.7.2.	Quick Training on Deep Fake Detection	87
	4.7.3.	Fingerprints	89
		4.7.3.1. StyleGAN	89
		4.7.3.2. StyleSwin	91
		4.7.3.3. StyleNAT	93
	4.7.4.	Attention To Details	95

Ch	Chapter Page								
V.	DI	STILLA	ATION OF INVERTIBLE NETWORKS		98				
	5.1.	Model	Distillation		99				
	5.2.	Distill	ing Normalizing Flows		100				
		5.2.1.	Categories of Flow Distillations		101				
			5.2.1.1. Latent Knowledge Distillation		101				
			5.2.1.2. Intermediate Latent Knowledge Distillation		102				
			5.2.1.3. Synthesized Knowledge Distillation		102				
			5.2.1.4. All Together		104				
	5.3.	Distill	ation Experiments		105				
		5.3.1.	Density Estimation		106				
		5.3.2.	Image Generation		108				
	5.4.	Conclu	usion		111				
VI	. CC	NCLU	SION AND FUTURE DIRECTIONS		113				
	6.1.	Summ	ary		113				
	6.2.	Future	e Directions		114				
		6.2.1.	Core Challenges		115				
		6.2.2.	Scaling		116				
		6.2.3.	Ingress and Egress of Data		117				
			6.2.3.1. Parameterization		117				
			6.2.3.2. Automated Preprocessing		118				
			6.2.3.3. Making The Most of it		118				
		6.2.4.	Core Processing Architectures		119				
			6.2.4.1. Flexible Learning		120				
			6.2.4.2. Is Beauty in the Eye of the Beholder?		120				
		6.2.5.	Structurally Aware Architectures		121				

Chapter													ł	Page
6.3. Conclusion														122

## LIST OF FIGURES

# Figure

# Page

1.	Domain, Range, Image, Preimage diagram
2.	Vision Transformer Architecture
3.	Taxonomy of Generative Models
4.	Diagram of Injection, Surjection, and Bijection
5.	Architectural design of Compact Transformers
6.	Variations of ViT Architectures
7.	Vision Transformer Salient Maps
8.	Accuracy of ViTs on Restricted Samples per Class
9.	Vision Transformer Resolution Based Performance
10.	StyleNAT Samples (FFHQ-256, FFHQ-1024, LSUN Church)
11.	NAT vs Swin Vs ConvNext ImageNet Performance
12.	Neighborhood Attention Transformer (NAT)
13.	StyleNAT Architecture
14.	StyleNAT: FID vs. Throughput vs. Parameters
15.	StyleNAT Samples: FFHQ & LSUN Church
16.	StyleNAT FID vs Iteration
17.	StyleGAN3 Visual Artifacts
18.	StyleSwin Visual Artifacts
19.	StyleNAT Visual Artifacts
20.	StyleNAT and StyleSwin Attention Maps
21.	StyleNAT and StyleSwin Attention Maps
22.	Illustration of Knowledge Transfer for Normalizing Flows

Figu	Ire								ł	Page
23.	Distilling Normalizing Flow CIFAR-10 Samples		•	•	•		•	•		110
24.	Distilling Normalizing Flow CelebA Samples									111

### LIST OF TABLES

Page

Table

#### 1. Terminology of Mathematical Sets 252. CCT Hyperparameters 48 . 3. 49CCT Main Results 4. 505. CCT Ablation Study 516. CCT Positional Embedding Comparison 557. CCT ImageNet Accuracy 578. CCT Flowers-102 Accuracy . . . . . . . . . . . . . . . . . . . 589. 6210. StyleNAT Configurations 7880 11. 12.83 13. Distilling Normalizing Flow Density Estimation Metrics 14. 10715.Distilling Normalizing Flow Model Configurations for Image Generation . 108 16. 17. Distilling Normalizing Flow Image Generation Metrics . . . . . . . . 109

### CHAPTER I

### INTRODUCTION

I don't believe in empirical science.I only believe in a priori truth.

Kurt Gödel

### 1.1 Motivation

This thesis focuses on the development of efficiently training machine learning algorithms, primarily applied to Computer Vision. Our focus is to develop methods which allow for a reduction in computational resources required to train and deploy models.

Machine Learning is a subfield of Artificial Intelligence which aims to process data and automate the discovery of structures within the data. This process reduces the burden of needing to derive explicit formulations, instead allowing automation through optimization. This process allows algorithms to "learn" by "training" on the data.

Computer Vision applies to a wide range of problems related to perception. Traditionally associated with image and video processing, the field extends to processing of other data, such as LIDAR, radio, depth estimation, and other forms of signal processing. The domain involves a broad range of tasks, including: regression, which models quantitative relationships between variables; discrimination, the processing distinguishing relevant objects or patterns; and generation, or data synthesis. The primary focus of this thesis revolves around discrimination and generation of images.

Image processing presents unique challenges, often due to the high dimensionality of the embeddings. This high dimensionality causes difficulties in formulating explicit descriptions of our data and the underlying structures within it. The goal of computer vision is to create the machinery necessary to automate this process for us, as efficiently as possible. While we may not be able to create fully formulate descriptions, the descriptions we provide our algorithms can both help and hinder them. For example, images usually have spatial relationships, with pixels that are local spatially having high probabilities of being related to one another. This has led to the use Convolutional Neural Networks, as their architecture is able to exploit this natural bias. But such relationships may not always hold. For example, a QR code contains sharp transitions, where neighboring pixels do not aid the prediction of one another. More flexible architectures, such as attention, can better process such imagery by reducing the importance of locality. Therefore, to efficiently process data we must consider the biases implicit to the neural architectures that we use.

The modern success of these algorithms has presented additional challenges. It has been found that many of these methods can be improved through simple means: making them larger and providing them with more training data [142]. While this has led to dramatic improvements, it has similarly led to dramatic increases in the computational resources necessary to train and deploy these models. Once trained, these models may still be quite difficult to deploy, with their high computational demands, greatly limiting where they can be used. This has led many researchers to consider how these models can be more efficiently trained, requiring: less data, less time to train, and fewer computational resources. Similar challenges exist with respect to the deployment of these models.

### **1.2** Research Goals and Approaches

The focus of this thesis revolves around two primary questions:

- How do we reduce the model's data dependence?
- How do we reduce the model's computational demands?

These questions are fundamentally intertwined, necessitating solutions which address the problems simultaneously. Naturally, by reducing the amount of data that a model must ingest reduces the amount of time that a model must be trained for. Conversely, by making a model more efficiently extract information from its data, the less data it will need to achieve a given performance level. This is because model parameters do not just determine its information capacity, but also play an integral role in the solution space during training [133]. Many works have found that once trained models are often significantly over-parametrized, meaning only a subset of their parameters are being used to model the data [33, 97]. These findings are further evidenced by the continued increasing performance of smaller models [66], and strongly suggest our models can be trained more efficiently.

Our motivation to reduce a model's data depends exists beyond our desire to be cost effective. Real world large datasets provides two primary challenges which require our models to be data efficient. First, many important structures within the data are subtle and difficult to recover. Second, data is often *heavytailed*, meaning we do not have many samples. Fundamentally, these require our models to generalize relationships with minimal examples. While we may focus on explicitly constrained data to aid the interpretation of our work, it provides benefits as our models and data expand in size.

These feats are primarily accomplished the development the development of neural architectures and optimization methods. This thesis focuses on the former, specifically, studying the design of Computer Vision architectures which reduce: parameters, data dependence, and system resources. These goals must

be simultaneously optimized. Our objective is not to develop models with a small number of parameters if they also require substantially greater costs during training or deployment. Similarly, this would undermine our own goals if we reduce a model's data dependence with significant cost to its performance.

This thesis investigates three critical aspects of our neural architectures and structure it to follow a natural progression in complexity. The first work focuses on the understanding how our core neural architecture takes in data and how to efficiently extract the relationships it uncovers. Without efficiently providing and extracting data to/from a model, they become wasteful and this hinders the ability to develop more efficient core architectures. The second work focus on the core architecture, which perform the majority of the data processing. This section studies these two aspects as applied to vision transformers, directly building off one another. The third work revolves around knowledge distillation of Normalizing Flows. These models are structurally aware, explicitly designed to preserve the structures within the data. From these three lenses this thesis seeks to better understand how to build neural architectures that are smaller, faster, and cheaper.

### **1.3** Dissertation Outline

This dissertation is organized as follows:

Chapter 2 provides the necessary background and foundational information necessary to understand the research objectives. This background is necessary for understanding how the works are connected and the ways we seek to resolve underlying issues.

Chapter 3 presents the work *Escaping the Big Data Paradigm with Compact Transformers* [51], and focuses on efficiently embedding and extracting data from Vision Transformers.

Chapter 4 presents the work *Efficient Image Generation with Variadic* Attention Heads [156], as well as the works it builds upon: Neighborhood Attention Transformer [52].

Chapter 5 presents the work *Distilling Normalizing Flows*, which provides a framework for knowledge distillation with Normalizing Flow architectures and studies the categorical distillation methods.

Chapter 6 provides an overview of the findings and recommendations for future work.

### 1.4 Co-Authored Material

The research presented herein involves previously published material. Below is a listing of the prior works in relation to the chapter material. Details of division of labor is provided in the preface to each chapter.

- Chapter 2: This chapter includes material that was part of Steven Walton's Area Exam [154].
- Chapter 3: This work was contains materials from *Escaping the Big Data* Paradigm with Compact Transformers [51]. This work was a collaboration with Ali Hassani, Nikhil Shah, Abulikemu Abuduweili, Jiachen Li, Humphrey Shi, and myself.
- Chapter 4: This chapter contains materials from both Neighborhood Attention Transformer [52] and Efficient Image Generation with Variadic Attention Heads [156], with focus around the latter. The former is a collaboration between Ali Hassani, Jaichen Li, Shen Li, Humphrey Shi, and myself. The latter was a collaboration between Ali Hassani, Xingqian Xu, Zhangyang Wang, Humphrey Shi, and myself.

Chapter 5: This chapter contains material from a collaboration between
 Valeriy Klyukin, Maksim Artemev, Denis Derkach, Nikita Orlov, Humphrey
 Shi, and myself.

### CHAPTER II

### BACKGROUND

Mathematicians do not deal in objects, but in the relationships among objects.

Henri Poincaré

**Nota Bene:** Some of the text and figures from this section were part of Steven Walton's Area Exam [154], which has been publicly released by The University of Oregon. Steven was the sole author of this work.

This section covers the background necessary for understanding the motivation and purpose of the work performed. There is includes some necessary discussion about how machine learning algorithms work, how data is processed, and the inherent biases of different learning architectures. The latter of which is the main focus of this thesis. While subsequent chapters will have lower mathematical notation and formulation, those herein provide important context and intuition for the work ahead. To reach our goal of making our machine learning models smaller, faster, and cheaper, we need to have some core understandings as to how these models work. It is not enough to treat them as black boxes; rather we have to look inside. Much of machine learning terminology has not been standardized, thus this section may be used to contextualize these terminologies and the usage within this thesis.

### 2.1 Learned Data Mappings

The procedure can be understood through mapping between two sets, where our neural network is a learned mapping, f(x). Deep neural networks are Universal Approximators [18, 67, 112], where every multivariate continuous function can,

Name	Relation	Set
Domain	D	$\{\forall \ x \in D\}$
Codomain	C	$\{\forall \ y \in C\}$
Range	$\widetilde{C} \subseteq C$	$\{y \in C \mid \exists x \in D : f(x) = y\}$
Image	$\widetilde{C} \subseteq C$	$\{y \in C \mid \exists x \in \widetilde{D} : f(x) = y\}$
Preimage	$\widetilde{D}\subseteq D$	$\left  \{ x \in D \mid \exists \ y \in \widetilde{R} : f(x) = y \} \right $

Table 1. Explanation of important set terms denoting their relationships and what elements are in their set.

in principle, be approximated by the superposition of a sequence of continuous functions.

With this in mind, it helps to revisit some of the basics of functions and set theory. We can view the *Domain*, D, as all valid inputs to the neural network. In the study of Computer Vision this is *any valid image*, regardless of whether this image is meaningful to humans or not. We can then define our *Range*, R, as all possible outputs that our neural net can produce. In the case of Image Classification this would be all labels that we are trying to learn. Our *Codomain*, C, is a super-set to our Range,  $R \subseteq C$ , and may include elements that our map cannot reach. In our example of Image Classification our Codomain would represent *all* possible labels.

In practice, we are likely only interested in studying some subset of our domain,  $\widetilde{D} \subseteq D$ . This subset can be arbitrary and may be something like our training set, the set of images interesting to humans, or even some subset of our training data. Regardless of what this subset is, when they are passed through our mapping function then we call the outputs an *Image*,  $\widetilde{R} \subseteq R$ . A "reverse" of this function may then be defined, called the *Preimage*,  $f^*[\widetilde{R}]$ . The Preimage is defined as the set of elements in the domain that map to some image in the codomain. It



Figure 1. The diagram illustrating concepts from Set Theory, explaining the Domain (D), Codomain, Range (R), Image ( $\tilde{R}$ ), and Preimage.

is important to note that the Preimage is not the *inverse* of the image. Many texts use the notation  $f^{-1}$ , but we will use  $f^*$  to avoid confusion.<sup>1</sup> Table 1 and Figure 1 are included to help explain these concepts.

We will define a *Target*,  $\mathcal{T}$ , as the set of data we *intend* to model. Unfortunately, this distribution may be unobtainable and is often intractable. That is, we are unable to provide a formal description of the distribution. An example, which we will use in Chapter 4 and Chapter 5, is "the set of all possible human faces." We do not have a proper mathematical description this set, making it *intractable*, nor is it possible for us to completely sample from this set as it would require infinite time<sup>2</sup>. We instead collect a set of sample data  $\Omega \subseteq \mathcal{T}$ , which may be used to train the model (e.g. FFHQ [79] or CelebA [108]). It is important to

<sup>&</sup>lt;sup>1</sup>This notation is used for a pullback, which is a nearly identical concept.

 $<sup>^{2}</sup>$ This set would include all faces that were and all faces that will be.

note that we may not know how well  $\Omega$  approximates  $\mathcal{T}$ , especially when  $\mathcal{T}$  is intractable. Our model processes data from the  $\Omega$  to generate output,  $\mathcal{O}$ . When performing Classification/Discrimination tasks, our output may be a (or a list of) label(s) but in generative tasks we instead seek to approximate the target distribution,  $\tilde{\mathcal{T}}$ . We should keep this model in mind when evaluating our work, so we can best understand what our models can and cannot do. Our data are discrete and sampled from the distributions we are trying to approximate, and great care must be taken to determine what is in our distribution or not.

### 2.2 Scale Is Not All You Need

In March of 2019 Richard Sutton wrote a short article titled The Bitter Lesson [142]. This article had a large impact on the machine learning community. Sutton makes the argument that methods based predominantly on leveraging human knowledge are ill-founded and that our historical progress has shown us that focusing on search has resulted in success. Sutton acknowledges the benefits of leveraging human knowledge as well as how in practice this can often be constraining, preventing our machines from leveraging more general computation. Either through misinterpretation by Sutton or through readers, a popular belief rose through the community: "Scale Is All You Need". This notion need be addressed, for if the belief is true to face then the only work need be done is that of scaling compute and data gathering. Some will interpret this in that scaling is sufficient, and that there may be more efficient methods, but we will show that scaling alone is insufficient. We do not disagree that scale is a necessary and essential component, but that it alone is insufficient to both explain recent progress as well as provide direction for further advancement. These claims let critical conditions remain implicit, assuming shared assumptions among readers. These

subtle details are consequential to generating efficient machine learning models, as understanding what data increases performance allows us to also better design algorithms to maximally incorporate information.

Two aspects of scaling must be addressed: that of scaling data (Chapter 2.2.1) and that of scaling compute (Chapter 2.2.2).

2.2.1 Scaling Data. Undeniably one of the reasons for major advances has resulted with scaling of data. There is a simple argument that may suggest scaling data will be sufficient. We need to look at this to understand where it works and doesn't.

Our goal in machine learning is to learn some distribution, which we will call our *Target Distribution*,  $\mathcal{T}$ . If we uniformly and randomly sample from our target distribution, one can conclude that with scale we will also increase our covering of the distribution. We may view this another way: if we select some arbitrary point in our target distribution, as we continue to sample then the distance between it and some data point,  $d_i$ , in our set of sampled points will decrease.  $\exists \varepsilon \in \mathbb{R} \text{ s.t. } ||d_i - d_j||_p^p < \varepsilon | \forall d_i, d_j \in \mathcal{T}$ . Where  $|| \cdot ||_p^p$  represents an arbitrary  $L_p$  distance.

We can refine this more generally, which will better help us as we increase complexity. We can partition our distribution  $\mathcal{T}$  into disjoint continuous partitions  $\{P_0, \ldots, P_n\}$ . That is:  $P_i \cap P_j = \{\emptyset\} \mid \forall i \neq j \text{ and } \bigcup P_i = \mathcal{T}$ . We can reach a similar natural conclusion: as the number of samples increases, the probability that there does not exist a sample belonging to partition  $P_i$  goes to zero.  $\lim_{n\to \inf} Pr(s \in$  $P_i) = 0$ .

This generalization helps us in two ways. Our partitions can be of arbitrary size and shape, allowing us to use them as abstractions, such as semantic representations.<sup>3</sup> Where a semantic representation may represent categories of our data. For example, if our model is generating human faces we may consider hair color as a semantic representation. This formulation can also be repeated for each partition, which allows us to extend the notion to a more realistic setting where data is discrete (i.e. discretization).

While this logic may be natural, it relies on assumptions that are not true in practice. Notably, it assumes that both the data is independent and identically distributed (i.i.d) and that our sampling process is unbiased. These assumptions are not representative of the real world data, nor of the way in which we sample. In practice, as we increase the number of samples we increase the diversity of our data. This diversity, or variance, in data has a large impact on our models' ability to generalize. We will see in Chapter 3 that introducing data augmentation to our models results in a significant improvement in their performance. These augmentations create additional variance in the data and help the model to not overfit.

Scaling of data in the way we typically gather data can grow the variance to a greater degree than our typical data augmentation methods can. But this represents a fundamental limitation as well. We cannot scale infinitely, and as we gather more data inevitably we turn from increasing variance to contracting the variance. There are only so many unique things in the world. To understand this, we may think about randomly throwing a dart at a dartboard. As we start, every new dart likely lands with a high distance from one another. But as we continue we increase our coverage over the dartboard and our new darts land close to an

 $<sup>^{3}</sup>$ We still need to maintain care to ensure our semantic representations are disjoint. This does not allow us to pick arbitrary semantic representations.

existing dart. This variance contraction means that we cannot rely on scaling data indefinitely.

Additionally, an extra challenge comes from scaling data. Once the data is so large, we are unable to properly investigate it. This means we will not be able to properly verify that our model is not trained on the data it is being tested on. In this manner, we want to use the minimum amount of data required to train our models, to reduce our burden of verification.

In practice, our data is heavy tailed, with many samples being underrepresented. Ultimately, despite high amounts of data, subsets exists in a low data regime. Our models may benefit from shared similarities, via a superposition of representations, but we are still motivated to develop models which work better when data is sparse. By better understanding how to make our models efficiently learn in limited data regimes we hope to build techniques that allow our larger models to efficiently model data that is within the long tail.

2.2.2 Model Size. We face similar complexities when it comes to the scaling of our models. Inherently our model parameters change our loss landscape [100], with larger models providing more ways for data to be disentangled [95, 45, 29]. It can be shown that different by using different loss functions that we may even trick ourselves into believing our models have found emergent capabilities [159] when they may have not [133].

With increased model parameters our models are more likely to overfit our data, making it difficult to generalize. With such sizes in terms of data and parameters it becomes difficult to distinguish between our models memorizing the data vs modeling the data. In practice, we benefit from physical limitations,

which also puts pressure on making our models as small as possible. The larger our models are, the more expensive they are to run.

### 2.3 The Foundations That Shape Us

To cost effectively train our models we want them to both be parameter efficient and data efficient. With too much data, we are may spend disproportionate times loading from disk and simply ingesting the data. With too many parameters, we must split, or shard, our model across large supercomputing infrastructures.

Key to Sutton's Bitter Lesson was that models should be powerful and flexible. With our trend in scaling, we have also seen tremendous improvements in the algorithms that we use, such as the advent of the transformer [150]. Scale cannot be enough to explain our progress, as we have found that as research progresses, many smaller models end up significantly outperforming larger models [66], and this thesis is further demonstration of that.

These algorithms may be referred to as our neural architectures, as we build them to work together. In the following sections we introduce some of the key architectures that will be used throughout this work. There exist far more frameworks methods [46, 112] and we focus only on what are used herein.

2.3.1 Transformers. The transformer model has become the backbone of modern machine learning models. This is due to its high flexibility, being able to form a relationship between all elements it attends to. Unlike many other architectures, the transformer is not limited by the locality of the data, with it being able to discover relationships between data regardless of its position in a sequence. This greater flexibility comes at an increased computational complexity,

but enables the model to form relationships that could not be efficiently formed through other previous architectures.

These models are fairly simple in construction, having two main components: attention [43, 114, 150], and a feed-forward layer.



*Figure 2.* The Transformer model architecture from Vaswani et. al. Diagram depicts dot-product self attention.

In Figure 2 depicts part of the transformer model from Vaswani et al.'s work, showing the dot-product self attention (DPSA) variant, which is used throughout this work. The figure depicts a "post-norm" configuration, with the normalization layers appearing after the attention and feed-forward units, but modern configurations usually use "pre-norm" due to increased stability. The core of the transformer model is attention, defined as:

Softmax 
$$\left(\frac{QK^T}{\sqrt{d_k}}\right)V$$
 (2.1)

Where Q, K, and V represent queries, keys, and values, respectively. These are learnable parameters, most usually parameterized by a single layer feed-forward network. In the DPSA configuration, these networks share the same input.  $d_k$  in



Figure 3. Taxonomy of Generative Models, based on Goodfellow's Taxonomy [40]

this equation is a softmax temperature scale, which is the inverse square root of the embedding dimension (a user defined hyperparameter). The queries and keys are multiplied together, learning a similarity matrix. The softmax of this is then referred to as the "score", as its values are defined by a probability distribution. The value tensor is then weighted by the score, defining our attention function.

Commonly, this configuration is done in a "multi-headed" manner. Instead of performing a single attention we may instead project our Q,K, and V tensors into an embedding so that we may process multiple attention calculations in parallel. The conclusion of the attention mechanism concatenates these tensors. This tends to make our models more efficient as each head is independent and can learn unique representations, as we will see in Chapter 4.

The transformer model typically includes the usage of positional encoding, which adds extra data to the model to indicate the position of tokens, or data, in a sequence.

**2.3.2** Adversarial Generation. Generative Adversarial Networks are a form of generative models introduced by Goodfellow et al. [41] which first enabled the generation of high quality synthetic imagery. Not necessarily restricted

to image synthesis, these models enable unsupervised learning by simultaneously training two models at once. If our goal is to train an image generator, we both a model to generate images and a model to discriminate real and fake images. The discriminator model requires labeled data, but only the binary distinction of real data or synthesized data. These models then competitively train, being able to play a minimax game, which often leads to high quality generation.

$$\min_{G} \max_{D} \mathbb{E}_{x \sim p_{\text{data}}} \left[ \log D(x) \right] + \mathbb{E}_{z \sim p_z} \left[ \log \left( 1 - D \left( G(z) \right) \right) \right].$$
(2.2)

G learns a differentiable map  $z \mapsto x$  that pushes forward a simple prior (usually spherical Gaussian) toward the data manifold, while D learns to spot discrepancies. While these models have shown great success and pushed the bounds of what is possible, they are not without problems. Training is notoriously unstable—mode collapse, vanishing gradients, and catastrophic forgetting are common.

In addition, many generative models have greatly increased in size. These size increases have resulted in more impressive images but also become harder to train, costlier to train, and become slower in throughput. There then must be a trade-off of capabilities and performance, depending on the applications. In Chapter 4 we will use a GAN to demonstrate an improved variant of an attention mechanism, improving throughput and quality while decreasing the total number of parameters.

**2.3.3** Normalizing Flows. Normalising flows provide *exact* loglikelihoods by composing a sequence of bijective, differentiable transforms  $f = f_1 \cdots f_k$ :

$$p_x(\mathbf{x}) = p_u(\mathbf{u}) \left| \det J_f(\mathbf{u}) \right|^{-1}$$
(2.3)

Here  $p_u$  is a tractable base distribution and det  $J_f$  denotes the Jacobian determinant. The Jacobian determinant allows for a change of variable, allowing data from one distribution ( $u \in U$ ) to be expressed in another coordinate system ( $x \in X$ ). A simplified example that many readers may be more familiar with is the change of coordinates from a Cartesian space into Polar coordinates

$$J = \det \frac{\partial(x, y)}{\partial(r, \theta)}$$

$$= \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{bmatrix}$$

$$= r \cos^2 \theta + r \sin^2 \theta$$
(2.4)

Given the Jacobian determinant it becomes trivial to convert from a Cartesian coordinate to Polar by the equation:  $\iint f(x, y) dx dy = \iint f(r, \theta) r dr d\theta$ 

= r

This idea extends greatly, with far more complex formulations of coordinate transforms. The importance of these transforms is that they generate an isomorphic mapping from one space to another, where every element in one coordinate precisely maps to a unique element in the other. Through the composition of these transformations we can then define a nice tractable distribution, such as a Gaussian, and learn a coordinate transform that maps our data. This, in effect, allows us to turn our intractable distribution into a tractable one. We should remain careful, as there are still some pitfalls and our distribution is still only an approximation. What makes this different from Approximate Density models, such as VAEs and Diffusion models, is that those models do not generate isomorphic functions. Like flows, they are able to generate a probability density function, making them "explicit" (Figure 3), but these models are by nature lossy. Where Flows are bijective, diffusion and VAEs are not.



Figure~4. Visual representation of injections, surjections , and bijections. Source: Wolfram~Mathworld

The two most common forms of Normalizing Flows, which are also used within this thesis, are:

Affine coupling flows. : Partition input x into two units,  $(x_0, x_1)$ , such that  $f(x_0, x_1) = (x_0, x_1 \odot e^{s(x_0)+t(x_1)})$ , which make computationally inexpensive triangular Jacobians (e.g. RealNVP [26], Glow [86]).

Autoregressive flows. : Parameterise each dimension conditioned on previous ones, yielding a composition of triangular Jacobians (MAF/IAF [121, 88]).

Unlike transformer models, the architecture to Normalizing flows are highly restrictive. These restrictions come with the benefits of increased interpretability, but at the cost of additional computation and less flexibility. Where to make these trade-offs is difficult but it remains a challenge in determining the capability of these models. Unfortunately these models tend to be greatly under studied, with
only a handful of models having been trained with > 100M parameters, which is fairly small by modern standards.

### 2.4 The Tyranny of Measurements

As a final note, we must be ever vigilant of the metrics that we use. Qualitative metrics are a critical part of the scientific method, evidencing our hypotheses and theories. Yet, metrics are only guides, proxying the things we wish to measure. We must stress the importance of this distinction as it is necessary to properly evaluate our models and interpret what they are doing. Within this thesis several of our works face the challenges of interpreting our metrics and the absence of them. In Chapters 4 and 5 perform image synthesis tasks, where our models create new data that is representative of what they trained on. There are no metrics that properly convey what is a good image or not.

For example, a common metric is for measuring the capabilities of image models is the Fréchet Inception Distance (FID) [60]. This metric was shown to correlate with human judgement of image quality, but was developed when image quality was much worse. For comparison, the paper that introduced FID demonstrated models with an FID around 12.5 on the CelebA dataset, while the current state of the art is 3.15 [146]. These correlations are helped improve the state of art systems, but not being perfectly aligned with an actual measurement of realism the discrepancies grow as our models improve.

The rapid success of machine learning is double edged sword. Our approximations that helped us make our progress may no longer be sufficient. With all metrics, we must constantly check their alignment, to ensure that we are progressing in the directions we intend. This is quite similar to the gradient decent process we use in machine learning, where early on we may make large

improvements with highly suboptimal steps towards the optima. Yet, as our model becomes better, we tend to make smaller steps to ensure we are progressing in the right direction.

## CHAPTER III

# ESCAPING THE BIG DATA PARADIGM

The first principle is that you must not fool yourself and you are the easiest person to fool.

Richard Feynman

**Nota Bene:** This chapter is based on the previously published co-authored work *Escaping the Big Data Paradigm with Compact Transformers* [51] and the associated blog post published through PyTorch's Medium page [155].

- Ali Hassani and Steven Walton are joint primary authors of this work.
   Together they wrote the majority of the code, performed the majority of experiments and writing of the paper. The majority of code was written during pair-programming sessions between the two.
- Steven Walton worked a bit more on designing the experiments and developing the theory, ensuring claims were thoroughly evidenced and finding relevant literature.
- Ali Hassani worked a bit more on code and launching experiments, increasing code quality and ensuring experiments were launched effectively, maximizing machine utilization.
- Nikhil Shah helped manage launching experiments and contributed to the paper writing.
- Abulikemu Abuduweili provided code and feedback for the NLP experiments.

– Humphrey Shi was the advisor, contributing overall guidance on the research as well as funding for the work. Humphrey also contributed to the writing of the paper and ensuring research stayed on track.

Critical to any data analysis is the preparation of that data. The ways in which we encode our data has significant impacts on the way that data is processed. It is not sufficient to simply apply the right modeling tools to the data, but one first needs to ensure that the data is properly processed. In machine learning systems, this processing is typically done by both man and machine. The ingress and egress of data is critical, and will influence what structures in the data can ultimately be recovered.

In this chapter we introduce the work *Escaping the Big Data Paradigm with Compact Transformers* [51]. This work demonstrates that Vision Transformers do not need large amounts of data to be performant, instead being able to be trained from scratch and be effective in limited data regimes. Our results run counter to conventional wisdom around scaling, demonstrating that scale may *decrease* performance, rather than increase. On small datasets, like CIFAR-10, our small models are able to achieve comparable performance to much larger ViT models that also have large pretraining. On medium datasets, like ImageNet, we are able to outperform ViTs of comparable sizes, and achieve accuracies only slightly lower than large models with large pretraining.

### 3.1 Vision Transformers

With Vaswani et al.'s[150] demonstration of a dot-product self-attention based transformer architectures in language, there were several attempts to integrate them into vision models [6, 129, 69, 68]. Cordonnier et al. [16] first showed that by downsampling and adding a positional encoding layer, that a



Figure 5. Architectural design of Compact Transformers

Bert [24] style Transformer architecture could learn convolutional filters, given a sufficient number of attention heads. Unfortunately, these researchers were memory bound and were using 2 × 2 invertible down-sampling. Dosovitskiy et al. [28] improved upon this work, claiming "An Image is Worth 16×16 Words", introducing the Vision Transformer. Instead of using a 2 × 2 down-sampling, they used larger 16 × 16 patches, giving the paper it's name. Additionally, Dosovitskiy et al. significantly increased scaled both data and compute. While Cordonnier et al.'s network was ≈12M parameters, Dosovitskiy et al. used 3 networks, 86M, 307M, and 632M. While Cordonnier et al. exclusively trained on CIFAR-10 and CIFAR-100 [147], Dosovitskiy et al. performed pretraining with the proprietary JFT-300M dataset [141], ImageNet-21k, and ImageNet-1k [23]. Their work showed that with large-data pretraining that one could outperform ResNet [55] trained models, although later work showed that by training ResNets with modern training procedures that classification accuracy becomes similar [161]. Dosovitskiy et al.

patch embedding and fine-tuning at higher resolutions than pretraining [148, 89]. Their results suggested that only through large pretraining and large models could ResNets be beat.

Dosovitskiy et al.'s work made an important claim: Transformers lack some of the inductive biases inherent to CNNs, such as translational equivariance and locality, and therefore <u>do not generalize well when trained on insufficient amounts</u> <u>of data.</u> However, the picture changes if the models are trained on larger datasets (14M-300M images). We find that large scale training trumps inductive biases.

If this problem could not be resolved then this would greatly limit research contributions by labs without large compute infrastructures <sup>1</sup>. The community was quick to challenge Dosovitskiy et al.'s claim.

Touvron et al.'s Training Data-Efficient Image Transformers & Distillation Through Attention [149], quickly followed in an attempt to address the claim, introducing the DeIT model. In particular, they criticized the large pretraining and sought to counter the claim that transformers do not generalize when trained on insufficient amounts of data. Their work similarly uses 3 models for training, but are a tiny (5M parameters), small (22M parameters), and base (86M parameters). The ViT was modified to introduce a knowledge transfer<sup>2</sup> token, and the training scheme was modified to include distillation from a pretrained convolutional based network. For their convolutional network they selected a RegNetY-16GF [127] network (84M parameters) as the default teacher network.

<sup>&</sup>lt;sup>1</sup>Often called "GPU Poor"

<sup>&</sup>lt;sup>2</sup>We use the phrasing *knowledge transfer* instead of *distillation* for increased clarity; as the "teacher" network having fewer parameters than the "student" network

#### **3.2** Data Efficient Vision Transformers

While we recognize the importance of these works we believe alternative conclusions are possible. The ViT results could be explained by several alternative hypotheses, including the size of the network and through training techniques. DeIT's results showed that part of the claim must be false, as even smaller models could achieve better performance, but this relied upon inheriting the local inductive biases transferred by a CNN rather than learning them themselves, which Cordonnier et al. had demonstrated is possible. The critical question remained: *Can transformer models, be trained to outperform ResNets when model size and data were held equal*? Both works suggested that the answer was no. On the other hand, Transformers are universal approximators and Cordonnier et al.'s work suggests there's no reason one should believe this data threshold requirement. Additionally, we believed ViT and DeIT were rejecting valuable information by only passing a slice of the transformer's outputs to the classification sub-network.

In an effort to resolve this, we proposed three hypothesis:

- Non-overlapping image patches bias the transformer networks due to information loss at the boundaries.
- A learned transformation to map the transformer's outputs to the classification sub-network will improve performance.
- Transformer networks rely more on data variance than data quantity.

**3.2.1 Convolutional Tokenizer.** The first hypothesis was believed due to the discussion in the background section (Chapter 2.2.1), where these models were gaining more benefit from data variance than data quantity. While diversity is a common side-effect of scaling, it is a distinct phenomena. The second

was inspired by subword tokenization that is commonly used by many language models [35, 135, 24, 150] and experience with computational modeling. The belief here is that by using non-overlapping patches we weaken the network's ability to incorporate information along the boundaries of the images. Such boundary conditions often plague computational models, requiring ghost cells and other forms of boundary communication techniques to de-bias calculations.



Figure 6. A comparison of the Vision Transformer variants used throughout this study. On the left is the batching and embedding process (tokenization). On the right is the main neural architecture. The Transformer Encoder blocks and Linear Layers (classification sub-network) are identical for all models. CVT follows ViT, removing the class token and introducing SeqPool. In CCT we modify the tokenization process, building from CVT.

ViT uses a simple patch and embedding procedure, where the image is evenly divided into patches. This in illustrated on the right half of Figure 5, under Compact Vision Transformer (CVT). The process is to do a Group Normalization, ReLU, MaxPool, patch, and embed. Notably, Dosovitskiy et al. did the patching and embedding simultaniously with a convolution, matching strides to the kernel size <sup>3</sup>. This same strategy is used for our ViT-Lite and Compact Vision Transformer (CVT) models. This procedure can be seen in Figure 6.

We propose removing the restriction of making the convolutional kernels and strides match, allowing these patches to overlap. This would have an additional beneficial side-effect, allowing for better generalizability, by not requiring images to be integer multiples of the kernel size. This extends the embedding process to allow for arbitrary image sizes and aspect ratios. Additionally, we remove the Group Normalization layer from the ViT model, finding it unnecessary. Given an image or feature map  $x \in \mathbb{R}^{H \times W \times C}$  we can process our image as follows:

$$\mathbf{x}_0 = \operatorname{MaxPool}\left(\operatorname{ReLU}\left(\operatorname{Conv2d}(\mathbf{x})\right)\right) \tag{3.1}$$

Our convolution has a number of filters equal to the embedding dimension of the transformer backbone, and both our convolution and pooling operations allow for overlapping, which can introduce local inductive biases.

**3.2.2 SeqPool.** In order to map the sequential output of a transformer to the linear representation required by a feed-forward classification network ViT uses a singular class index, or token, similar to language models like BERT [24]. This class token is learnable and then allows for the output of the transformer to be sliced along the learned index. Unfortunately, this underutilizes the relationships learned by the transformer encoding layers. This method makes the assumption that the transformer encoder can, and will, decouple the relationships of the training data. This disentanglement is the main task of the classification subnetwork, thus forcing our Transformer to also perform this likely leads to underutilization and overly constrains the encoding layers.

<sup>&</sup>lt;sup>3</sup>This can be seen at github.com/google-research/vit\_jax/models\_vit.py:264

We propose SeqPool, an attention inspired pooling method. The method is based on the assumption that the transformer encoder's output sequence contains information relevant to classification. While this method is more computationally complex than slicing, it can reduce overall computation due to removal of an additional token that must be processed by the entirety of the network. We use a network to generate a contraction  $S : \mathbb{R}^{b \times n \times d} \mapsto \mathbb{R}^{b \times d}$ , which then is an appropriate shape to be processed by the classification sub-network.

Softmax 
$$(g(\mathbf{x}_L)^T) \mathbf{x}_L$$
 (3.2)

Unlike dot-product attention we are not using keys, queries, and values, but instead learning a weighting of our sequence. Our function g is a single feed-forward layer mapping  $g : \mathbb{R}^{b \times n \times d} \mapsto \mathbb{R}^{b \times d}$ . We score this contraction and weight our original input producing the flattened output. This process can be seen as a learnable submersion, incorporating across sequential data better, seemingly allowing us to take advantage of neuron polysemanticity [134, 62] and superpositionality [31].

#### 3.3 Experiments

We perform a variety of experiments in order to test our research hypotheses. We name our models similar to those of ViT, using the more explicit format:

$$[model] - [N \ layers] / [patch \ size] \times [N \ convolutions].$$
(3.3)

The original ViT-B/16 model has 12 transformer encoder layers and a patch size of 16, where we make the number of layers explicit: ViT-12/16. We use this convention for all ViT and CVT models, dropping the number of convolutions. For CCT we specify the number of convolutions, even if only one. This section is organized to first provide details of our experiments and resources. Chapter 3.3.4

contains our main results, demonstrating high performance Vision Transformer models on small datasets. Chapter 3.3.5 includes details of our ablations, detailing the effects of our changes to the architecture. Chapter 3.3.6 provides a scaling study, investigating the scaling of both data and parameters. Finally, Chapter 3.3.7 includes our NLP experiments, to demonstrate that these results generalize to language models.

**3.3.1 Datasets.** Our primary focus is on small datasets, where we train on CIFAR-10, CIFAR-100 [147], MNIST [94], and Fashion-MNIST [164]. We also test our models on Oxford Flowers-102 [120] <sup>4</sup> for generalizability due to its large similarity between classes and high variance for intra-class similarity. We also use ImageNet [23] to test the scalability of our approach, allowing for more direct comparisons to ViT and DeiT. We also test our approach in Natural Language Processing, using AG-News [172], TREC [101], SST [138], IMDb [116], and DBpedia [2].

**3.3.2 Computational Resources.** For most experiments we use a machine with an Intel(R) Core(TM) i9-9960X CPU @ 3.10GHz and 4 NVIDIA RTX 2080Tis (11GB). The exception was the CPU test which was performed with an AMD Ryzen 9 5900X, where we found you could reach 90% accuracy in under 30 minutes. Our ImageNet experiments were performed on a single machine with either 2 AMD EPYC) 7662s and 8 NVIDIA RTX A6000 (48GB) or 2 AMD EPYC 7713s and 8 NVIDIA A100s (80GB).

**3.3.3 Hyperparameters.** We used the Pytorch Image Models library (timm) [160] to train our models for all image experiments. Our augmentations include CutMix [167], Mixup [170], RandAugment [19], and Random Erasing [178].

 $<sup>^4\</sup>mathrm{We}$  used the dataset from Kaggle, which has a different data split than torchvision. Further discussion is provided later.

Model	# Layers	# Heads	Ratio	Dim
ViT-Lite-6	6	4	2	256
ViT-Lite-7	7	4	2	256
CVT-6	6	4	2	256
CVT-7	7	4	2	256
CCT-2	2	2	1	128
CCT-4	4	2	1	128
CCT-6	6	4	2	256
CCT-7	7	4	2	256
CCT-14	14	6	3	384

Model	# Layers	# Convs	Kernel	Stride
ViT-Lite-7/8	7	1	8×8	8×8
ViT-Lite- $7/4$	7	1	$4 \times 4$	$4 \times 4$
CVT-7/8	7	1	$8 \times 8$	$8 \times 8$
CVT-7/4	7	1	$4 \times 4$	$4 \times 4$
CCT-2/3x2	2	2	$3 \times 3$	$1 \times 1$
CCT-7/3x1	7	1	$3 \times 3$	$1 \times 1$
CCT-7/7x2	7	2	$7 \times 7$	$2 \times 2$

(a) Transformer Hyperparameters

(b) Tokenizer Hyperparameters

Table 2. Hyperparameters used in different model configurations. Table 2a (left) shows transformer hyperparameters while Table 2b (right) shows those for tokenizers.

We performed hyperparameter sweeps for our differing methods and report the best results we achieved. All hyperparameter experiments were trained for 300 epochs, use a learning rate of  $5 \times 10^{-4}$ , a cosine learning rate scheduler, and weighted Adam optimizer ( $\beta = [0.9, 0.999]$ )[85, 177]. For CNN models we found that some performed best with AdamW while others were more performant with SGD with momentum 0.9. For reproducibility we release our checkpoints corresponding to the reported numbers and YAML files corresponding to our experimental settings. These can be found on our public GitHub repository <sup>5</sup>.

3.3.4 Transformers On Small Datasets. The main results of this work are the success of training Vision Transformers on small datasets. We follow the aforementioned training procedure, except our best model we further train as it did not appear to be saturated. Our full results can be read in Table 3, where we show a comparison of various ResNet based models, ViTs, CVT, and CCT, testing our small vision datasets with comparisons of model size and required compute. Notably, on CIFAR-10, we are able to achieve a 10% improvement over similarly

<sup>&</sup>lt;sup>5</sup>https://github.com/SHI-Labs/Compact-Transformers

Model	CIFAR-10	CIFAR-100	FashionMNIST	MNIST	# Params	FLOPs
Convolutional Net	tworks (Desig	ned for Image	Net)			
ResNet18	90.27%	66.46%	94.78%	99.80%	11.18 M	0.04 G
ResNet34	90.51%	66.84%	94.78%	99.77%	21.29 M	$0.08~\mathrm{G}$
ResNet50	91.63%	68.27%	94.99%	99.79%	$23.53 \mathrm{M}$	$0.08~{\rm G}$
MobileNetV2/0.5	84.78%	56.32%	93.93%	99.70%	0.70 M	$< 0.01 \ \mathrm{G}$
MobileNetV2/1.0	89.07%	63.69%	94.85%	99.75%	2.24 M	$0.01~\mathrm{G}$
MobileNetV2/1.25	90.60%	65.24%	95.05%	99.77%	3.47 M	$0.01~\mathrm{G}$
MobileNetV2/2.0	91.02%	67.44%	95.26%	99.75%	8.72 M	0.02 G
Convolutional Net	tworks (Desig	ned for CIFAI	?)			
ResNet56[56]	94.63%	74.81%	95.25%	99.27%	0.85 M	$0.13~\mathrm{G}$
ResNet110[56]	95.08%	76.63%	95.32%	99.28%	1.73 M	$0.26~\mathrm{G}$
ResNet164-v1[57]	94.07%	74.84%	_	_	1.70 M	$0.26~\mathrm{G}$
ResNet164-v2[57]	94.54%	75.67%	_	_	1.70 M	$0.26~\mathrm{G}$
ResNet1k-v1[57]	92.39%	72.18%	_	_	$10.33 { m M}$	$1.55~\mathrm{G}$
ResNet1k-v2[57]	95.08%	77.29%	_	_	$10.33 { m M}$	$1.55~\mathrm{G}$
$ResNet1k-v2^{\star}[57]$	95.38%	_	_	_	$10.33 { m M}$	$1.55~\mathrm{G}$
Proxyless-G[12]	97.92%	—	—	—	$5.7 \mathrm{M}$	-
Vision Transform	ers					
ViT-12/16	83.04%	57.97%	93.61%	99.63%	85.63 M	0.43 G
ViT-Lite-7/16	78.45%	52.87%	93.24%	99.68%	3.89 M	$0.02~{ m G}$
ViT-Lite-6/16	78.12%	52.68%	93.09%	99.66%	$3.36 \mathrm{M}$	$0.02~{\rm G}$
ViT-Lite-7/8	89.10%	67.27%	94.49%	99.69%	3.74 M	0.06 G
ViT-Lite-6/8	88.29%	66.40%	94.36%	99.73%	$3.22 \mathrm{~M}$	$0.06~{\rm G}$
ViT-Lite-7/4	93.57%	73.94%	95.16%	99.77%	3.72 M	0.26 G
ViT-Lite-6/4	93.08%	73.33%	95.14%	99.74%	$3.19 \ { m M}$	$0.22~\mathrm{G}$
Compact Vision	Transformers					
CVT-7/8	89.79%	70.11%	94.50%	99.70%	3.74 M	0.06 G
CVT-6/8	89.50%	68.80%	94.53%	99.74%	3.21 M	$0.05~{\rm G}$
CVT-7/4	94.01%	76.49%	95.32%	99.76%	3.72 M	0.25 G
CVT-6/4	93.60%	74.23%	95.00%	99.75%	3.19 M	$0.22~\mathrm{G}$
Compact Convolu	tional Transf	ormers				
$CCT-2/3 \times 2$	89.75%	66.93%	94.08%	99.70%	<b>0.28</b> M	0.04 G
$CCT-4/3 \times 2$	91.97%	71.51%	94.74%	99.73%	0.48 M	0.05 G
$CCT-6/3 \times 2$	94.43%	77.14%	95.34%	99.75%	3.33 M	$0.25~\mathrm{G}$
$CCT-7/3 \times 2$	95.04%	77.72%	95.16%	99.76%	$3.85 \mathrm{M}$	0.29 G
CCT-6/3×1	95.70%	79.40%	95.41%	99.79%	3.23 M	1.02 G
$CCT-7/3 \times 1$	96.53%	80.92%	<b>95.56</b> %	<b>99.82</b> %	3.76 M	$1.19~\mathrm{G}$
$CCT-7/3 \times 1^*$	98.00%	82.72%	_	_	3.76 M	$1.19~\mathrm{G}$

Table 3. Comparisons of various models when trained on small datasets.  $\star$  was trained for longer, see Table 4 for additional details. Our 3.76M parameter CCT model is about to outperform both ResNets and ViTs across all datasets, with longer training only being necessary to outperform the 5.7M Proxyless-G model on CIFAR-10.

sized ViT-Lite models (ViT-Lite-7/8) and an 18% improvement over the ViT-12/16 (ViT-B/16) model while our model has a 95.6% reduction in the number of parameters. Our best model only contains a single convolutional layer within the embedding process, meaning that the transformer architecture is performing the main computation, achieving an accuracy of 98% while using only 3.76M parameters. This result is only slightly less than Vaswani et al.'s much larger models that include JFT-300M or ImageNet-21k pretraining and outperforms VIT-12/32, ViT-24/16, and ViT-24/32 when using ImageNet-1k pretraining and finetuning at 384 resolution (Table 5 of Vaswani et al.). We found that an increase in convolutions tended to harm model performance.

# Epochs	Pos. Emb.	CIFAR-10	CIFAR-100
300	Learnable	96.53%	80.92%
1500	Sinusoidal	97.48%	82.72%
5000	Sinusoidal	98.00%	82.87%

Table 4. Training of CCT- $7/3 \times 1$  with an increased number of epochs.

These results show that our CCT based model is able to outperform both standard Vision Transformers as well as ResNet models. We demonstrate that neither large scale pretraining nor knowledge distillation are needed to overcome the biases found in smaller scale data. Furthermore, we strongly suspect that the underlying issue is due to the tokenization process of overlapping patches. We include a comparison of Salient Maps [32, 137] in Figure 7, comparing visualizations on ImageNet. Saliency maps operate by looking visualizing the gradient accumulations across the network. We should take care as to fully interpret the semantic meaning of these maps, but the visualizations do clearly indicate how the original patching may be recovered in the standard ViT model

Model	CLS	# Conv	Conv Size	Aug	Tuning	C-10	C-100	# Params	FLOPS
"Large" Models ( $\approx 85$ M Parameters)									
ViT-12/16	CT	×	×	X	×	69.82%	40.57%	85.63 M	$0.43~\mathrm{G}$
ViT-12/16	CT	×	×	1	1	80.72%	56.73%	85.63 M	$0.43~\mathrm{G}$
CVT-12/16	SP	×	×		1	80.84%	58.05%	85.63 M	0.34 G
ViT-12/8	CT	X	×	1	1	90.24%	69.81%	85.20 M	$1.45~\mathrm{G}$
ViT-12/4	CT	X	×	1	$\checkmark$	94.07%	76.08%	85.12 M	$5.61~\mathrm{G}$
$CCT-12/7 \times 1$	SP	1	$7 \times 7$	1	1	93.72%	76.21%	85.20 M	$5.55~\mathrm{G}$
$CCT-12/3 \times 2$	SP	2	$3 \times 3$	1	1	94.50%	77.05%	85.53 M	$5.63~\mathrm{G}$
Small Mod	lels~(pprox	4M Parar	neters)						
ViT-Lite-7/16	CT	X	×	X	X	71.78%	41.59%	3.89 M	$0.02~\mathrm{G}$
ViT-Lite-7/8	CT	X	×	×	X	83.38%	55.69%	3.74 M	$0.06~{\rm G}$
ViT-Lite-7/4	CT	X	×	X	×	83.59%	58.43%	3.72 M	$0.26~\mathrm{G}$
CVT-7/16	SP	X	×	X	×	72.26%	42.37%	3.89 M	$0.02~\mathrm{G}$
CVT-7/8	SP	X	×	X	X	84.24%	55.49%	3.74 M	$0.06~\mathrm{G}$
CVT-7/8	SP	×	×	1	×	87.15%	63.14%	3.74 M	$0.06~\mathrm{G}$
CVT-7/4	SP	X	×	X	X	88.06%	62.06%	3.72 M	$0.25~\mathrm{G}$
CVT-7/4	SP	X	×	~	X	91.72%	69.59%	3.72 M	$0.25~\mathrm{G}$
CVT-7/4	SP	X	×			92.43%	73.01%	3.72 M	0.25 G
CVT-7/2	SP	X	X	X	X	84.80%	57.98%	3.76 M	1.18 G
$CCT-7/7 \times 1$	SP	1	$7 \times 7$	X	X	87.81%	62.83%	3.74 M	$0.26~\mathrm{G}$
$CCT-7/7 \times 1$	SP	1	$7 \times 7$	1	×	91.85%	69.43%	3.74 M	$0.26~\mathrm{G}$
CCT-7/7×1	CT	1	$7 \times 7$	1	1	91.67%	72.07%	3.74 M	$0.26~\mathrm{G}$
$CCT-7/7 \times 1$	SP	1	$7 \times 7$	1	1	92.29%	72.46%	3.74 M	0.26 G
$CCT-7/3 \times 2$	CT	2	$3 \times 3$	1	1	93.36%	74.77%	3.85 M	0.29 G
$CCT-7/3 \times 2$	SP	2	$3 \times 3$	1	1	93.65%	74.77%	3.85 M	$0.29~\mathrm{G}$
$CCT-7/3 \times 1$	SP	1	$3 \times 3$	1	1	94.47%	75.59%	3.76 M	1.19 G

while we have a much smoother representation in CCT, evidencing the first

research hypothesis.

Table 5. Ablation study, transforming ViT into CCT. We measure CIFAR validation accuracy across each modification as well as the number of model parameters and computation (MACs). All ViT models use a class token (CT), while CVT and CCT use SeqPool (SP). We report the number of convolutions used during embedding (# Conv), its kernel size, if we utilized image augmentation (Aug), and tuning.

**3.3.5** Ablations. We include ablations of our parameters to better understand the impact of our changes to the ViT model. In Table 5 we step through the process of converting our ViT model into CCT.In our table we denote if we used a class token (CT) or SeqPool (SP), the number of convolutions



*Figure 7.* Salient maps of ViT, CCT, and NAT based on ImageNet-1k. It can be seen that CCT removes the blocking artifacts from ViT. CCT sometimes creates displacement, but this is resolved by NAT (presented in Chapter 4).

user (overlapping patches), the kernel size, if image augmentations were used, and additional tuning. Our tuning includes dropout, attention dropout, and stochastic depth. We separate our models into two sections, with "Large" models, with approximately 85M parameters and small models, with approximately 4M parameters.

By directly comparing similar ViT-Lite models to our CVT models we can see the effect of our SeqPool method. In all cases we see that there is a minor performance improvement due to this, with a much lower effect with the large 85M parameter models. When comparing on CIFAR-10, models with 7 transformer encoders, a patch size of 16 we observe a 0.7% increase, 1.0% for a patch size of 8, a 5.3% increase with a patch size of 4. For the larger 12 transformer layer models with a patch size of 16 we only notice a 0.1% increase, but these models included tuning and augmentation, likely reducing the impact.

In the smaller models we see that the larger contribution to performance increase is due to decreased patch size. For ViT models, decreasing from a patch size of 16 to 8 increased model performance by 16.2%, but reducing to a patch size of 4 only accounted for an additional 0.3% increase. For CVT the decrease to a patch size of 8 showed a similar 16.6% improvement, but further reduction to a patch of 4 gave another 4.5% increase. Larger impacts can be observed when looking at CIFAR-100, except in the case of a patch size of 8 where SeqPool appears to have a slight negative (< 0.5%) impact. We see a +1.8%, -0.4%, and +6.2% difference for SeqPool, for our 3 patches. On ViT the patch reduction accounts for a 33.9% and 4.9% improvements while CVT shows 31.0% and 11.8% improvements. With decreased patch sizing the transformer appears to be able to overcome the primary issues presented by smaller training sets. Our SeqPool

method still demonstrates greater performance, especially as patch size decreases, showing greater network utilization.

The largest gains come from moving to CCT, which can also better take advantage of data augmentations, showing better capacity for generalization. For example, ViT-Lite-7/8, CVT-7/8, and CCT-7/3×1 all have 3.74M parameters, but their CIFAR-10 scores are 83.38%, 84.24%, and 87.81% respectively. Where CVT shows a 1% improvement, CCT shows 5.3%. We can see that CVT-7/8 improves to 87.15% (2.91%), while CCT-7/3×1 improves to 91.85% (4.04%) when introducing augmentation. We can also see in our CCT experiments that by removing SeqPool and reintroducing the class tokens that we drop performance by 0.62%, demonstrating that SeqPool does not account for these differences. A similar pattern can be found with larger models, though our comparisons are not as thorough. These results show that the overlapping patches and better extraction of data from the transformer architecture result in significant improvements, evidencing our first two hypotheses.

We also include a short study on Positional Embedding, in Table 6. Because our overlapping tokenization allowed us to debias some of the positional relationships within the data we test to find the importance of positional embedding. While ViT and CVT benefit strongly from positional embedding, CCT only gets minor benefits. This further demonstrates the bias introduced by patching in ViT. Some additional positional embedding comparisons can be found in Figure 9.

**3.3.6 Scaling Study.** While the previous results demonstrate that pretraining is unnecessary for Vision Transformers to be effective on small datasets,

Model	PE	CIFAR-10	CIFAR-100
Conventional Vision Tre	ansformers a	re more dependent on L	Positional Embedding
ViT-12/16	Learnable Sinusoidal None	$\begin{array}{c cccc} 69.82\% & (+3.11\%) \\ 69.03\% & (+2.32\%) \\ 66.71\% & (baseline) \end{array}$	$\begin{array}{rrr} 40.57\% & (+1.01\%) \\ 39.48\% & (-0.08\%) \\ 39.56\% & (baseline) \end{array}$
ViT-Lite-7/8	Learnable Sinusoidal None	83.38% (+7.25%) 80.86% (+4.73%) 76.13% ( baseline)	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$
CVT-7/8	Learnable Sinusoidal None	84.24% (+6.52%) 80.84% (+3.12%) 77.72% ( baseline)	55.49%       (+7.23%)         50.82%       (+2.56%)         48.26%       ( baseline)
Compact Convolutional	Transformer	s are less dependent on	Positional Embedding
CCT-7/7	Learnable Sinusoidal None	$\begin{array}{c cccc} 82.03\% & (+0.21\%) \\ 81.15\% & (-0.67\%) \\ 81.82\% & (baseline) \end{array}$	63.01% (+3.24%) 60.40% (+0.63%) 59.77% ( baseline)
CCT-7/3×2	Learnable Sinusoidal None	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{rrrr} 65.88\% & (+2.82\%) \\ 64.12\% & (+1.06\%) \\ 63.06\% & (baseline) \end{array}$
$CCT-7/3 \times 2^{\dagger}$	Learnable Sinusoidal None	$\begin{array}{ccc} 95.04\% & (+0.64\%) \\ 94.80\% & (+0.40\%) \\ 94.40\% & (baseline) \end{array}$	$\begin{array}{rrrr} 77.72\% & (+0.20\%) \\ 77.82\% & (+0.30\%) \\ 77.52\% & (baseline) \end{array}$
$CCT-7/3 \times 1^{\dagger}$	Learnable Sinusoidal None	96.53% (+0.29%) 96.27% (+0.03%) 96.24% ( baseline)	$\begin{array}{rrrr} 80.92\% & (+0.65\%) \\ 80.12\% & (-0.15\%) \\ 80.27\% & (baseline) \end{array}$
CCT-7/7×1-noSeqPool	Learnable Sinusoidal None	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{cccc} 62.61\% & (+3.31\%) \\ 61.04\% & (+1.74\%) \\ 59.30\% & (baseline) \end{array}$
CCT-7/3×2-noSeqPool	Learnable Sinusoidal None	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{cccc} 66.57\% & (+1.40\%) \\ 64.71\% & (-0.46\%) \\ 65.17\% & (baseline) \end{array}$

Table 6. Validation accuracy comparison comparing Positional Embedding method. Augmentations and training techniques such as Mixup and CutMix were turned off for these experiments to better highlight differences. The numbers reported are best out of 4 runs with random initializations. † denotes model trained with extra augmentation and hyperparameter tuning.

we need to understand the relationship of model size, data quantity, and data quality.

In order to address the *Scale is All You Need* arguments, we begin with the study of model size. Our main study of model size can be seen in our ablations (Table 5), where we observe that our larger 85.53M parameter model outperforms out 3.76M parameter model on both CIFAR-10 and CIFAR-100, showing very minor improvement on CIFAR-10 and a 2% increase on CIFAR-100.

This result runs counter to ViT, where the larger model has a performance decrease of up to 16.5% and 30.5%, respectively. When given additional augmentation, the larger ViT model is only able to outperform our largest ViT-Lite-7/16 model, which did not use tuning or augmented training. The two slightly smaller ViT-Lite models are still able to outperform this large model without the inclusion of additional augmentation or training, demonstrating that the smaller patch sizes play a more significant role, as discussed in Chapter 3.3.5. We believe that the smaller window sizes allow the transformer architecture to better integrate data across patches, learning convolutions similar to what Cordonnier et al. had shown, but further study is required to confirm or deny. The increase relationship between patch size and performance applies to both large and small ViTs, with the large ViT approaching the performance of CCT (surpassing ViT-Lite models) once the patch size is reduced to 4, yet still do not surpass the performance of small CCT models on CIFAR-10.

Under most configurations, CVT also shows a decrease in performance, again with improved performance primarily being attributed to the path size. Performance decreases at a patch size of 2, similar to Cordonnier et al.'s configuration, showing that the patches can be too small. In a way, this

demonstrates that scale plays an important role, but these trends run counter to the conventional wisdom. These results demonstrate the importance of the embedding process and that naïvely scaling architectures may instead hinder performance. Careful design of the neural architecture trumps scaling.

Model	Top-1	# Params	FLOPS	Epochs
ResNet50	77.15%	$25.55 { m M}$	4.15 G	120
ResNet50 (2021)	79.80%	$25.55~\mathrm{M}$	$4.15~\mathrm{G}$	300
ViT-S	79.85%	<b>22.05</b> M	$4.61~\mathrm{G}$	300
$CCT-14/7 \times 2$	80.67%	$22.36~{\rm M}$	$5.53~\mathrm{G}$	300
DeiT-S 🐔	81.16%	22.44M	<b>4.63</b> G	300
$ ext{CCT-14}/7{ imes}2$ $ extsf{m}$	81.34%	<b>22.36</b> M	$5.53~\mathrm{G}$	300

Table 7. ImageNet Top-1 validation accuracy comparison (no extra data or pretraining). Models with  $\Re$  denotes distillation and follow the knowledge distillation process as described in Touvron *et al* [149]. ResNet50 (2021) is reported from [161] which has the same training recipe as ours.

To study relation of data to model performance we perform multiple scaling studies. In order to complete our parameter scaling study, we test our model's performance on larger amounts of data, with ImageNet, but leave further large model and large data scaling studies to labs with resources similar to Vaswani et al. In Table 7 we train a 14 layer (22M param) model, and compare it to ViT-S and DeiT-S models from Touvron et al. [149]. It is difficult to get these models to be exactly the same parameter size, but our model is able to still outperform ViT on ImageNet-1k without any pretraining. We also compare to DeiT-S  $\Re$ , where our model is slightly smaller, following the same knowledge distillation process. Our model again shows improvements, demonstrating that our procedure does not produce negative effects with increased data scale.

Model	Resolution	Pretraining	Top-1	# Params	FLOPs
$CCT-14/7 \times 2$	224	-	97.19%	22.17 M	$18.63~\mathrm{G}$
DeiT-B	384	ImageNet-1k	98.80%	$86.25 {\rm M}$	$55.68 { m ~G}$
ViT- $L/16$	384	JFT-300M	99.74%	$304.71 {\rm \ M}$	$191.30 { m G}$
ViT- $H/14$	384	JFT-300M	99.68%	$661.00 {\rm M}$	$504.00~\mathrm{G}$
$CCT-14/7 \times 2$	384	ImageNet-1k	99.76%	$22.17~\mathrm{M}$	$18.63~\mathrm{G}$

Table 8. Flowers-102 Top-1 validation accuracy comparison. CCT outperforms other competitive models, having significantly fewer parameters and GFLOPs. This demonstrates the compactness on small datasets even with large images.

We also test our 22M parameter model on the Flowers-102 dataset, which is designed for high data variance and to test model generalizability. For this we are able to achieve an accuracy of over 97% without the use of any pretraining data or higher resolution tuning. These results can be found in Table 8. When using ImageNet-1k pretraining and including higher resolution tuning, following the procedure of DeiT, we are able to achieve state of the art results, outperforming models that included more than a magnitude more parameters and a more than a magnitude amount of pretraining data. It should be noted that we used the Flowers-102 dataset provided from Kaggle and that this uses a different data split than that which is included in the torchvision version<sup>6</sup>. This was brought to our attention through a GitHub issue,<sup>7</sup> where a user was unable to replicate our results. We retrained our CCT- $7/7 \times 2$  (4M params) and CCT- $14/7 \times 2$  models at 224 resolution and obtained 68.26% and 68.85% accuracy, respectively. When applying the same procedure to ViT-S/16 we obtained a result of 48.63%, only showing our model having better performance applied to this dataset.

 $<sup>^{6}\</sup>mathrm{The}$  torchvision dataset collection did not include Flowers-102 when initially trained.

 $<sup>^7\</sup>mathrm{A}$  wandb report showing training results can be found alongside the issue here: https://github.com/SHI-Labs/Compact-Transformers/issues/65



*Figure 8.* Comparison of models with restricted number of samples per class. At 10% models are trained on only 5000 images. Transformer based models demonstrate better scalability than ResNet based models.

Moving on to further test the scalability of our model with respect to data, we study the performance with respect the number of samples as well as the size of our images. In Figure 8 we restrict the number of samples in each class within the CIFAR-10 dataset. We compare the performance of CCT, ViT, ResNet18, and MobileNet when using only 10% of CIFAR-10 up to the full dataset. With only 10% of CIFAR-10, CCT is still able to achieve 77.7% accuracy, compared to ViT's 67.9%. CCT is able to outperform the other models regardless of the data reduction. ViT shows worse performance with data scaling, only beating ResNet18 when including 70% or more of the data.

Additionally, we include a short study where we modify the image sizes of CIFAR-10 to understand the dependence on resolution, found in Figure 9. With smaller resolution images models will likely be less able to rely upon local structures within the data, as they will be merged. When upscaling, we use a standard bicubic interpolation. In the first row of the graphs we train our models



Figure 9. Comparison of ViT-Lite and CCT accuracy on CIFAR-10 with differing image resolutions. In first row, models are trained from scratch. In second row, models are inference and trained on  $32 \times 32$  images. Fig. 9a is without positional embedding, Fig. 9c with sinusoidal positional embedding, and Fig. 9b with a learnable positional embedding. Inference with learnable positional embedding cannot be extended to larger images without modifying model parameters.

from scratch, allowing them to discover these associations. In the second row, we only run inference, testing our models' capacity to generalize to novel resolutions. We also show comparisons without positional embedding, with Sinusoidal Positional Embedding, and with Learnable Positional Embedding. In our inference results Learnable Positional Embedding models are unable to process larger resolution images than they were trained on, creating a significant limitation to this method. In all cases, except inference with Sinusoidal Positional Embedding, CCT is able to out perform ViT, further demonstrating data generalizability.

3.3.7 Natural Language Processing. Finally, we test our method on small natural language processing datasets. This network needs slight modification, incorporating GloVe [125] to provide word embeddings for our model. We do not train these embedding parameters and we do not include GloVe in our model parameter sizes, which is about 20M. To process the data we treat the text as single channel data, use an embedding dimension of 300, and a convolution kernel of size 1. We also perform masking in the typical manner.

By using CCT on these datasets we are able to achieve up to a 3% improvement when comparing to vanilla transformers. Additionally, our CCT model is able to do this while using fewer parameters. Our CCT models that are able to perform best have less than 1M parameters, making GloVe a significantly larger part of the network. We report a comparison of vanilla transformers, ViT, CVT, and CCT in Table 9

#### 3.4 Conclusion

In this work we saw the importance of properly embedding information into our machine learning models. We need to ensure that this is done properly or we may severely limit our model's capabilities. Even small seemingly trivial differences

Model	AGNews	TREC	$\mathbf{SST}$	IMDb	DBpedia	# Params		
Vanilla Transformer Encoders								
Transformer-2	93.28%	90.40%	67.15%	86.01%	98.63%	1.086 M		
Transformer-4	93.25%	92.54%	65.20%	85.98%	96.91%	$2.171 { m M}$		
Transformer-6	93.55%	92.78%	65.03%	85.87%	98.24%	$4.337~\mathrm{M}$		
Vision Transformers (ViT)								
ViT-Lite-2/1	93.02%	90.32%	67.66%	87.69%	98.99%	0.238 M		
ViT-Lite-2/2	92.20%	90.12%	64.44%	87.39%	98.88%	$0.276 {\rm ~M}$		
ViT-Lite-2/4	90.53%	90.00%	62.37%	86.17%	98.72%	$0.353 \mathrm{~M}$		
ViT-Lite-4/1	93.48%	91.50%	66.81%	87.38%	99.04%	$0.436 { m M}$		
ViT-Lite-4/2	92.06%	90.42%	63.75%	87.00%	98.92%	$0.474~{\rm M}$		
ViT-Lite-4/4	90.93%	89.30%	60.83%	86.71%	98.81%	$0.551 {\rm ~M}$		
ViT-Lite-6/1	93.07%	91.92%	64.95%	87.58%	99.02%	$3.237 {\rm M}$		
ViT-Lite- $6/2$	92.56%	89.38%	62.78%	86.96%	98.89%	$3.313 {\rm M}$		
ViT-Lite-6/4	91.12%	90.36%	60.97%	86.42%	98.72%	$3.467~{\rm M}$		
Compact Vision	Transformers	(CVT)						
CVT-2/1	93.24%	90.44%	67.88%	87.68%	98.98%	$0.238 { m M}$		
CVT-2/2	92.29%	89.96%	64.26%	86.99%	98.93%	$0.276 {\rm ~M}$		
CVT-2/4	91.10%	89.84%	62.22%	86.39%	98.75%	$0.353 \ \mathrm{M}$		
CVT-4/1	93.53%	92.58%	66.64%	87.27%	99.04%	$0.436 {\rm M}$		
CVT-4/2	92.35%	90.36%	63.90%	86.96%	98.93%	$0.474~{\rm M}$		
CVT-4/4	90.71%	90.14%	61.98%	86.77%	98.80%	$0.551 \ { m M}$		
CVT-6/1	93.38%	92.06%	65.94%	86.78%	99.02%	$3.237 {\rm M}$		
$\mathrm{CVT}$ -6/2	92.57%	91.14%	64.57%	86.61%	98.86%	$3.313 \mathrm{~M}$		
CVT-6/4	91.35%	91.66%	61.63%	86.13%	98.76%	3.467 M		
Compact Convolutional Transformers (CCT)								
CCT-2/1x1	93.40%	90.86%	68.76%	88.95%	99.01%	$0.238 { m M}$		
CCT-2/2x1	93.38%	91.86%	67.19%	89.13%	99.04%	$0.276 {\rm ~M}$		
CCT-2/4x1	93.80%	91.42%	64.47%	88.92%	99.04%	$0.353 \ \mathrm{M}$		
CCT-4/1x1	93.49%	91.84%	68.21%	88.71%	99.03%	$0.436 {\rm M}$		
CCT-4/2x1	93.30%	93.54%	66.42%	88.94%	99.05%	$0.474~{\rm M}$		
CCT-4/4x1	93.09%	93.20%	66.57%	88.86%	99.02%	$0.551 { m M}$		
CCT-6/1x1	93.73%	91.22%	66.59%	88.81%	98.99%	$3.237 \mathrm{\ M}$		
CCT-6/2x1	93.29%	92.10%	65.02%	88.74%	99.02%	3.313 M		
CCT-6/4x1	92.86%	92.96%	65.84%	88.68%	99.02%	$3.467 {\rm M}$		

Table 9. Top-1 validation accuracy on text classification datasets. The number of parameters does not include the word embedding layer, because we use pretrained word-embeddings and freeze those layers while training.

can have tremendous effects on these models, making it important to care when designing our neural architectures. If great care is not taken we will make the wrong conclusions and hinder our own progress.

While pretraining can help with model performance, when working with very large datasets it becomes difficult to deduplicate data, and works have shown that despite attempts to deduplicate these datasets may still be reduced upwards of 50% [1]. These duplications reduce model performance and generalizability, as they push the models to over attend to certain semantics. While reducing the requisite dataset size doesn't solve this problem, it certainly makes it a much more tractable problem. Given such results it makes it difficult to distinguish if large pretrained models are generalizing or simply memorizing data.

An important result of this work was the ability to achieve comparable performance while using orders of magnitude fewer parameters. While there are still a large number of parameters, having fewer decreases a model's ability to overfit. Smaller models also enable them to be used by more people, with fewer computational resources, and in more domains. Despite the rapid advancement of computational power, such small models are still critical tools for many areas of science, which may not have access to multiple GPUs or the ability to obtain large datasets. While datasets like CIFAR-10 are considered to be small by machine learning standards, they are often orders of magnitude larger than datasets available within other research domains. This work makes transformer models available to these researchers.

# CHAPTER IV

### VARIADIC NEIGHBORHOOD ATTENTION

Random numbers should not be generated with a method chosen at random.

## Donald Knuth

**Nota Bene:** This chapter is based on the previous published co-authored work *Efficient Image Generation with Variadic Attention Heads* [156], formerly released as *StyleNAT: Giving Each Head a New Perspective*. Additionally, this chapter involves content from *Neighborhood Attention Transformer* [52] (NAT) in order to facilitate the discussion of StyleNAT, but is not the focus of this chapter.

- Steven Walton programmed the majority of the source code for StyleNAT and ran the majority of experiments. This includes creating all the research questions and designing all the necessary experiments to evidence them. His contributions also include all the visual analysis as well as the development of the attention maps to visualize restricted attention mechanisms. He was also the main writer of the paper. Steven also made significant contributions to the work of NAT, helping develop the theory (primarily around generalization), made contributions to the source code, provided advice, and help write the paper.
- Ali Hassani developed the *NATTEN* CUDA kernel that was used in both StyleNAT and NAT. He provided important insights, especially with the rapidly changing *NATTEN* code, made contributions to the source code, helped perform experiments, and provided key insights for the development of

the restricted attention visualization. Ali Hassani was also the primary author of the NAT paper, writing the majority of code, performing the majority of experiments, and was the largest contributor to the paper's text.

- Xingqian Xu contributed advice and insights around the underlying StyleGAN architecture.
- Zhangyang Wang provided guidance during the research and feedback for the project.
- Jaichen Li provided feedback for the NAT design and contributed to the writing of the paper.
- Shen Li provided general design feedback for the *NATTEN* CUDA kernel and support for running the large scale experiments.
- Humphrey Shi was the advisor for both StyleNAT and NAT, contributing overall guidance on the research as well as funding for both works. Humphrey also contributed to the writing of the paper and ensuring research stayed on track.

While Chapter 3's success with CCT demonstrated that ViTs could be significantly improved in terms of data and computational efficiency, it left the core neural architecture untouched. These impacts come from preparing the data for processing, but further improvements can be made by also improving the processing. Our ViT models still struggle with their  $O(n^2)$  complexities, in both time and space, so making improvements to these layers can have significant impacts. Still, the work showed that transformers did not need big data nor big models to be successful. This motivates further work into improving these architectures themselves.

Transformers were born with language in mind, but had been adapted for vision. The computational challenges are particularly challenging in Computer Vision due to the multi-dimensional data that must be processed,  $c \times w \times h$  which frequently leads to out-of-memory (OOM) issues [174, 96, 171]. The *de-facto* solution to this problem had been to use Convolutional Neural Networks (CNNs)[94, 93, 41]. This is because CNNs provide memory efficiency by operating only on a localized context window as well as naturally incorporating multi-dimensional spatial relationships.

On the other hand, transformer networks attend over the entire data, allowing for arbitrary connections to be made. As previously discussed (Chapter 3.1), transformers are capable of learning convolution filters, so it should be possible for them to be just as powerful. These benefits come at a cost of  $\mathcal{O}(n^2)$ both in computational complexity as well as memory complexity, but our previous work demonstrated that smaller ViTs could outperform CNNs. This then begs the question if ViTs can be better adapted to vision tasks. Are we able to achieve  $\mathcal{O}(n)$ performance while also being able to incorporate both local and global structures within our data?



*Figure 10.* Samples form FFHQ-256 (left) with FID: 2.05, FFHQ-1024 (center) with FID: 4.17, and Church (right) with FID: 3.40 generated by our StyleNAT network, using Hydra Neighborhood Attention.

This chapter studies the core architecture of the network, by introducing *Efficient Image Generation with Variadic Attention Heads* [156], which allows the vision transformer to do more with less. The primary modification for this work is simple, yet powerful: allow attention heads to attend to independent receptive fields. Our results demonstrate that some simple modifications to our attention heads can allow our Vision Transformers to better integrate local and global relationships during image generation. The result of this is the ability to train a StyleGAN [79] based model, using a modified version of Neighborhood Attention [52, 50], which pushes the Pareto Frontier for image generation on FFHQ-256. Our model makes significant improvements in terms of visual fidelity while being smaller and has a higher throughput than other comparative models.

### 4.1 Localized Attention

In an effort to address the computational challenges of transformers, researchers looked to a number of different solutions. One such solution is to only perform attention on some localized region instead of the whole input. This formulation is natural as analysis of attention maps shows that there is strong correlation between neighboring tokens [90, 3, 152], or having Attention Sinks [163]. Works like Image Transformer [123] and Stand Alone Self-Attention (SASA) [129] use localized context windows for their transformer algorithms, similar to the ideas proposed in Longformer [7]. These methods reduced the computational burden of attention mechanisms, approximating  $\mathcal{O}(n)$  complexity, but had issues generalizing as the window size increased. Other works like HaloNet [151] and the Window Self-Attention (WSA) from Swin Transformer [109, 110] partitioned the query and context sets, independently performing self-attention. These blocks become highly parallelizeable but does not account for cross-block interactions.

Swin tried to address this issue by introducing shifted windows (SWSA), where subsequent attentions would shift their windows. With a hierarchical structure the network can is able to attend to every pixel in an image to attend to one another, but incorporates biases around boundaries, similar to the issues faced in the nonoverlapping blocks of ViT (Chapter 3).



#### 4.2 Neighborhood Attention

*Figure 11.* Comparison of Neighborhood Attention, Swin, and ConvNeXt on ImageNet classification.

To resolve these issues, Hassani et al. developed the Neighborhood Attention Transformer (NAT) [52]. The architecture is similar to SASA but resolved the generalization issue, ensuring that when the window size was equal to the image size that Neighborhood Attention (NA) would be identical to the traditional dotproduct self-attention mechanism. Like a convolution, NA considers a context window around each individual input *queries*, Q. The keys, K, then evaluate over the surrounding neighborhood (a square). If a (relative) positional bias [68, 128], B, is used then this must also be modified to account for the key location. Similarly, the *value*, V, must be updated to correspond with the local neighborhood. We can describe this attention variant as follows:

$$\mathbf{A}_{i}^{k} = \begin{bmatrix} Q_{i}K_{\rho_{1}(i)}^{T} + B_{i,\rho_{1}(i)} \\ \vdots \\ Q_{i}K_{\rho_{k}(i)}^{T} + B_{i,\rho_{k}(i)} \end{bmatrix} \qquad \mathbf{V}_{i}^{k} = \begin{bmatrix} V_{\rho_{1}(i)}^{T} \\ \vdots \\ V_{\rho_{k}(i)}^{T} \end{bmatrix}$$
(4.1)

For an input token, i, we consider a window of size k and its neighborhood,  $\rho$ . Specifically,  $\rho_j(i)$  denotes the i's j<sup>th</sup> nearest neighbor. We can then consider the full attention about a token as:

$$\operatorname{NA}_{k}(i) = \operatorname{Softmax}\left(\frac{\mathbf{A}_{i}^{k}}{\sqrt{d}}\right) \mathbf{V}_{i}^{k}$$

$$(4.2)$$

Testing Neighborhood Attention in discriminative settings showed that it was able to outperform other attention variants, such as Swin, as well as modernized Convolutional variants such as ConvNext [111], show in Figure 11. Importantly, for a fixed number of parameters, NA outperformed others on accuracy, memory, and flops, but did not on throughput. Further development of NA led to a Dilated variant (DiNA) [50] and improving the GPU kernel [49, 53] and generalizing the architecture [54], allowing for arbitrary attention configurations, similar to tools like Flash Attention [22, 21, 136] and xFormers [98]. These improvements led to significant improvements in speed, with over a  $100 \times$ improvement in the forward pass and  $80 \times$  for both forward and backwards passes, at FP16 and on an NVIDIA A100 GPU. These improvements led to further adoption of the model throughout other works [76, 75, 29].

While localized attention provides significant advantages in reducing the computational load, they have limitations due to the restricted context window over which they attend to. This creates a similar to CNNs, gaining advantages of the localized structure of the data at the cost of global structures. Similar to CNNs, this can often be resolved by using a hierarchical model, where downsampling and pooling allow for full token mixing. For example, NAT takes in an input image sized  $\mathbb{R}^{H \times W}$ , then embeds this into  $\mathbb{R}^{\frac{H}{4} \times \frac{W}{4}}$  through an overlapping tokenizer, which uses the same embedding process as CCT [51]. There are 3 transformer blocks before another overlapping downsampling is performed and another 4 transformer layers process the image at  $\mathbb{R}^{\frac{H}{8} \times \frac{W}{8}}$ . This process repeats with 18 layers at  $\mathbb{R}^{\frac{H}{16} \times \frac{W}{16}}$  and 5 layers at  $\mathbb{R}^{\frac{H}{32} \times \frac{W}{32}}$ . Figure 12 depicts the architecture. This formulation works especially well for discriminative tasks, due to the network map's endomorphic formulation. In the case of classification the network is learning the map  $f : \mathbb{R}^{C \times H \times W} \mapsto \mathbb{N}^0$ . Through this hierarchical mapping and overlapping downsampling all tokens become sufficiently mixed and there is an assurance that all inter-relations can be accounted for.



*Figure 12.* Diagram depicting the Neighborhood Attention Transformer [52] as applied to vision classification.

## 4.3 Variadic Attention Heads

A subtle feature of multi-headed attention [150] is that attention heads are independent of one another. This property allows each head to attend to different features within the data, analogous to feature maps in CNNs. This feature plays a key role in the performance of attention models [119, 145], with a few specialized attention heads being the primary drivers [153]. We thus propose the following hypothesis: *Decoupling attention heads will improve the performance of Neighborhood Attention*.

An important feature of Neighborhood Attention is that it generalizes to standard attention and allows for dilated receptive fields [50]. With this in mind, we are able to reduce the locality bias imparted by local receptive fields. By allowing attention heads to attend to different receptive fields we allow for the intermixing of global and local information, similar to the standard attention. Thus, we modify the standard Neighborhood Attention mechanism to allow attention heads to have independent kernel sizes and dilations. This modification can be explicitly represented as follows:

$$\mathbf{A}_{i,h}^{k} = \begin{bmatrix} Q_{i,h(k,d)} K_{\rho_{1}(i),h(k,d)}^{T} + B_{i,\rho_{1}(i),h(k,d)} \\ \vdots \\ Q_{i,h(k,d)} K_{\rho_{k}(i),h(k,d)}^{T} + B_{i,\rho_{k}(i),h(k,d)} \end{bmatrix} \qquad \mathbf{V}_{i,h}^{k} = \begin{bmatrix} V_{\rho_{1}(i),h(k,d)}^{T} \\ \vdots \\ V_{\rho_{k}(i),h(k,d)}^{T} \end{bmatrix}$$
(4.3)

With this variation we specify that each query, Q, and key, K, are independent calculations, split across each attention head, h. Where the head is a function of the window size, k and dilation, d. The positional bias may also need be offset in a head-wise fashion.

$$NA_{k(i)} = Softmax\left(\frac{\mathbf{A}_{i,h}^{k}}{\sqrt{d}}\right) \mathbf{V}_{i,h}^{k}$$
(4.4)

This formulation maintains the computational and memory advantages of Neighborhood Attention, but mitigates the losses to architectural flexibility. Within this formulation arbitrary combinations of window sizes and dilations may be used, allowing for higher flexibility and integration of information. Consequently, these hyper-parameters, k and d, need not be fixed.

### 4.4 Generating The Right Experiment



#### StyleNAT

Figure 13. StyleNAT Architecture

To evidence our hypothesis presents a challenge, due to the nature of most neural architectures accounting for these limitations and mixing data as dept increases. With limited compute infrastructure there is significant pressure to design the right experiment to properly test the research questions and ensure we limit any observed effects to our procedure. Most architecture changes are demonstrated through discriminative tasks; such as classification, detection [103, 58, 13], and segmentation [179], as performed in our Neighborhood Attention papers. Many of the small (e.g. CIFAR-10 [147, 91]) and mediumsized (e.g. ImageNet-1k [23]) datasets, which would be within our computational budget, are nearly saturated; with many results similar to the labeling error rates
in the data. Consequently, improvements tend to me minor, often with only a percent or lower difference. This makes it difficult to evaluate the effect of the modifications, even with exhaustive search, as the performance gains become difficult to distinguish from many other factors. Their hierarchical nature also makes it difficult to isolate our contribution as this already performs local-global token mixing. To combat these issues while remaining mindful of computational budgets, we must intentionally design an experiment to limit the variables of interest and ensure proper variable isolation.

While discriminating tasks often learn a mapping from some  $\mathbb{R}^m \to \mathbb{R}^n$ where n < m it is common for generative tasks to learn maps where  $n \ge m$ , or even a map onto itself (Chapter 2.1). By placing focus on these formulations we can better isolate our variables of interest. Specifically, the StyleGAN [79, 81, 80, 82, 96] architecture uses a progressive [78] structure, generating an image starting from a small noise sample, relying on the Latent Manifold Hypothesis, making the assumption that the necessary information for the generation of samples is smaller than the dimensionality of the images themselves. This progressive nature is designed for the local receptive fields of CNNs, with low resolution images capturing more global structures and as the resolution grows the neural net can learn more appropriate upsampling methods that account for finer detail synthesis. This is effectively in reverse to the structure of hierarchical classification models, benefiting locality.

This structure also has the benefit that, in general, it synthesizes an image at progressive image resolutions, allowing for greater computational efficiencies. Unfortunately, due to these biases the architectures commonly struggle with long range fine detail synthesis, commonly resulting in features like heterochromia

73

(eyes differing in color) when doing human face synthesis. Such fine grain detail may not exist at low resolutions and only appear later when the image resolution is significantly larger than the convolution window. Thus, locality is a double edged sword for these architectures: providing high utility at low resolutions but becoming detrimental as the image grows. The prolific nature of these architectures also yields a large number of comparitors. The well studied nature of GANs allows for better isolation and allows us to assume that these architectures are reasonably optimized. This setting makes it a great platform for variable isolation and allowing us to test our hypothesis.



Figure 14. StyleNAT represents the Pareto Frontier for FID (y-axis), Parameters (bubble size), and throughput (x-axis) on FFHQ-256. StyleNAT has the lowest FID of 2.05, with fewer parameters (48.92M) than similarly performing models and is capable of generating images in real-time (59.90 imgs/s). This makes StyleNAT smaller, faster, and better than its competitors. (Note: StyleNAT's NATTEN kernel is not optimized and may still be improved for higher throughputs.)

Additionally, there were many attempts to integrate attention and transformers [144, 171, 71, 72] to resolve the issues in long range detail generation. These works also needed to ensure that methods were computationally efficient, as performing full attention across the entire image is computationally intractable at high resolutions. Specifically, the work Hit-GAN [174] and StyleSwin [169]had created similar formulations, allowing for clearer experimental control. Both these networks base their architectures off of the StyleGAN architecture. Additionally, HiT-GAN and ViT-GAN [96] found that transformers struggled with the generation of images above  $64 \times 64$  resolution, allowing our work to demonstrate its capabilities at the much more common, and computationally reasonable, scale of  $256 \times 256$ .

HiT-GAN divided their generative architecture into two sub-networks, one for low-resolution ( $\leq 32 \times 32$ ) and the other for high-resolution. To generate spatial mixing in the low-resolution network extended the method of Nested Hierarchical Transformer [173], dividing the input into non-overlapping patches, applying positional encoding, and bifurcating the attention heads such that each operates on a different spatial axis (height and width). They note that they make an assumption that proper spatial mixing in low-resolution stages will allow for high resolution stages to focus on image synthesis, replacing their attention modules with MLPs. Additionally, they replaced the common AdaIn [70] and modulated layers [81] with cross-attention due to excessive memory issues and being unable to produce images above  $64 \times 64$  resolution.

StyleSwin takes a different approach, much more closely following the original StyleGAN architecture. The typical Swin Transformer [109, 110] architecture performs spatial mixing by using non-overlapping patches, performing

75

a cyclic permutation on subsequent layers. Through multiple layers this ensures there is spatial mixing, allowing for some long range attention across edges. StyleSwin instead splits their attention heads such that half utilize the window patches and the other half utilizes the shifted windows, removing the necessity of communication across layers for spatial mixing. Otherwise the architecture closely follows that of StyleGAN-2 [81, 80], using 2 synthesis layers per resolution level, incorporating wavelets [36], TTUR [60], and bCR [175]. For their attention they use an attention dimension equivalent to the number of channels in StyleGAN convolutions. They divide this by 32 to get the number of attention heads, setting a minimum number of heads to 4. This results in 16 heads for resolutions 4 - 64, 8 heads at 128 and 4 at 256. Additionally, a learnable parameter size  $512 \times 4 \times 4$  is used to seed the synthesis network. StyleSwin faced "blocking artifacts" and similar to HiT-GAN found that self-attention could be removed at higher resolutions but that it failed to model high-frequency details.

With the progressively growing structure we have a good platform that can clarify the impact of our architectural changes. The high rates of visual artifacts also meets the goals that this architecture is designed to mitigate. Furthermore, HiT-GAN and StyleSwin provide strong control baselines that can be used to isolate our variables of interest. Given this setting, we utilize StyleSwin as our experimental platform.

Due to computational budget limitations, we must minimize experimentation and datasets to the most impactful. This model has a large number of potential configurations, as our kernels can range from a size of 3 to the nearest odd integer smaller than the resolution,  $\mathcal{R}$ . Fixing the hyper-parameters determining window size and dilation reduces the computational search space,

76

helping stability, which is a common problem with GANs. Fixed kernel sizes and dilations can also help increase interpretability, allowing for more direct visual analysis (Chapter 4.7). This will also help us better attribute effects of the context mixing, making it a more direct comparison to StyleSwin. For simplicity we assume an image is square, thus, for a given resolution,  $\mathcal{R}$ , we will have  $\frac{\mathcal{R}}{2} - 1$  potential kernels. Each dilation must also be positive and while the dilation can make the effective kernel size larger than the image, this results in attending to no information. Using this restriction results in  $\frac{\mathcal{R}}{4}$  kernels that can have dilations, and the max dilation for a given kernel is  $\lfloor \frac{\mathcal{R}}{k} \rfloor$ . The total number of configuration of attention heads, per resolution, is:

$$N_c = \sum_{i=1}^{\mathcal{R}/2-1} \left\lfloor \frac{\mathcal{R}}{2i+1} \right\rfloor$$
(4.5)

$$= \frac{\mathcal{R}}{4} + \sum_{i=1}^{\mathcal{R}/4-1} \left\lfloor \frac{\mathcal{R}}{2i+1} \right\rfloor$$
(4.6)

Following the StyleSwin architecture, for an image size  $3 \times 256 \times 256$  ( $\mathcal{R} = 256$ ) this results in  $2 \times ((16 \times (4 + 14 + 37 + 97)) + (8 \times 237) + (4 \times 565)) = 13176$  possible configurations and exceeds 47k configurations for high resolution generation ( $\mathcal{R} = 1024$ ).<sup>1</sup> Given our interpretation of the Bitter Lesson, this flexibility is promising to the architecture, but is not feasible to exhaustively test within a modest computational budget. We leave such work for bigger labs and instead focus on evidencing the research hypothesis.

4.4.1 Datasets. For our main dataset we use the Flickr-Face-HQ Dataset (FFHQ), introduced in the original StyleGAN work [79]. FFHQ is widely used across generative modeling research, has a high resolution ( $1024 \times 1024$ ) variant, and a wide diversity of faces and accessories (e.g. glasses, hats, jewelry).

<sup>&</sup>lt;sup>1</sup>Neither kernels nor dilations need be square, further increasing the potential configurations

Level	Kernel	Dilation	Dilated Size	Level	Kernel	Dilations
4	-	-	-	4	-	-
8	7	1	7	8	7	1
16	7	2	14	16	7	1,2
32	7	4	28	32	7	1,2,4
64	7	8	56	64	7	1,2,4,8
128	7	16	112	128	7	1,2,4,8,16
256	7	32	224	256	7	1,2,4,8,16,32
512	7	64	448	512	7	1,2,4,8,16,32,64
1024	7	128	896	1024	7	1,2,4,8,16,32,64,128

(a) Split Head configuration

(b) Progressive configuration

Table 10. Configurations for different attention head configurations. Table 10a shows our 2 headed configuration, used in FFHQ, where half the heads are dense kernels and half are sparse dilated. We show the dilations and their effective size. Table 10b shows the progressive head configuration that was used in some LSUN Church experiments.

Importantly, by generating human faces we are better able to analyze the images as human are naturally attuned for facial perception, being able to detect subtle distinctions [77]. For our second dataset, we use the Church Outdoor class from the Large Scale image database (LSUN) [166]. This dataset is relatively common and presents a significant challenging, containing many features with straight lines, often not found in biological objects.

4.4.2 Hyperparameters. To ensure that we only measure the effects of the architecture we follow the same training procedure as StyleSwin: using TTUR, a discriminator learning rate of  $2 \times 10^{-4}$ , bCR with  $\lambda_{real} = \lambda_{fake} = 10$ , and  $r_1$  regularization [118] with  $\gamma = 10$ , as well as model hyper-parameters. The only deviations we make is the iteration we begin our LR-decay, which is experimentally driven, and we doubled the batch size in later experiments, to 64, finding we were able to maintain stability at this level. This difference may be due to our usage of 80GB A100 GPUs as opposed to the 32GB V100s that StyleSwin

used, as our batch size required 36GB of VRAM. We include all hyper-parameters in our GitHub repository<sup>2</sup> as well as save these values to our model checkpoints, including our seed values, for reproducibility. We do not perform hyper-parameter optimization, seeking to evaluate our model on architectural effect rather than ultimate performance. A minor architectural change is made such that at the lowest resolution,  $4 \times 4$  we utilize a standard Multi-Headed Attention (MHA) layer as opposed to a Neighborhood Attention layer. At this layer StyleSwin uses a  $4 \times 4$ window size and NA is restricted to odd window sizes. Experimentally we found no difference when using a  $3 \times 3$  window size, but the MHA layer slightly increases the computational efficiency due to the unoptimized performance of NA at the time.<sup>3</sup> Elsewhere StyleSwin uses a window size of  $8 \times 8$  while StyleNAT uses  $7 \times 7$ , giving a slight context range advantage to StyleSwin. Table 11 shows a full comparison of our results, comparing different generative models by generative performance, the number of model parameters, and the rate at which they can generate images. We determine parameter size from officially released model checkpoints, removing any non-generative parameters such as discriminators, hyper-parameters, or exponential moving averages [126, 74, 59]. Similarly, we gather all throughput measures, ensuring consistent GPU architecture and versions of Python and PyTorch [124]. To ensure throughput is properly calculated, we first warm-up the models to ensure they are properly cached, generating 50 samples, and then generate an additional 100 samples, which we find the average of. For all GANs we use a batch size of 1 and for diffusion models we maximize the batch size for available memory <sup>4</sup>. We

<sup>&</sup>lt;sup>2</sup>https://github.com/SHI-Labs/StyleNAT

 $<sup>{}^{3}\</sup>mathcal{N}ATTEN$  has undergone significant optimizations since the time of these experiments.

<sup>&</sup>lt;sup>4</sup>Doing otherwise results in significantly decreased throughputs while GANs show little to no deviation.

include our procedure in our public repository for additional transparency. For evaluation we will primarially rely on the Fréchet Inception Distance (FID) [60], but include more discussion and evaluation in Chapter 4.7.

Anab	Madal	$  \mathbf{FFHQ} \mathbf{FID} \downarrow  $		Church	Church   Usage Metrics	
Arcii	Model	256	1024	<b>256</b>	img/s	Params (M)
	StyleGAN2 [81]	3.83	2.84	3.86	84.85	30.03
	StyleGAN3-T [82]	-	2.70	-	$108.84^{\star}$	$23.32^{\star}$
Convolution	Projected GAN [131]	3.39	-	1.59	143.64	105.84
	INSGen [165]	3.31	-	-	89.00	24.94
	StyleGAN-XL [132]	2.19	2.02	-	14.29	67.93
	GANFormer [71]	7.42	-	-	-	32.48
	GANFormer2 [72]	7.77	-	-	-	-
	HiT-S [174]	3.06	-	-	$86.64^{\dagger}$	$38.01^{\dagger}$
Attention	HiT-B [174]	2.95	-	-	$52.09^{\dagger}$	$46.22^{\dagger}$
	HiT-L [174]	2.58	6.37	-	$20.67^{\dagger}$	$97.46^{\dagger}$
	StyleSwin [169]	2.81	5.07	2.95	62.48	48.93
Attention	StyleNAT (ours)	2.05	4.07	3.40	59.90	48.92
	DDPM [63]	-	-	7.89	-	256.00
	D.StyleGAN2 [158]	-	2.83	3.17	-	23.94
	D.Proj.Gan [158]	-	-	1.85	-	105.85
Diffusion	LDM [130]	4.98	-	4.02	1.28	329.32
	LFM [20]	4.55	-	5.54	4.18	457.06
	UDM [84]	5.54	-	-	-	65.58
	Unleashing [9]	6.11	-	4.07	6.65	159.96

Table 11. FID50k results. Usage Metrics are evaluated at 256  $\times$  256 resolution for fair comparison and were collected ourselves. StyleNAT does not utilize any FID enhancing processing, such as StyleGAN's truncation trick. <sup>†</sup>HiT-L was optimized for TPU and there is no existing PyTorch version to compare. There is no public checkpoints for Hit-GAN [174] and we use their reported V100 values. While most architectures are built off of the official StyleGAN models, they may not all be able to utilize the custom CUDA kernels, which can significantly increase throughput [81]. We use no truncation or tempering for StyleNAT.

## 4.5 When Faced With Sparse Attention

To gather a baseline value we first replace the Swin layers in StyleSwin with an unmodified Neighborhood Attention Transformer, focusing on the FFHQ dataset. This modification results in a minor improvement of 0.07 FID. Following this, we incorporate Hydra-NA, using a kernel size of  $7 \times 7$  for all attention heads,



(a) FFHQ-256 Samples(b) LSUN Church SamplesFigure 15. Samples generated by StyleNAT. We do not use truncation, softmax

tempering, nor any other such enhancement techniques.

but set half the heads to have a dilation increasing by a power of 2  $(d = 2^N)$ , maximally for the resolution at a given level, where  $N = \lfloor \log_2 \left(\frac{R}{k}\right) \rfloor$ .<sup>5</sup> This method allows for dense local receptive fields as well as highly sparse global receptive fields to intermix through the attention mechanism.

This improved the performance by an additional 0.5, strongly suggesting that this method is better able to learn the data generating function. Notably, this result is only outperformed by StyleGAN-XL [132], which is  $\approx 40\%$  larger and 24% the throughput, and a variant StyleSAN-XL [144] that introduces a novel training objective. At the time of our work, StyleGAN-XL was the state of the art network on FFHQ-256 and this result caused StyleNAT to push the Pareto Frontier in both FID vs model size as well as FID vs throughput. We noticed that StyleSwin had utilized random horizontal flips when training on LSUN Church,

<sup>&</sup>lt;sup>5</sup>At a  $256 \times 256$  image resolution and a kernel of size 7 this gives us a dilation of size 32, making a highly sparse receptive field across 224 pixels.



*Figure 16.* FFHQ training: FID vs Iteration (in thousands). We see that the FID performance has not converged. This suggests the models are not optimally trained.

but was not used on FFHQ and decided to perform this training as NA had shown to be quasi-equivariant to translations and rotations [52], and given transformers' preference for augmentation, that this would improve the score while demonstrating better generalization capabilities. This model trained for 10<sup>6</sup> iterations, beginning the LR-Decay at 740k iterations, and achieved out best result at 940k iterations (60.2M images). Notably the model was continuing along a decreasing trajectory, as show in Figure 16. Since our result had surpassed the state of the art at the time, StyleGAN-XL, we chose to move on, considering computational restraints. Our goal is not to achieve state of the art performance, but rather to demonstrate the integration of local and global structures within data.

Subsequently, we tried a few other dilation patterns but did not see significant changes. Additionally, we attempted further partitioning of the attention heads, including two intermediate dilations, but observed a decline in performance and frequent model collapse. We believe this is due to only having 4 attention heads at resolutions  $\geq 256 \times 256$  and 8 attention heads at  $128 \times 128$ , requiring undue burden for each head. We believe that increasing the number of heads and head embedding dimension may lead to increased performance. The results of these ablation studies can be found in Table 12a

FFHQ Ablation								
Ablation	$\mathbf{FID}\downarrow$	$\Delta \downarrow$						
StyleSwin	2.81	—						
+ NA	2.74	-0.07						
+ Hydra-NA	2.24	-0.50						
+ Flips	2.05	-0.19						
$+ \operatorname{Prog} \operatorname{Di} (4)$	2.55	+0.50						

Church Ablation								
$\begin{array}{ c c c c c } \hline {\rm Splits} & {\rm Heads} & {\rm FID} \downarrow & \Delta \ \downarrow \\ \hline \end{array}$								
2	4	23.33	—					
4	4	6.08	-17.25					
6	8	5.50	-0.58					
8	8	3.40	-2.10					

(a) Ablation study comparing models on FFHQ-256 dataset. Starting with StyleSwin [169] we first add unmodified Neighborhood Attention (NA) [52], then Hydra-NA, horrizontal flipping data augmentation, and progressive dilations.

(b) Comparison for number of head
partitions (splits) when learning LSUN
Church. Min heads represents the minimum
number of heads in our transformer. Early
layers begin with 16 heads and halve until
minimum beginning at $128 \times 128$ resolution.

Table 12. Ablation studies of StyleNAT architecture, studying different configurations. Results for FFHQ-256 and LSUN Church, respectively.

To test the scalability of this work we also perform a single training for high resolution, at  $1024 \times 1024$ . Identical training procedures were utilized, but this time we started our LR-Decay at 500k iterations and stopped training at 900k iterations, achieving an FID of 4.17. We did not perform any parameter search at this scale due to the costly computational budget but believe this demonstration demonstrates scalability as the result significantly outperforms all other transformer based architectures. While we did not surpass the FID of StyleGAN3 our model is able to produce images of higher visual fidelity and does not contain many of the visual artifacts that StyleGAN3 creates. Further discussion is provided in Chapter 4.7.

### 4.6 A Bump While Headed To Church

We also train our model using the LSUN Church dataset, which includes images of cathedrals, churches, temples, and towers. This dataset presents significantly different challenges, images containing both biological and nonbiological features and with much more complex scenes. While FFHQ has images center cropped around human faces and minimize backgrounds, this dataset has diverse foregrounds and backgrounds (usually the sky). This creates strong dependence on localized features and lower dependence on global ones, as many long range features may be determined entirely through local ones (e.g. the sky). The highly asymmetric nature of the images also reduces these global dependencies, with features such as windows frequently appearing in different sizes and shapes. This frequently results in generators having significant performance gaps between FFHQ and LSUN Church. While StyleGAN-XL demonstrated that Style-based generators could scale with data diversity, this requires significant architectural changes and additional parameters. Despite these challenges, this dataset can help to better understand the biases of our architectural changes and how well it can adapt to more complex environments.

We initially follow identical training procedures and architecture, splitting heads between dense local windows and sparse global windows. We observe that this model quickly diverges, resulting in mode collapse. We then increase the number of partitions, following the architecture that diverged in FFHQ. This variant dramatically improves FID and stability, showing the dataset's stronger dependence on localization. We further increase the partitions to 6 and change our minimum head count, which only affects the final layer, to 8. This necessitates a decrease in the head dimension and results in a more modest increase in FID. A final configuration is attempted increasing the number of partitions to 8, assigning 2 heads to each partition in layers operating on resolutions below 128 and 1 attention head for those larger. This results in a larger FID gain, and while the result is not as impressive as those in FFHQ the result is highly competitive. The results of this ablation can be found in Table 12.

84

### 4.7 Metrics Are Not Enough

While the FID results in Table 11 show substantial effects, it is important to recognize the biases and limitations of the evaluation metrics (Chapter 2.4). *The main issue is that most of these metrics were developed when the quality of generation was substantially lower.* The authors of the metrics were not deceived by the correlations they found, but the rapid success of generative research forces us to face their limitations. They still provide utility but we must be careful to not become overly reliant upon them as they are not perfectly aligned with the things we wish to measure.

FID uses a Fréchet Divergence, which measures the difference between two Gaussians,  $G_0, G_1$ 

$$d(G_0(\mu_0, \Sigma_0), G_1(\mu_1, \Sigma_1)) = \sqrt{||\Delta \mu||_2^2 + Tr\left(\Delta \Sigma - 2\sqrt{(\Sigma_0 \Sigma_1)}\right)}$$
(4.7)

Where  $\mu$ ,  $\Sigma$  is the mean and covariance, respectively, and Tr is the trace of the matrix. The *Inception* part of this is metric refers to the fact that the Gaussians are drawn from the final pooling layer of a Inception-V3 Network [143] that has been trained on ImageNet [23]. While the performance was sufficient at the time, the accuracy is sub-par by today's standards. Other work has demonstrated that FID can create distortions [92] or there can be flaws in evaluation in subtle effects like through the image downsampling method used [122]. These subtle effects can make evaluation difficult, vary dramatically between libraries and even library versions. Simply updating the Inception Network to a different model can provide improvements, as shown by Kynkäänniemi et al. [92], this does not resolve the underlying problem.

After the pre-print of this paper was released Stein et al. [140] performed a large study to determine which metrics strongly correlated with human preference. The work involved the largest human preference study to date and used StyleNAT in their analysis due to its state of the art performance on FFHQ-256 at the time.<sup>6</sup> Their work sought to better understand the biases of many different image evaluation metrics. To determine this, they crafted a rather straight-forward experiment, measuring participants ability to determine if a given image was genuine or a deep fake. This metric serves as a proxy to determining if images are *photo-realistic* or not. Participants were paid for their, being given bonuses based on their accuracy, and at a minimum had a Bachelor's level education. Their results found that there was not a strong relationship between metrics such as FID and participants ability to distinguish deep fakes from real imagery.

The result of this forces us to carefully analyze our images and investigate our network to better determine if our architectural changes actually caused the improvements we sought. To do this we perform two forms of visual analysis to better understand our network is doing. The first, Chapter 4.7.1, compares visual fidelity of images from StyleGAN3, StyleSwin, and StyleNAT on FFHQ-1024 images. The second, Chapter 4.7.4, dives deeper into the attention maps and what our models are actually attending to. These visual analyses are limited, but can give us much deeper clues as to what is happening within these networks.

4.7.1 The Face Says It All. Given our metric limitations we visually inspect samples from images of our networks. On FFHQ-1024, we compare StyleGAN3, StyleSwin, and StyleNAT which have FIDs of 2.79, 5.07, and 4.17, respectively. We use our high resolution 1024 images because this allows us to

<sup>&</sup>lt;sup>6</sup>We have no affiliation with Stein et al. nor have had any communication.

better inspect subtle features. Local features can be more easily seen without the need to zoom and global features have more difficulties being generated. We use FFHQ because of our biological aptitudes at recognizing faces. The human brain was designed to recognize faces, making us apt at identifying subtleties.

4.7.2 Quick Training on Deep Fake Detection. Readers who are untrained or inexperienced in detecting deep fakes may wish to pay special attention to some key areas. Often visual artifacts can be quickly identified by looking at ears, eyes, neck, and hair, often in that order. FFHQ has biases where faces will look much more in the style of profile photos: that is, they are facing towards the camera. In general, this results in a full face being visible, ears and all.

Ears make for quick detection due to their large distance within the image and natural tendencies for faces to be highly symmetric (across the vertical axis)<sup>7</sup>. The localization bias is thus used to our advantage. Ears are not typically focused on by a typical viewer, so may be easily missed. Both ears also may not appear in samples, as may not always be relied upon.

Eyes are said to be the window to the soul, and are surprisingly complex. Issues can often be easy to detect but cultural biases may cause in how natural this detection is.<sup>8</sup> These are great features due to this complexity, their long range, and high symmetry. In particular, pupils (the black center of the eye) may not appear round in generated images. Difficulties in capturing long range symmetry result in high rates of medical conditions such as anisocoria (unequal pupil size), dyscoria (misshapen pupils), ectropion uveae (displacement). or other such effects.

<sup>&</sup>lt;sup>7</sup>Faces are not perfectly symmetric, but in general, they are far more symmetric than asymmetric

<sup>&</sup>lt;sup>8</sup>e.g. some ethnic groups have high variance in iris colors while others don't. This plays a role in cultural attention to eye color.

Irises, the colored part of an eye, can also exhibit features like heterochromia (differing eye colors or differing color in the same eye), aniridia (absence of iris), or others. Additionally, eyes often contain reflections, which can quickly give away the synthetic nature. Detailed reflections make this easy to spot, but the bright spot of a light source will often be non-physical.

Necks can provide more subtle clues. There is higher variance in necks within these images, where some photos taken with body facing the camera and the neck will be straight while others will have their body slightly turned with their head facing the camera. This can cause issues if we pay attention to the depth in an image and especially around the chin. In addition to this, mouth and teeth can exhibit these phasing artifacts, as shown in the StyleGAN2 work [81]. High variances result in our detectors being worse at these features so they often slip through. Hair also provides substantially high variance, but may require more detailed attention. These may be simple issues like rapidly changing texture and color or one may need to carefully follow some strands of hair.

Finally, accessories like jewelry, hats, glasses, and so on make for quick identification. These have lower sampling rates within the data and high variance, so are far less likely to be coherent. Ablations and subtle artifacts can appear, which are more difficult for the detector to catch.

With this in mind we encourage the reader to carefully inspect our images. We embed our images at high resolutions to make it possible to zoom in for careful inspection.

It is also important to highlight the biases in the FFHQ dataset. Many mistakes that these models make can be much better understood by understanding the data they are trained on. Some works have found that there are

88

disproportionate representation of certain demographics [99]. In particular, there are higher rates of women than men, in particular of Caucasian and Latin descent. The images are also center cropped, primarily containing a single individual, and usually facing the camera. Often people are smiling in these photos, eyes are open, looking at the camera, in a portrait style. There are still high variances within the dataset and subjects commonly may be wearing glasses, hats, have artistic face painting or cultural face painting (e.g. Bindi or Ashes), hands on their face, microphones, wearing costumes, and a wide variety of situations exist. In order to perform a serious evaluation generative researchers are strongly encouraged to manually inspect the dataset so that they can better understand what they are modeling. Without manual inspection researchers will most certainly make false assumptions about this data.<sup>9</sup>

4.7.3 Fingerprints. These results are highly subjective but still can provide substantial value. To ensure that we are not completely unfair in our comparisons we try to present the best samples from these generators. We wish to error in the direction of a steelman rather than a strawman.

Our goal is not to determine which image generator is better, but find patterns in the unique flaws. These flaws provide clues into how our networks interpret the data and can provide hints at how to improve our generators. Understanding these systematic flaws is critical to understanding what future architectural changes need to be made.

4.7.3.1 StyleGAN. For StyleGAN3 we carefully searched through the public set of curated images which is linked on their GitHub Repository under the directory StyleGAN3-r-ffhq-1024x1024. Being curated this is already biased towards

 $<sup>^9 \</sup>rm Samples$  may be available online, such as: <code>https://huggingface.co/datasets/pravsels/FFHQ\_1024</code>

higher quality samples. We then manually search through this for what we believed was the best sample. For StyleSwin and StyleNAT we instead generate 50 samples, discard any with obvious artifacts (colloquially referred to as "GAN Monsters") and select the best example. This potentially creates a bias towards StyleGAN3, given the additional level of curation.



(a) 1024 FFHQ Sample from StyleGAN3



(b) Forehead bead pattern. Two bands at top and bottom third.



(c) Glasses with hexagonal artifacts around edges.

Figure 17. Visual artifacts from StyleGAN3 FFHQ-1024 samples (using image 0068). Sample highlights banding effects, hexagonal patterns, and other artifacts common to this generator. Within the StyleGAN images we notice a string of beads like artifacts. These structures may be difficult to notice at first glance but become difficult to ignore after noticed. These appear most prominently between the two "wrinkles" in the middle of the forehead of the sample image. We noticed that such patterns appear throughout the face and were quite visible in all images we looked at. We do not know the cause of these patterns but found that they were noticeable in other datasets, including AFHQ, which contains images of animals. This suggests this is a fingerprint of the architectural design rather than of the dataset, potentially being a

more advanced droplet artifacts discussed in StyleGAN2 [81]. Those patterns were often masked by an animal's fur and more easily detected when looking at noses or tongues. <sup>10</sup> <sup>11</sup>

In addition to this we noticed extremely high rates of geometric artifacts in glasses. Most visible around the edges of the glasses, but careful inspection will show that these appear throughout. These may be due to difficulties in capturing reflections. The glasses also are non-physical, with the temples simply vanishing. Another strong band can be found where the temples should be, and indicate that these are statistical artifacts (like the droplets), fooling the detector into thinking the temples exist. Additionally, there is some non-physicality to the nose pads. More inspection can reveal many other artifacts, including around the mouth, melding teeth, hair, fused neck, and tear duct. Specifically, the person in the photo appears to be missing a jaw, which appears surprisingly frequently among the curated samples.

Despite StyleGAN3's high FID score these artifacts are trivially detectable if one knows what to look for, but may easily be missed if only given a passing glance. In particular, StyleGAN3's errors typically highlight larger failures when it comes to long range coherence. While still highly symmetric, there are more symmetry errors than one would expect of an average human.

4.7.3.2 StyleSwin. StyleSwin holds the lowest FID, and unfortunately produced a large number of low quality samples at this scale. The authors of the paper noted some "block" like artifacts, which can be clearly seen in Figures 3 and 5 of their work [169]. We notice similar artifacts in all the samples we generated.

 $<sup>^{10} \</sup>rm stylegan 3-t-afhqv 2-512 x 512/0175. png$ 

 $<sup>^{11}</sup>$ stylegan3-t-afhqv2-512x512/0138.png

Detection can be a bit difficult depending on a reader's screen, but by zooming into the forehead concentric rectangles become quite visible. We believe that these artifacts are due to the Swin Transformer, and provide further discussion alongside our attention maps.



(a) 1024 FFHQ Sample from StyleSwin



(b) Forehead squares

(c) Right ear texture

*Figure 18.* Visual artifacts from StyleSwin FFHQ-1024 samples (we generated these). Sample highlights rectangular geometric patterns on face, and poor texture on ears.

We found that StyleSwin's integration of sliding windows (SWA) and shifted windows (SWSA) does not properly integrate long range features. This is most apparent by looking at the eyes in our sample. All parts of the eye differ in size: both iris and pupil. The effect is as if the right eye is closer to the camera than the left eye, yet this depth does not correctly correspond to the direction that the nose, eyes, and mouth point in. The eyes exhibit heterochromia, being different shades of blue (right is almost green), anisocoria, dyscoria, and ectropion uveae. The reflections within the eyes are also

substantially different, as if looking at completely different scenes.

Additionally, we found common issues with facial textures, easily noticeable in the ear. High rates of speckling can be seen by zooming in on the cheek, where some non-physical banding may also be found. While this image does not have the fused neck like StyleGAN, it has minor issues with generating realistic depth and some artificial lines can be seen along the neck. Similar depth issues may be visible by looking at the nose, which blends into the cheek.



(a) 1024 FFHQ Sample from StyleNAT





(c) Eye spotting

*Figure 19.* Visual artifacts from StyleNAT FFHQ-1024 samples (We generated these). Sample highlights minor skin texture issues, some chromatic aberrations, and unnatural blue speckling around eyes. While this sample has many artifacts and may be more easily identified than the StyleGAN3 sample, there are some aspects that perform better. The concentric rectangles are often less noticeable compared to the beading in StyleGAN, as well as the neck and jawline appear more realistic.

## 4.7.3.3 StyleNAT.

StyleNAT has a FID  $\frac{2}{3}$  the distance between StyleGAN3 and StyleSwin, being much closer to StyleSwin on the metric. Yet, we noticed that images were consistently much better than StyleSwin, and the fidelity was much closer to StyleGAN3. We believe StyleGAN3 still produces better images at a higher frequency and our work could likely have greatly benefited from tuning and continued training. Despite producing many high quality images, our images are still not without error.

The eyes are a bit of interest and may demonstrate some aniridia (absence of iris). Careful inspection makes this unclear, as there is some brown and even a bit of blue-gray. Interestingly the eyes look quite similar, with both eyes following the same pattern. We are unable to differentiate if this is sectoral heterochromia (partitioned) or central heterochromia (radial). The colors being a brown and dark blue, especially on a male, make this much more natural but it should not be assumed the model learned that causal relationship.<sup>1213</sup>

Like the other images, our model still struggles with the neck. While the image much better captures depth there are some non-physical features. Below the jaw the neck bulges and might be mistaken for an Adam's Apple or a weird camera angle. There is also some slight banding around the lower part of the neck, half way between the jawline and shirt collar. Additionally, the jaw bulges, as if merging a forward facing face and a slightly turned face.

Interestingly, we do not observe systematic depth errors like StyleGAN and StyleSwin. Yet we do notice our images also create unique skin textures, different than StyleGAN3 or StyleSwin. These are most noticeable around the lips and the subject's nose. Larger lines may be found around the forehead and like StyleSwin, we can trace these to the attention maps (Chapter 4.7.4). Additionally, we find a systematic blue speckling, most easily noticed around the eye or the beard of

<sup>&</sup>lt;sup>12</sup>Irises do not have blue or green pigments. Instead these colors are created through eye structure. Both are brown eyes with lower amounts of melanin.

 $<sup>^{13}{\</sup>rm Green}$  and Blue eyes are sexually dimorphic. Blue is more common among men, green is more common among women.

the subject's chin. This may pattern can be difficult to detect depending on the reader's monitor.

4.7.4 Attention To Details. To better understand the cause of these systematic issues we visualize the attention maps across our generator. We modify the standard roll-out attention map to account for the localize windows and have to undo the shifting for both methods. This method is open sourced with our project and appears to be the first method for visualizing attention maps for either Swin or  $\mathcal{N}ATTEN$ . For quick reference Figure 20 illustrates these intermediate layers for both StyleNAT (Fig. 20a) and StyleSwin (Fig. 20b).

When looking at the final layer of StyleNAT (Figure 21) we observe many of the same patterns that we found during visual inspection. The matching patterns help validate our interpretation of these attention maps and our method of extraction. In the first transformer we are able to observe the same banding lines around the face. Using these as reference can aid in their detection if this was previously unclear. We also notice that these appear in the dense heads in the second transformer. Similarly, we are able to observe the speckling, especially around the chin. In most of these maps we are able to observe a circular shape in the forehead, which corresponds to a hair curl. Careful inspection of this curl in Figure 19 shows that this is too perfectly circular and may actually me more similar to a statistical droplet that is better masked. Given this, similar explanations might apply to the blue speckling.

95



(a) We progressively see the face form and notice the first head captures local features while the last head captures global features. Structural features appear early on while details are generated at higher resolutions.



(b) Low resolutions show decoherence and artifacts are not removed in progressive resolutions. We do not observe strong differentiation between local and global features. These maps explain the blocking artifacts discussed in section 1 of StyleSwin [169].

Figure 20. Visualization of the first and last attention head progressing through StyleNAT. We start at a resolution of  $16 \times 16$  and grow to  $1024 \times 1024$ . We generate 50 samples from each network and choose the best image from the sample to make comparisons as fair as possible. The top row shows the first attention head, with 2 transformers per resolution level. The bottom row shows the last attention head. Fig. 20a visualizes for StyleNAT (**ours**) and Fig. 20b follows StyleSwin [169].

Critical to the verification of our hypothesis, we observe that the attention maps form two distinct groups, directly corresponding to our partitioning. This grouping occurs in different transformers and at different resolutions, directly matching the head partitioning regardless of the total number of heads. Our dense kernels have much smoother attention maps, suggesting they are attending locally. Our sparse global kernels have more patterns and the highlighted regions (such as ears and background) correspond to the long range patterns we expect. While the dense maps also highlight some of the backgrounds their boundaries closely correspond to similar coloring, which would be a local feature rather than global. These maps strongly suggest we have achieved our goals, even if our method is not fully optimized.



(c) Transformer 0

(d) Transformer 1

Figure 21. Visualization of Attention maps (Figs. 21a and 21b StyleNAT, Figs. 21c and 21d StyleSwin) for transformers at the 1024 resolution. Top row corresponds to localized dense kernels (k = 7, d = 1), second row corresponds to the sparse dilated kernels (k = 7, d = 128). Banding and blue speckling on images directly corresponds to those in the attention maps. We observe divergent attention maps across heads, matching our exception.

# CHAPTER V DISTILLATION OF INVERTIBLE NETWORKS

I know numbers are beautiful. If

they aren't beautiful, nothing is.

Paul Erdős

**Nota Bene:** This work is based on the previously published co-authored work Distilling Normalizing Flows [157].

- Steven Walton was the primary author of the source code and performed the majority of experiments. Steven was also the primary author of the paper.
- Valeriy Klyukin made significant contributions to the source code and to the experiments. Valeriy also provided feedback and contributed to the writing of the paper.
- Maksim Artemev provided software engineering expertise and feedback influential to the design and experiments. Maksim also contributed to the writing of the paper.
- Denis Derkach provided general support and feedback for the project.
- Nikita Orlov provided general support and feedback for the project. He also helped provide access to the hardware used for our experiments.
- Humphrey Shi was the advisor, contributing overall guidance on the research as well as funding for the work. Humphrey also contributed to the writing of the paper and ensuring research stayed on track.

A frequent task of interest for generative models is the "*reversibility* problem" (GAN inversion, etc) [5, 39]. That is, determining the map from the image

to pre-image (Figure 1), or can be seen by mapping the model's representations back to the data. This is sometimes referred to as the "inverse problem," but an inverse does not always uniquely exist, so we avoid such nomenclature unless one does. Of particular concern are Tractable Density Models (Figure 3), which allow for a formal, mathematical, description of the image's probability density function. These models are of special interest to many scientists as the formalization allows for better interpretability [83, 15, 34, 48, 73]. Reversible models increase utility by allowing manipulation of the data generating process, while invertible models extend this further as manipulation on the image corresponds to a unique modification in the pre-image (and vice versa).

### 5.1 Model Distillation

Unfortunately these models are not as easily trained due to their more restrictive architectures. As discussed in Chapter 2.2.2, larger models allow for more smoother solution spaces, and thus can reduce difficulties in optimization. Fortunately, there usually exist multiple trajectories that provide a mapping from the domain to range, and any such mappings are equivalent. This encourages the training of large models, but their size makes their usage cumbersome. Deployment may be limited, as they may require greater system resources than available on many systems, as is common with LLMs [10, 24]. Methods like quantization [38] and reduced precision can help reduce the computational burdens but may themselves require specialty hardware or instruction sets. Early works by Buciluă et al. [11] showed that an ensemble of models [25] could be compressed into a single model. Further work by Hinton et al. [61] showed that smaller "student" models could reduce test error by matching the logits of a larger "teacher" and more accurate model, effectively distilling the large model's knowledge. These ideas

99

expanded, demonstrating the effectiveness of to other architectures, studying what kinds of information transfers, and how to optimize such knowledge transfer.

## 5.2 Distilling Normalizing Flows

While knowledge distillation has been widely studied, these efforts have not extended to the architectures of Compositional Normalizing Flows. There only exists limited studies of knowledge distillation for Normalizing Flows, such as Baranchuk et al. [4], the work used a conditional normalizing flow for the teacher but removed constraints of invertibility and thus the student network is no longer a normalizing flow. Such works do not take advantage of the unique properties that these architectures have, which similarly limits their capabilities. Flow models are naturally invertible, learning compositional diffeomorphisms to produce their final mappings. Given a network, f, they may be broken down into k sub-networks that are each diffeomorphic themselves:

$$f = f_1 \circ \dots \circ f_k \tag{5.1}$$

The common formulation of these architectures is to use the Change of Variables formula, where each subnetwork contains the same information but in a different coordinate system. For probability distributions, we can specify such a coordinate change as follows:

$$p_x(\mathbf{x}) = p_u(\mathbf{u}) \left| \det J_f(\mathbf{u}) \right|^{-1}$$
(5.2)

where we are mapping from density  $p_u(\mathbf{u}) \mapsto p_x(\mathbf{x})$ . Here det  $J_f(\mathbf{u})$  denotes the absolute value of the determinant of the Jacobian of  $\mathbf{u}$ . Given the compositional nature of these flows we may similarly calculate the final Jacobian determinant through the product of those in each transform:

$$\det J_{\mathbf{f}}(\mathbf{x}) = \prod_{i=1}^{n} \det J_{\mathbf{f}_{i}}(\mathbf{x}_{i})$$
(5.3)

Unfortunately the Jacobian determinant is often computationally expensive, and much research has been dedicated to finding expressive architectures with more computationally efficient determinant [86, 26, 27, 30, 64, 8, 42] calculations.

Due to the unique construction of these models, there are unique opportunities for transfer between a teacher and student model. We seek to formalize these relationships and encourage further studying. We show that there are three main categories in which we may transfer knowledge between teacher and student and formalize these relationships.

5.2.1 Categories of Flow Distillations. We present these categories of knowledge transfer in a general sense, noting that arbitrary loss functions,  $\mathcal{L}$ , may be used between them. We note that since each layer in a Compositional Flow represents a probability distribution, that this presents unique conditions that may not be present within other networks. Given the comparison between two distributions it is often natural to use a Kullback-Leibler (KL) divergence.

$$\mathcal{L}(\theta) = D_{KL}\left[p_x(\mathbf{x})||p_u(\mathbf{x};\theta)\right]$$
(5.4)

$$= \sum p_x(\mathbf{x}) \log\left(\frac{p_x(\mathbf{x})}{p_u(\mathbf{x})}\right)$$
(5.5)

Though it is not necessary to make such restrictions and any divergence or metric may be used for minimization. One may have different interests in what actually is desired to be minimized, potentially more interested in probabilistic constraints or geometric.

5.2.1.1 Latent Knowledge Distillation. We define Latent Knowledge Distillation,  $\mathcal{L}_{LKD}$ , to be the distillation between the final learned distributions of the teacher and student. This may be thought of knowledge distillation in the

traditional sense, similar to that of Hinton et al. Specifically, we define this as knowledge transfer when data processing in the normalizing direction.

$$\mathcal{L}_{LKD}(t, s, x) = \mathcal{L}_r(t(x), s(x)) \tag{5.6}$$

5.2.1.2 Intermediate Latent Knowledge Distillation. Due to the compositional nature of these flows, there forms more natural relationships between intermediate layers. For example, if we wish our student to be half the size of the teacher network we may view every two flow layers in the teacher as equivalent to a single flow layer in the student. In this manner we would compress two teacher layers into a single layer in the student. This framing may not be work similarly with other architectures, and may require significantly more complex maps to be found which have no guarantees of invertibility.

$$\mathcal{L}_{ILKD}(t, s, x) = \sum_{i} \mathcal{L}_{r}(t_{i}(x), s_{i}(x))$$
(5.7)

Due to the bidirectional nature of these flows, such intermediate knowledge may be transferred when data processing in either direction. We believe this form of distillation is deceptively simple but may provide rich areas of study, especially in the domain of Optimal Transport.

5.2.1.3 Synthesized Knowledge Distillation. The invertible nature of flows allows for symmetry in our models. While LKD performs knowledge transfer in the normalizing direction, SKD is performed in the generating direction. We can view SKD as the inverse of LKD.

$$\mathcal{L}_{SKD}(t, s, z) = \mathcal{L}_r(t^{-1}(z), s^{-1}(z))$$
(5.8)



Figure 22. Illustration of knowledge transfer between two Glow [86] based models.  $\mathcal{L}_{LKD}$  represents the Latent Knowledge transfer between the learned representations.  $\mathcal{L}_{ILKD}$  is the knowledge transfer between intermediate representations.  $\mathcal{L}_{SKD}$  is the knowledge transfer via synthesized data.

Importantly, this form of knowledge transfer need not be performed via conditional generation. This means, unlike cycle loss [180] or similar styles, we do not need a ground truth label. Instead, we can simply ensure that given the same sample from the learned distribution, the generative outputs are aligned. In this manner we should treat both the teacher and student as if having the same learned distribution.

Currently, the generative capabilities of Normalizing Flows makes this form of distillation more difficult to analyze. Specifically, image generation has not matched the performance of other architectures, such as GANs and Diffusion. Only a few flow models have been trained with large numbers of parameter which has shown great promise in the capabilities. Prior to TarFlow [168] and StarFlow [47], DenseFlow [44] and MaCow [115], were the largest trained Normalizing Flows, having 130M and 177M parameters, respectively. Even TarFlow, having variants at  $\approx$ 475M parameters and  $\approx$ 820M parameters, is much smaller than many diffusion models which have well over a billion parameters [87, 17, 65, 176, 37] while StarFlow is, to the best of our knowledge, the only multi-billion parameter Normalizing Flow. Additionally, there has been recent success with Flow Matching [20, 105, 107, 14, 104] has presented promising results in this area but are restricted to continuous flows operating on conditional velocity fields. The principles should similarly apply but in this work we focus on more general approaches. With this in mind, we should expect some complications with unconditional generation.

5.2.1.4 All Together. We can combine all these distillations together to create a stronger and unifying distillation method. We may provide weights to each distillation type, using hyperparamter  $\lambda_i$ . Combining with our standard flow loss, we can write our final loss as:

$$\mathcal{L}(t, s, x, z) = \lambda_0 \log(p_s(x)) + \lambda_1 \mathcal{L}_{LKD}(t, s, x) + \lambda_2 \mathcal{L}_{ILKD}(t, s, x) + \lambda_3 \mathcal{L}_{SKD}(t, s, z)$$
(5.9)

We may compress this format by writing  $\mathcal{L}_{(I)LKD}$  recognizing that the LKD loss may be viewed as another step. We write explicitly due to its importance as a boundary condition.

#### 5.3 Distillation Experiments

Given these classes of knowledge transfer we can see that we can use both directions of data processing with these networks to better align their mapping trajectories. Our goals are to determine the capabilities of these differing distillation methods and better understand their strengths and weaknesses. Given this framework there is a large search space. We do not intend to provide a complete search, but focus on demonstration and forming the foundations. For simplicity, we will use Lasso Regression for losses within this work,  $\mathcal{L} = L_1$ . For hardware, all experiments were performed using a single NVIDIA Tesla V100 GPU.

To evidence our hypotheses and the utility of our framework we use two classes of data. Our first will focus on density estimation (Chapter 5.3.1) and then focus on synthetic image generation (Chapter 5.3.2). The latter of which is a significantly more challenging task for these architectures. For our models, we use the Masked Autoregressive Flow (MAF) [121] and Generative Flow with Invertible  $1 \times 1$  Convolutions (GLOW) [86]. For our GLOW models, we use the affine coupling setting. Unfortunately, the autoregressive nature of MAF makes sampling intractable, so we do not perform SKD distillation with this mode.

These two models are commonly used and have been much more thoroughly studied. These models also significantly differ in architectures, which will help us determine the capabilities of these distillation methods. For our GLOW models we perform our ILKD distillation between flow levels, matching the diagram in Figure 22. For MAF we match across depths, pushing every two depths from the teacher into the student. These are not necessary choices but we believe present natural points for communication between teacher and student. In all studies we seek to make large reductions in model parameters, as this will best demonstrate our ability to compress knowledge into our student networks. Our MAF students are half the size of their teachers while we do not let our GLOW student contain more than 30% as many parameters as their teachers.

GLOW	Level (L)	Hidden	
Student	3	32	
Teacher	3	64	
MAF	Depth (K)	Hidden	
MAF Student	Depth (K)	Hidden 32	

Table 13. Model configurations for generation of density estimation. Provided for GLOW and MAF architectures. Number of levels (L) is equal to 1. Notation is taken from the original paper [86]. 5.3.1 Density Estimation. For our tabular data experiments we perform density estimation on five common datasets. We use *Metric*, *POWER*, *GAS*, *HEPMASS*, and *MINIBOONE* from the UCI Machine Learning Repository [102] and BSDS300 from the Berkeley Segmentation Dataset and Benchmark [117].

Our model configurations are presented in Table 13. For our MAF model we focus on expanding the teacher's depth, letting the teacher model have twice the depth. For

GLOW we let our models have the same number of levels and flow steps but double the number of hidden neurons in the teacher. For MAF this results in the teacher having approximately double the number of model parameters as the student. For GLOW, this results in the teacher having approximately five times the number of parameters as the student. GLOW is a much more powerful model than MAF and thus we expect the ability to greatly reduce model parameters.

For each model we train the flow for a fixed  $10^4$  iterations with a batch size of 65, 536 (2<sup>16</sup>). We use a learning rate of 5 ×  $10^{-5}$ , applied to the AdamW optimizer [85, 113]. The results of these runs can be found in Table 14. Rows

Architecture	Model	POWER	GAS	HEPMASS	MINIBOONE	BSDS300
	Student	-0.228	5.967	-22.668	-17.251	147.298
	LKD Student	-0.132	6.008	-22.332	-17.136	162.103
GLOW	ILKD Student	-0.133	6.191	-22.187	-17.008	163.148
	SKD Student	-0.078	6.515	-21.852	-16.130	163.953
	Teacher	0.143	6.604	-19.938	-13.597	165.702
	Student	-0.152	4.385	-21.904	-15.314	155.463
	LKD Student	-0.149	4.473	-21.389	-15.217	155.629
MAF	ILKD Student	-0.145	4.502	-21.223	-15.184	155.785
	SKD Student			Intractable		
	Teacher	0.133	5.887	-20.662	-13.488	159.442

Table 14. Averaged test log-likelihood (in nats) for unconditional density estimation (higher is better) across multiple runs.

labeled "Student" and "Teacher" contain no distillation and are the baseline values of or models. For LKD and ILKD we set  $\lambda_0 = 0.9$  and  $\lambda_1 = \lambda_2 = 0.1$ . For SKD we again decrease  $\lambda_0 = 0.85$  and set the rest to 0.075. These weights were chosen to set control the weights as percentages of the whole loss, and are likely non-optimal. We find that most performance comes from the introduction of the LKD student, with +10% for GLOW but only +1% for MAF on BSDS300. We get continued improvements with ILKD, +6% for GLOW and +1% for MAF. With GLOW we can cleanly sample from our distribution and find an additional +5% gain, for a total improvement of +12.7% improvement above our student model.

On BSDS300, our final student GLOW model is has  $\approx 25.5$  as many parameters as the teacher model while achieving 98.94% the accuracy. We also compare the computational performances differences of our teacher and student models in Table 15, directly comparing the number of model parameters and their throughput. This result suggest strong distillation capabilities, with the teacher passing nearly all its "knowledge" to its student. We do not believe our parameters are near optimal, but this provides significant evidence to our theory that suggests

it may be possible to fully distill the teacher's knowledge into the student, under the assumption that the student's latent representation is at least as large as the latent data manifold.

Arch	Model	Metric	POWER	GAS	HEPMASS	MINIBOONE	BSDS300
	Ct. Jant	Time (ms)	$2.32\pm0.16$	$2.46\pm0.10$	$2.55\pm0.35$	$2.47\pm0.07$	$2.45\pm0.07$
CLOW	Student	Params $(K)$	13.8	14.20	17.4	24.9	34.4
GLOW	Teacher	Time (ms)	$3.65\pm0.26$	$3.88\pm0.09$	$4.41\pm0.28$	$3.95\pm0.11$	$3.89\pm0.14$
		Params $(K)$	86.7	87.8	96.3	114.2	134.7
	Student	Time (ms)	$2.00\pm0.21$	$1.98\pm0.19$	$1.82\pm0.05$	$1.82\pm0.05$	$1.91\pm0.22$
MAF	Student	Params $(K)$	5.0	5.6	9.4	15.9	21.8
MAF -	Toochor	Time (ms)	$3.34\pm0.22$	$3.22\pm0.18$	$3.34\pm0.23$	$3.36\pm0.26$	$3.45\pm0.20$
	reacher	Params $(K)$	10.1	11.2	18.9	31.8	43.6

Table 15. Time consumption for a single batch inference averaged across multiple batches and the number of parameters (in thousands). Average time (ms) and number of parameters (in thousands) are reported.

CIFAR-10				CelebA		
	Levels $(L)$	Hidden	Params	Levels (L)	Hidden	Params
Student	8	512	11.0M	16	256	7.9M
Teacher	32	512	44.2M	32	512	61.2M

Table 16. Model configurations image generation tasks (GLOW). Notation is taken from the original paper [86]. All models have a depth (K) of 3.

5.3.2 Image Generation. To demonstrate capacity in image synthesis we demonstrate our methods on using the CelebA [108] and CIFAR-10 [147] datasets. Due to MAF's low performance we only perform these experiments using the GLOW model. We use the same settings as before, except reduce the batch size to 32. Model configurations and sizes can be found in Table 16. All models have a constant depth of 3, like before. In the CIFAR-10 student we only quarter the number of levels but in the CelebA student we halve the levels and halve the number of hidden parameters.
In our experiments, we found that SKD appeared to be helping with model distillation but that the results were unstable, and we were unable to complete training. We suspect that this was due to the poor image generation quality, as can be seen in Figures 23 and 24. For CIFAR-10 our teacher model obtained a Bits per Dimension (bpd) of 3.423, which is slightly worse than Kingma and Dhariwal's work, while our CelebA model reached 2.474 bpd.

The results of our experiments can be found in Table 17. In terms of bpd, the ILKD student showed an improvement of 0.5%, but note that the teacher is only 2% better. On CelebA we see a similar 0.16% improvement, while the teacher is only 0.2% better. When looking in terms of FID we instead see a 2.5% (of 3.8%) on CIFAR-10 and 20% (of 45%). We note that our CIFAR-10 student is  $\approx 25\%$  the size of its teacher and the CelebA is  $\approx 13\%$ .

	CIFAR-10		CelebA	
	bpd	FID	bpd	FID
Student	3.498	71.177	2.479	68.127
ILKD Student	3.481	69.371	2.475	54.480
Teacher	3.423	68.503	2.474	37.460

Table 17. Metrics for the image generation task for the GLOW architecture using ILKD on the test set: bits per dimension and FID (lower is better).

To ensure the knowledge distillation does not corrupt the hidden space, we need to ensure that random samples from the students still maintain similar quality images. With high dimensional information, it is possible for Normalizing Flows, and other models, to have a small KL-Divergence but also have poor sampling quality. This appears to be the case in our results, where our teacher and student have similar bpds but very different FIDs. Similar to Chapter 4.7, we need to be careful in how we analyze our results given the biases of our metrics.



Figure 23. CIFAR-10 samples from teacher model (Fig. 23a), student model (Fig. 23b), and student model with no knowledge distillation (Fig. 23c). All images are generated at  $32 \times 32$  resolution and with a temperature of 0.7.

Considering the biases of FID, high quality samples can only happen if there is a sufficiently good enough cover, determined by the Inception Network, within the learned latent space. Thus, we propose to measure the quality of the inferred samples for randomly chosen images  $\boldsymbol{u}, \boldsymbol{v}$  and an  $\alpha \in [0, 1]$ , where  $\alpha$  is the interpolation fraction. The preserved norm of the latent vector can be defined as:

$$\boldsymbol{f}(\boldsymbol{u},\boldsymbol{v},\boldsymbol{\alpha}) = \left((1-\alpha)\boldsymbol{f}(\boldsymbol{u}) + \alpha\boldsymbol{f}(\boldsymbol{v})\right) \cdot \frac{(1-\alpha)||\boldsymbol{f}(\boldsymbol{u})|| + \alpha||\boldsymbol{f}(\boldsymbol{v})||}{||(1-\alpha)\boldsymbol{f}(\boldsymbol{u}) + \alpha\boldsymbol{f}(\boldsymbol{v})||}$$
(5.10)

The results of this method are provided for CelebA dataset in Table 18. This table shows that the ILKD Student performs significantly better than the student without knowledge distillation, independent of temperature. Both the student and ILKD student show substantially larger improvements in FID with reduced temperatures showing their improperly configured latent representations. In both cases, the ILKD student shows a 30% improvement over the student.

While the CIFAR-10 (Figure 23) samples are more difficult to differentiate, it is clear that in the CelebA generation (Figure 24) that our distilled images are significantly better than those in the original student.

	T = 1.0	T = 0.7
Student ILKD Student	40.159 <b>28.413</b>	28.432 <b>19.688</b>
Teacher	19.062	16.382

Table 18. CelebA FID values of images obtained by interpolation in the latent space of trained models.



Figure 24. CelebA samples from teacher model (Fig. 24a), student model (Fig. 24b), and student model with no knowledge distillation (Fig. 24c). All images are generated at  $64 \times 64$  resolution and with temperature=0.7.

### 5.4 Conclusion

Our work we sought to build foundations for investigating the capabilities of knowledge distillation in Normalizing Flows. Our work is not intended to be comprehensive, but to demonstrate how effective these methods are and motivate further study. These architectures are underrepresented, but the theory and practice shows that these models may offer unique capabilities to the field of machine learning. The success of large Normalizing Flows like TarFlow [168, 106] and StarFlow [47] show promise in their capabilities, and that similar scaling success may be found here too.

With a large variety of flow types [154], there are many avenues open to build upon this work. With many flow architectures often presenting computational challenges, this may provide an avenue to resolve some of them. We believe our foundation will extend ensembles of models, letting students benefit from the training advantages of differing flow types. We also believe that our process will extend to distillation between differing architectures. This may offer unique capabilities, such as replacing computationally difficult flow steps with simpler ones. While these models are often overlooked due to their mathematical formulations, we believe that continued study will show these models to be highly capable and researchers will highly benefit through their greater interpretability.

# CHAPTER VI

## CONCLUSION AND FUTURE DIRECTIONS

People think of education as something they can finish.

Isaac Asimov

#### 6.1 Summary

This work explored the importance of neural architectures and how they influence not only a machine learning model's capacity to learn, but also how to do so in computationally constrained environments. Even as compute infrastructure grows, there exists strong pressures to use what we have more efficiently. If we are able to do so, then our progress can outpace our growth in compute.

Chapter 2 gave an overview of the subject matter necessary understand to influence neural designs. This serves not to only help the reader understand the problems that need be addressed, as well as illustrates the many pitfalls and subtleties that exist. The remains many challenges when scaling models (Chapter 2.2) and data (Chapter 2.2.1), highlighting the importance of algorithmic or architectural improvements (Chapter 2.3). Chapter 2.4 also discussed the difficulties when defining objectives and developing adequate measures, known as "The Alignment Problem," and the critical relationship which influences neural designs.

Chapter 3 focuses on the importance of data encoding and decoding. Specifically in how to improve these designs for Vision Transformers, allowing them to better automate discovery of underlying data structures. Our work demonstrates that without efficient encoding and decoding, we may inadvertently hinder the performance of these models. These inefficiencies, ideally, may be overcome through brute force scaling, but through careful design we may reduce our costs, allowing us to do more with less.

Chapter 4 shifts focus to modifying the core architectural designs. Through understanding the ways in which architectures operate and how leveraging structures within the data allows for more informative decisions in the design of core processing units. Neighborhood Attention modifies the Vision Transformer architecture to increase computational and memory performances, leveraging the natural localization biases of the data while still being able to recover global structures. Through our improved design, allowing attention heads to operate over independent receptive fields we are able to reduce the sacrifices made and our models can uncover structures they previously could not.

Chapter 5 focuses on architectures with structurally focused designs. This studies the way in which information is processed through Normalizing Flows and how this can be used to create efficient knowledge distillation. Understanding the mathematical structures within architectures allows for better design and efficient methods which reduce model size and required computation.

Putting this all together, this dissertation positively answers the question: Can we design neural architectures to be smaller, faster, and cheaper without sacrificing performance?

#### 6.2 Future Directions

While this dissertation affirms that our models *can* be more efficient, we are unable to provide a complete answer as to *how*. Despite the significant strides the field has made in recent decades, we are only at the beginning. With the rapid development of machine learning, it is easy to lose sight of the larger goals. Therefore, we briefly discuss our core goals to ensure our future work remains aligned.

6.2.1**Core Challenges.** A core challenge still must be solved in order to efficiently design our neural architectures. Largely, machine learning deals with the problem of alignment. Without a strong mathematical foundation we are unable to verify how well our models are aligned to our intended goals. To draw an analogy, in Chapter 4 we discussed the limitations in our ability to determine the realism of the generated imagery. This hinges on our inability to describe the "realness" of our images in a rigorous way. Our Generative Adversarial Network cleverly trains an adversarial detector as a means to bypass this formalism. While this proxy has allowed us to dramatically improve the quality of the generated imagery, it is not uncommon for the generator to become misaligned. Instead of generating high quality imagery, it may instead produce incoherent images that have the right statistics to deceive the generator. Without formalism we must take great care to ensure that we do not fall for the same trap as the detector. Our metrics play a critical role in driving our research and designs, but we must not blindly follow a map that may drive us off a cliff instead of to our intended destination. Until such rigorous formalism is developed we must remain skeptical of ourselves. We cannot forget this fundamental problem while addressing more specific challenges.

Some practical advice may be offered by Donald Knuth: If you find that you're spending almost all your time on theory, start turning some attention to practical things; it will improve your theories. If you find that you're spending almost all your time on practice, start turning some attention to theoretical things; it will improve your practice.

**6.2.2 Scaling.** The subject of this thesis would be incomplete without revisiting the issue of scaling. The question remains: *Do these methods work as data increases and as model size increases?* This is, after all, the fundamental question addressed in Chapter 1. This research has been carefully designed to ensure that the answer will be *yes.* While we cannot confirm this conclusion without access to large compute infrastructures to verify these beliefs, we have strong evidence for this belief.

In Chapter 3 we made only minor changes to the network, so this should be surprising if it were to make the much larger networks substantially unstable. These ViT networks have shown success at scale and we believe our modifications should make difference, with respect to scalability. In this work, we showed that CCT has strong performance across small to medium scales, *strictly dominating* ViT at every step of the way. The saliency maps suggest we are removing a fundamental flaw found in ViTs, which should only lead to greater stability.

In Chapter 4 we similarly suspect that these methods are highly scalable. We trained our single  $1024 \times 1024$  run in an effort to show this, and Figure 16 suggests that neither our small resolution nor our large resolution training achieved peak performance. The design of the Hydra-Neighborhood attention specifically allows more configurations, and thus adaptability. The work of StyleGAN-XL mostly saw success to its own scaling, and we suspect the same here. While the LSUN church results did not perform as well, all results suggest that this is likely an embedding problem, with the attention dimension being too small. If this is a correct assumption, then scale should yield substantial improvements to this experiment.

In Chapter 5 we again see no blockers. The theory behind the methods suggest that training is better performed with large scale models but that these can also be reduced in size through distillation. Here, the question is not a matter of if the procedure can continue to scale, but by how much can we compress these large flows. With works like TarFlow and StarFlow resenting huge state of the art models, this only makes our methods more valuable.

At this time we do not have the computational resources to prove that these methods are scalable, but there is nothing that suggests that they won't be just as effective, if not more, at scale.

6.2.3 Ingress and Egress of Data. In Chapter 3 our work focused on making the most of our data, allowing our core architecture to make better utilize available data. It is key that we provide our models with data in the formats that best suit them. Similarly, we must be careful in how we extract the data from them, ensuring we do not lose useful relationships they have uncovered.

6.2.3.1 Parameterization. Our Compact Transformers improved upon the patching and embedding method of the original Vision Transformer by recognizing how non-overlapping patches removed structure from the data. By using small kernels and overlapping patches our embedding is able to better preserve the structure within our data. The size of these kernels, strides, and other parameters were determined through directed search optimizing a validation set. These specific relationships may not hold for data that has other inherent biases, and this process may need be done again. Similarly, there is no reason to believe that those we found are optimal. By revisiting former models and architectures many researchers have demonstrated that their performances can be improved in many ways [161, 111, 162]. The method itself does not prevent these hyper-

parameters from being learned. More optimal parameters may be found through HyperNets [139], or other optimization methods.

6.2.3.2 Automated Preprocessing. The CNN based structure itself causes some of the image structure to be lost and new methods should be investigated which can better embed these. Most importantly, the CNN places greater importance on pixels that are local spatially. While we expect this relationship to be strong it may not always be true, nor should we assume that in some cases we may wish to place greater importance on more global features. This is, after all, the same reasoning that led to the development of the transformer architecture. Given these problems attention needs to be given to develop ingestion methods that flexibly adapt to the data. Modern machine learning methods are becoming multi-modal, processing language, vision, and other manners of data. This necessitates new forms of embedding that can recognize and adapt to the data, performing the preprocessing for us.

Another benefit of the CNN architecture is that it is flexible to the data shape. CNNs mainly rely on a single dimension of our data, channels, allowing us to more easily accommodate images of varying dimensions. As we seek to make our models multi-modal this demands that we develop architectures to ingest data of differing types and dimensionality. We may inefficiently provide patchwork by padding or replicating data, but these may provide more hindrances than utility. It becomes important to investigate means of arbitrary data ingestion, that can embed our data without the loss of structures within the data.

6.2.3.3 Making The Most of it. On the other side our SeqPool method demonstrated that our network had learned useful relationships that could help in image classification but were unavailable to our classifier sub-network. This

demonstrates the ease in which we may underutilize our networks. Certainly our SeqPool method has not extracted all available information from our network. The method allows for the importance of each of our tokens but asks that a lot be done within this simple step. This constrains our core network which almost certainly adapts to this bottleneck. Other extraction methods should be explored which allow greater flexibility. While the classification network can, in some ways, act similar to the linear layers of the transformer, they do not have the same expansion layers that allow data untangling as the traditional transformer. Finally, while the classification sub-network remains the de-facto solution for converting our multidimensional relationships into linear ones, there likely exists better methods for this. While being seemingly less exciting, finding such architecture may lead to transformative impacts in the field.

Further improvement needs to be made to disentangle data. We use batching and pooling within our network and while this can speed up and even benefit training it can also entangle data, causing our networks to over aggregate. Our work used a learnable class token to constrain our network to disentangle these data, but further study to better understand and improve this disentanglement remains.

6.2.4 Core Processing Architectures. Our work in Chapter 4 demonstrated that even seemingly simple changes to the core neural architectures can have tremendous impacts on our performance. An idea that was seemingly simple, yet non-obvious due to the full attention mechanism naturally having the ability do this and other restricted attention methods, not necessarily sharing the flexibility of Neighborhood Attention. Our work placed greater focus on demonstrating our hypothesis around integrating local and global features than

on ensuring the method was computationally efficient. While significant advances have been made to the  $\mathcal{N}ATTEN$  kernel [53, 54], these improvements have not been made to optimize the independent head approach. The algorithm developed for the paper was not intended to be optimized for speed or memory and instead for better readability. Simple modifications can be made to better parallelize the data processing and further improve performance.

6.2.4.1 Flexible Learning. The number of configurations available made our method flexible, but it was not possible to exhaustively test these within our limited computational budget. Further work should be performed to tune these parameters. Initial testing has shown that we are able to modify these kernel and dilation parameters during the training of our networks. This suggests that these parameters may be optimized during training, allowing them to be adapted to the data being modeled. This may resolve the need to manually craft these hyper-parameters but research is still needed to determine the stability and effectiveness of this process.

The method has also not been tested on other architectures, such as diffusion model. Our focus was on demonstrating the utility of the method rather than developing the best image generator. Works like Hourglass Diffusion [17] have demonstrated substantial improvements due to integrating Neighborhood Attention but remain outside our computational budgets to integrate Hydra-NA. We believe that our results will apply much more broadly than just to GANs, but this has yet to be demonstrated.

6.2.4.2 Is Beauty in the Eye of the Beholder?. The work also highlighted the challenges of metrics and alignment. Our previous CCT model focused on classification tasks, where there exists a much clearer objective and

measure. In classification, the labels are not always accurate as some noise exists around identification, but there is at least a clear metric to define if the model produced some expected output or not.<sup>1</sup> In the case of generative imagery, no such metric exists and even may not exist. For millennial scholars have attempted to create formal definitions of beauty, but have yet to find success. Therefore, those studying generative modeling must then take great care concerning the alignment problem and we must be creative in determining how we may achieve better measures and importantly, optimization methods. Naïvely, we may unintentionally develop models which can only produce content that is appealing to limited groups. This requires us to be suspicious of ourselves, recognizing our own cognitive biases. Works like Stein et al. [140] help by using large human studies, but even these studies have population biases.

While diffusion models optimize towards a probability distribution function they often still make certain assumptions about the data, such as being *i.i.d.* This method limits our generation to only match distributions similar to those we train on but so far have been unable to demonstrate the ability to capture the depth and nuance that art is known for. Without incorporating seemingly small details our generators may be unable to escape an uncanny valley which is blatancy apparent to some but invisible to others. Through studying other forms of measure and optimization we most certainly will find better neural architectures and make steps to reducing these discrepancies in human preference.

**6.2.5** Structurally Aware Architectures. In Chapter 5 we moved from transformer architectures to study Normalizing Flow architectures. These have seen significant progress, especially with recent works in Flow Matching [105].

<sup>&</sup>lt;sup>1</sup>We should still take care to recognize that even a classification metric is not foolproof.

While our work may not directly apply to these methods due to their continuous nature, we show how our compositional flows layers can be compressed. This presents the question to determine if these compositional flows can be efficiently transformed into continuous ones. This may allow for more flexible relationships to be found or by turning our continuous flows into compositional ones we may gain more interpretability. Understanding how our models operates remains a core challenge, relating to our issues of alignment.

For our specific work there is quite a lot of potential future work we see here. None of our models were optimized and we were focusing on demonstration due to computational limitations. The SKD distillation should promise in the density estimation experiments but was unstable in the more difficult image generation tasks. As flow architectures become more powerful, this too is likely to increase in capacity. One may also wish to add schedulers to the weights of our distillation methods, and particularly with SKD.

More broadly, with the larger discussion of this thesis, the work demonstrates the importance of understanding the structures our neural architectures do and do not preserve. Normalizing Flows provide nice mathematical structures that are easier to study than other neural architectures, but this only highlights the need to better study the limits and capabilities of other architectures, which are significantly more difficult than Normalizing Flows.

#### 6.3 Conclusion

This thesis presents work that demonstrates the potential for making machine learning models more efficient through careful design of our neural architectures, specifically applied to Computer Vision problems. We show that we can train smaller models, from scratch, while greatly reducing compute, memory, and

the costs of gathering and labeling data. We also show that seemingly trivial modifications may be made that have significant impacts on performance. While many of these changes may seem obvious post-hoc our work only highlights how hidden such simple modifications are. Like our education, the pursuit of more efficient models is something that can never be finished, and many grand challenges still lay waiting to be discovered.

#### Bibliography

- Amro Abbas, Kushal Tirumala, Dániel Simig, Surya Ganguli, and Ari S Morcos. Semdedup: Data-efficient learning at web-scale through semantic deduplication. arXiv preprint arXiv:2303.09540, 2023.
- [2] Sören Auer, Christian Bizer, Georgi Kobilarov, Jens Lehmann, Richard Cyganiak, and Zachary Ives. Dbpedia: A nucleus for a web of open data. In *The semantic web*, pages 722–735. Springer, 2007.
- [3] Dzmitry Bahdanau, Kyunghyun Cho, and Yoshua Bengio. Neural machine translation by jointly learning to align and translate, 2016.
- [4] Dmitry Baranchuk, Vladimir Aliev, and Artem Babenko. Distilling the knowledge from conditional normalizing flows. In *ICML Workshop on Invertible Neural Networks, Normalizing Flows, and Explicit Likelihood Models*, 2021.
- [5] David Bau, Hendrik Strobelt, William Peebles, Jonas Wulff, Bolei Zhou, Jun-Yan Zhu, and Antonio Torralba. Semantic photo manipulation with a generative image prior. ACM Transactions on Graphics, 38(4):1–11, 2019.
- [6] Irwan Bello, Barret Zoph, Ashish Vaswani, Jonathon Shlens, and Quoc V. Le. Attention augmented convolutional networks. In Proceedings of the IEEE/CVF International Conference on Computer Vision (ICCV), 2019.
- [7] Iz Beltagy, Matthew E Peters, and Arman Cohan. Longformer: The long-document transformer. arXiv preprint arXiv:2004.05150, 2020.
- [8] Rianne van den Berg, Leonard Hasenclever, Jakub M. Tomczak, and Max Welling. Sylvester Normalizing Flows for Variational Inference, 2019. arXiv:1803.05649 [cs, stat].

- [9] Sam Bond-Taylor, Peter Hessey, Hiroshi Sasaki, Toby P Breckon, and Chris G Willcocks. Unleashing transformers: Parallel token prediction with discrete absorbing diffusion for fast high-resolution image generation from vector-quantized codes. In European Conference on Computer Vision, pages 170–188. Springer, 2022.
- [10] Tom B. Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, Sandhini Agarwal, Ariel Herbert-Voss, Gretchen Krueger, Tom Henighan, Rewon Child, Aditya Ramesh, Daniel M. Ziegler, Jeffrey Wu, Clemens Winter, Christopher Hesse, Mark Chen, Eric Sigler, Mateusz Litwin, Scott Gray, Benjamin Chess, Jack Clark, Christopher Berner, Sam McCandlish, Alec Radford, Ilya Sutskever, and Dario Amodei. Language models are few-shot learners, 2020.
- [11] Cristian Buciluă, Rich Caruana, and Alexandru Niculescu-Mizil. Model compression. In Proceedings of the 12th ACM SIGKDD international conference on Knowledge discovery and data mining, pages 535–541, 2006.
- [12] Han Cai, Ligeng Zhu, and Song Han. Proxylessnas: Direct neural architecture search on target task and hardware. In *ICLR*, 2018.
- [13] Zhaowei Cai and Nuno Vasconcelos. Cascade r-cnn: Delving into high quality object detection. In CVPR, 2018.
- [14] Ricky T. Q. Chen and Yaron Lipman. Flow matching on general geometries, 2024.
- [15] Ricky T. Q. Chen, Jens Behrmann, David Duvenaud, and Jörn-Henrik Jacobsen. Residual Flows for Invertible Generative Modeling, 2020. arXiv:1906.02735 [cs, stat].

- [16] Jean-Baptiste Cordonnier, Andreas Loukas, and Martin Jaggi. On the relationship between self-attention and convolutional layers. In *International Conference on Learning Representations*, 2020.
- [17] Katherine Crowson, Stefan Andreas Baumann, Alex Birch, Tanishq Mathew Abraham, Daniel Z Kaplan, and Enrico Shippole. Scalable high-resolution pixel-space image synthesis with hourglass diffusion transformers. In *Forty-first International Conference on Machine Learning*, 2024.
- [18] Balázs Csanád Csáji et al. Approximation with artificial neural networks. Faculty of Sciences, Etvs Lornd University, Hungary, 24(48):7, 2001.
- [19] Ekin D Cubuk, Barret Zoph, Jonathon Shlens, and Quoc V Le. Randaugment: Practical automated data augmentation with a reduced search space. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition Workshops, pages 702–703, 2020.
- [20] Quan Dao, Hao Phung, Binh Nguyen, and Anh Tran. Flow matching in latent space. arXiv preprint arXiv:2307.08698, 2023.
- [21] Tri Dao. Flashattention-2: Faster attention with better parallelism and work partitioning. In *ICLR*, 2023.
- [22] Tri Dao, Daniel Y Fu, Stefano Ermon, Atri Rudra, and Christopher Ré. Flashattention: Fast and memory-efficient exact attention with io-awareness. In *NeurIPS*, 2022.
- [23] Jia Deng, Wei Dong, Richard Socher, Li-Jia Li, Kai Li, and Li Fei-Fei. Imagenet: A large-scale hierarchical image database. In CVPR, 2009.
- [24] Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. Bert:Pre-training of deep bidirectional transformers for language understanding. In

Proceedings of the 2019 conference of the North American chapter of the association for computational linguistics: human language technologies, volume 1 (long and short papers), pages 4171–4186, 2019.

- [25] Thomas G Dietterich. Ensemble methods in machine learning. In International workshop on multiple classifier systems, pages 1–15. Springer, 2000.
- [26] Laurent Dinh, Jascha Sohl-Dickstein, and Samy Bengio. Density estimation using real nvp, 2017.
- [27] Hadi M. Dolatabadi, Sarah Erfani, and Christopher Leckie. Invertible Generative Modeling using Linear Rational Splines, 2020. arXiv:2001.05168 [cs, stat].
- [28] Alexey Dosovitskiy, Lucas Beyer, Alexander Kolesnikov, Dirk Weissenborn, Xiaohua Zhai, Thomas Unterthiner, Mostafa Dehghani, Matthias Minderer, Georg Heigold, Sylvain Gelly, Jakob Uszkoreit, and Neil Houlsby. An image is worth 16x16 words: Transformers for image recognition at scale. In International Conference on Learning Representations, 2021.
- [29] Emilien Dupont, Arnaud Doucet, and Yee Whye Teh. Augmented Neural ODEs. In Advances in Neural Information Processing Systems. Curran Associates, Inc., 2019.
- [30] Conor Durkan, Artur Bekasov, Iain Murray, and George Papamakarios. Cubic-Spline Flows, 2019. arXiv:1906.02145 [cs, stat].
- [31] Nelson Elhage, Tristan Hume, Catherine Olsson, Neel Nanda, Tom Henighan, Scott Johnston, Sheer ElShowk, Nicholas Joseph, Nova DasSarma, Ben Mann, Danny Hernandez, Amanda Askell, Kamal Ndousse, Andy Jones, Dawn Drain, Anna Chen, Yuntao Bai, Deep Ganguli, Liane Lovitt, Zac Hatfield-Dodds, Jackson Kernion, Tom Conerly, Shauna Kravec, Stanislav Fort, Saurav Kadavath, Josh Jacobson, Eli Tran-Johnson, Jared Kaplan, Jack Clark, Tom Brown, Sam McCandlish, Dario

Amodei, and Christopher Olah. Softmax linear units. *Transformer Circuits Thread*, 2022. https://transformer-circuits.pub/2022/solu/index.html.

- [32] Dumitru Erhan, Yoshua Bengio, Aaron Courville, and Pascal Vincent. Visualizing higher-layer features of a deep network, 2009.
- [33] Jonathan Frankle and Michael Carbin. The lottery ticket hypothesis: Finding sparse, trainable neural networks. In International Conference on Learning Representations, 2019.
- [34] Richard D. Fuhr and Michael Kallay. Monotone linear rational spline interpolation. Computer Aided Geometric Design, 9(4):313–319, 1992.
- [35] Philip Gage. A new algorithm for data compression. C Users J., 12(2):23–38, 1994.
- [36] Rinon Gal, Dana Cohen Hochberg, Amit Bermano, and Daniel Cohen-Or. Swagan: A style-based wavelet-driven generative model. ACM Trans. Graph., 40(4), 2021.
- [37] Shanghua Gao, Pan Zhou, Ming-Ming Cheng, and Shuicheng Yan. Masked diffusion transformer is a strong image synthesizer. In Proceedings of the IEEE/CVF International Conference on Computer Vision (ICCV), pages 23164–23173, 2023.
- [38] Amir Gholami, Sehoon Kim, Zhen Dong, Zhewei Yao, Michael W. Mahoney, and Kurt Keutzer. A survey of quantization methods for efficient neural network inference, 2021.
- [39] Lore Goetschalckx, Alex Andonian, Aude Oliva, and Phillip Isola. Ganalyze: Toward visual definitions of cognitive image properties. In Proceedings of the IEEE/CVF International Conference on Computer Vision (ICCV), 2019.
- [40] Ian Goodfellow, Yoshua Bengio, and Aaron Courville. Deep Learning. MIT Press, 2016.

- [41] Ian J. Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, and Yoshua Bengio. Generative adversarial nets. In Advances in Neural Information Processing Systems. Curran Associates, Inc., 2014.
- [42] Will Grathwohl, Ricky T. Q. Chen, Jesse Bettencourt, and David Duvenaud. Scalable reversible generative models with free-form continuous dynamics. In International Conference on Learning Representations, 2019.
- [43] Alex Graves, Greg Wayne, and Ivo Danihelka. Neural turing machines, 2014.
- [44] Matej Grcić, Ivan Grubišić, and Siniša Šegvić. Densely connected normalizing flows, 2021.
- [45] Ulf Grenander and Michael I Miller. Pattern theory: from representation to inference. OUP Oxford, 2006.
- [46] Albert Gu and Tri Dao. Mamba: Linear-time sequence modeling with selective state spaces, 2024.
- [47] Jiatao Gu, Tianrong Chen, David Berthelot, Huangjie Zheng, Yuyang Wang, Ruixiang Zhang, Laurent Dinh, Miguel Angel Bautista, Josh Susskind, and Shuangfei Zhai. Starflow: Scaling latent normalizing flows for high-resolution image synthesis, 2025.
- [48] Leonard Hasenclever, Jakub M Tomczak, and Max Welling. Variational Inference with Orthogonal Normalizing Flows. In *Bayesian Deep Learning*, 2017.
- [49] Ali Hassani. Neighborhood attention: Dynamic restriction of self-attention, 2023.
- [50] Ali Hassani and Humphrey Shi. Dilated neighborhood attention transformer, 2023.

- [51] Ali Hassani, Steven Walton, Nikhil Shah, Abulikemu Abuduweili, Jiachen Li, and Humphrey Shi. Escaping the big data paradigm with compact transformers, 2022.
- [52] Ali Hassani, Steven Walton, Jiachen Li, Shen Li, and Humphrey Shi. Neighborhood attention transformer. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), pages 6185–6194, 2023.
- [53] Ali Hassani, Wen-Mei Hwu, and Humphrey Shi. Faster neighborhood attention: Reducing the  $\mathcal{O}(n^2)$  cost of self attention at the threadblock level, 2024.
- [54] Ali Hassani, Fengzhe Zhou, Aditya Kane, Jiannan Huang, Chieh-Yun Chen, Min Shi, Steven Walton, Markus Hoehnerbach, Vijay Thakkar, Michael Isaev, Qinsheng Zhang, Bing Xu, Haicheng Wu, Wen mei Hwu, Ming-Yu Liu, and Humphrey Shi. Generalized neighborhood attention: Multi-dimensional sparse attention at the speed of light, 2025.
- [55] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 2016.
- [56] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In CVPR, 2016.
- [57] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Identity mappings in deep residual networks. In ECCV, 2016.
- [58] Kaiming He, Georgia Gkioxari, Piotr Dollár, and Ross Girshick. Mask r-cnn. In ICCV, 2017.
- [59] Kaiming He, Xinlei Chen, Saining Xie, Yanghao Li, Piotr Dollár, and Ross Girshick. Masked autoencoders are scalable vision learners. In *Proceedings of*

the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), pages 16000–16009, 2022.

- [60] Martin Heusel, Hubert Ramsauer, Thomas Unterthiner, Bernhard Nessler, and Sepp Hochreiter. Gans trained by a two time-scale update rule converge to a local nash equilibrium. In Advances in Neural Information Processing Systems. Curran Associates, Inc., 2017.
- [61] Geoffrey Hinton, Oriol Vinyals, and Jeff Dean. Distilling the knowledge in a neural network. arXiv preprint arXiv:1503.02531, 2015.
- [62] Geoffrey F. Hinton. Shape representation in parallel systems. In Proceedings of the 7th International Joint Conference on Artificial Intelligence - Volume 2, page 1088–1096, San Francisco, CA, USA, 1981. Morgan Kaufmann Publishers Inc.
- [63] Jonathan Ho, Ajay Jain, and Pieter Abbeel. Denoising diffusion probabilistic models. In Advances in Neural Information Processing Systems, pages 6840–6851. Curran Associates, Inc., 2020.
- [64] Emiel Hoogeboom, Victor Garcia Satorras, Jakub Tomczak, and Max Welling. The convolution exponential and generalized sylvester flows. In Advances in Neural Information Processing Systems, pages 18249–18260. Curran Associates, Inc., 2020.
- [65] Emiel Hoogeboom, Jonathan Heek, and Tim Salimans. Simple diffusion: End-toend diffusion for high resolution images, 2023.
- [66] Sara Hooker. On the limitations of compute thresholds as a governance strategy, 2024.
- [67] Kurt Hornik, Maxwell Stinchcombe, and Halbert White. Multilayer feedforward networks are universal approximators. *Neural networks*, 2(5):359–366, 1989.

- [68] Han Hu, Zheng Zhang, Zhenda Xie, and Stephen Lin. Local relation networks for image recognition. In Proceedings of the IEEE/CVF International Conference on Computer Vision (ICCV), 2019.
- [69] Jie Hu, Li Shen, and Gang Sun. Squeeze-and-excitation networks. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 2018.
- [70] Xun Huang and Serge Belongie. Arbitrary style transfer in real-time with adaptive instance normalization. In Proceedings of the IEEE International Conference on Computer Vision (ICCV), 2017.
- [71] Drew A Hudson and C. Lawrence Zitnick. Generative adversarial transformers.
  Proceedings of the 38th International Conference on Machine Learning, ICML 2021, 2021.
- [72] Drew A Hudson and C. Lawrence Zitnick. Compositional transformers for scene generation. Advances in Neural Information Processing Systems NeurIPS 2021, 2021.
- [73] Aapo Hyvärinen. Estimation of Non-Normalized Statistical Models by Score Matching. Journal of Machine Learning Research, 6(24):695–709, 2005.
- [74] Pavel Izmailov, Dmitrii Podoprikhin, Timur Garipov, Dmitry Vetrov, and Andrew Gordon Wilson. Averaging weights leads to wider optima and better generalization. arXiv preprint arXiv:1803.05407, 2018.
- [75] Jitesh Jain, Jiachen Li, Mang Tik Chiu, Ali Hassani, Nikita Orlov, and Humphrey Shi. Oneformer: One transformer to rule universal image segmentation. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), pages 2989–2998, 2023.

- [76] Jitesh Jain, Anukriti Singh, Nikita Orlov, Zilong Huang, Jiachen Li, Steven Walton, and Humphrey Shi. Semask: Semantically masked transformers for semantic segmentation. In Proceedings of the IEEE/CVF International Conference on Computer Vision (ICCV) Workshops, pages 752–761, 2023.
- [77] Nancy Kanwisher and Galit Yovel. Face Perception, chapter 43. John Wiley & Sons, Ltd, 2009.
- [78] Tero Karras, Timo Aila, Samuli Laine, and Jaakko Lehtinen. Progressive growing of GANs for improved quality, stability, and variation. In *International Conference on Learning Representations*, 2018.
- [79] Tero Karras, Samuli Laine, and Timo Aila. A style-based generator architecture for generative adversarial networks. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), 2019.
- [80] Tero Karras, Miika Aittala, Janne Hellsten, Samuli Laine, Jaakko Lehtinen, and Timo Aila. Training generative adversarial networks with limited data. In Advances in Neural Information Processing Systems, pages 12104–12114. Curran Associates, Inc., 2020.
- [81] Tero Karras, Samuli Laine, Miika Aittala, Janne Hellsten, Jaakko Lehtinen, and Timo Aila. Analyzing and improving the image quality of stylegan. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), 2020.
- [82] Tero Karras, Miika Aittala, Samuli Laine, Erik Härkönen, Janne Hellsten, Jaakko Lehtinen, and Timo Aila. Alias-free generative adversarial networks. In Advances in Neural Information Processing Systems, pages 852–863. Curran Associates, Inc., 2021.

- [83] Noble Kennamer, Steven Walton, and Alexander Ihler. Design amortization for bayesian optimal experimental design. Proceedings of the AAAI Conference on Artificial Intelligence, 37(7):8220–8227, 2023.
- [84] Dongjun Kim, Seungjae Shin, Kyungwoo Song, Wanmo Kang, and Il-Chul Moon. Soft truncation: A universal training technique of score-based diffusion model for high precision score estimation. In *Proceedings of the 39th International Conference* on Machine Learning, pages 11201–11228. PMLR, 2022.
- [85] Diederik P. Kingma and Jimmy Ba. Adam: A method for stochastic optimization, 2017.
- [86] Durk P Kingma and Prafulla Dhariwal. Glow: Generative Flow with Invertible 1x1 Convolutions. In Advances in Neural Information Processing Systems. Curran Associates, Inc., 2018.
- [87] Diederik P. Kingma and Ruiqi Gao. Understanding diffusion objectives as the elbo with simple data augmentation, 2023.
- [88] Diederik P. Kingma, Tim Salimans, Rafal Jozefowicz, Xi Chen, Ilya Sutskever, and Max Welling. Improving variational inference with inverse autoregressive flow, 2017.
- [89] Alexander Kolesnikov, Lucas Beyer, Xiaohua Zhai, Joan Puigcerver, Jessica Yung, Sylvain Gelly, and Neil Houlsby. Big transfer (bit): General visual representation learning, 2020.
- [90] Olga Kovaleva, Alexey Romanov, Anna Rogers, and Anna Rumshisky. Revealing the dark secrets of BERT. In Proceedings of the 2019 Conference on Empirical Methods in Natural Language Processing and the 9th International Joint Conference on Natural Language Processing (EMNLP-IJCNLP), pages 4365–4374, Hong Kong, China, 2019. Association for Computational Linguistics.

- [91] Alex Krizhevsky, Geoffrey Hinton, et al. Learning multiple layers of features from tiny images.(2009), 2009.
- [92] Tuomas Kynkäänniemi, Tero Karras, Miika Aittala, Timo Aila, and Jaakko Lehtinen. The role of imagenet classes in fréchet inception distance. In The Eleventh International Conference on Learning Representations, 2023.
- [93] Yann LeCun, Bernhard Boser, John Denker, Donnie Henderson, R. Howard, Wayne Hubbard, and Lawrence Jackel. Handwritten digit recognition with a backpropagation network. In Advances in Neural Information Processing Systems. Morgan-Kaufmann, 1989.
- [94] Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner. Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, 86(11):2278– 2324, 2002.
- [95] John M Lee and John M Lee. Smooth manifolds. Springer, 2003.
- [96] Kwonjoon Lee, Huiwen Chang, Lu Jiang, Han Zhang, Zhuowen Tu, and Ce Liu. ViTGAN: Training GANs with vision transformers. In International Conference on Learning Representations, 2022.
- [97] Namhoon Lee, Thalaiyasingam Ajanthan, and Philip Torr. SNIP: SINGLE-SHOT NETWORK PRUNING BASED ON CONNECTION SENSITIVITY. In International Conference on Learning Representations, 2019.
- [98] Benjamin Lefaudeux, Francisco Massa, Diana Liskovich, Wenhan Xiong, Vittorio Caggiano, Sean Naren, Min Xu, Jieru Hu, Marta Tintore, Susan Zhang, Patrick Labatut, Daniel Haziza, Luca Wehrstedt, Jeremy Reizenstein, and Grigory Sizov. xformers: A modular and hackable transformer modelling library. https://github. com/facebookresearch/xformers, 2022.

- [99] Roberto Leyva, Victor Sanchez, Gregory Epiphaniou, and Carsten Maple. Demographic bias effects on face image synthesis. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR) Workshops, pages 3818–3826, 2024.
- [100] Hao Li, Zheng Xu, Gavin Taylor, Christoph Studer, and Tom Goldstein. Visualizing the loss landscape of neural nets. In Advances in Neural Information Processing Systems. Curran Associates, Inc., 2018.
- [101] Xin Li and Dan Roth. Learning question classifiers. In COLING 2002: The 19th International Conference on Computational Linguistics, 2002.
- [102] M. Lichman. Uci machine learning repository, 2013.
- [103] Tsung-Yi Lin, Michael Maire, Serge Belongie, James Hays, Pietro Perona, Deva Ramanan, Piotr Dollár, and C Lawrence Zitnick. Microsoft coco: Common objects in context. In ECCV, 2014.
- [104] Yaron Lipman, Ricky T. Q. Chen, Heli Ben-Hamu, Maximilian Nickel, and Matthew Le. Flow matching for generative modeling. In *The Eleventh International Conference on Learning Representations*, 2023.
- [105] Yaron Lipman, Marton Havasi, Peter Holderrieth, Neta Shaul, Matt Le, Brian Karrer, Ricky T. Q. Chen, David Lopez-Paz, Heli Ben-Hamu, and Itai Gat. Flow matching guide and code, 2024.
- [106] Ben Liu and Zhen Qin. Accelerate tarflow sampling with gs-jacobi iteration, 2025.
- [107] Xingchao Liu, Chengyue Gong, and Qiang Liu. Flow straight and fast: Learning to generate and transfer data with rectified flow. In *The Eleventh International Conference on Learning Representations*, 2023.

- [108] Ziwei Liu, Ping Luo, Xiaogang Wang, and Xiaoou Tang. Deep learning face attributes in the wild. In Proceedings of International Conference on Computer Vision (ICCV), 2015.
- [109] Ze Liu, Yutong Lin, Yue Cao, Han Hu, Yixuan Wei, Zheng Zhang, Stephen Lin, and Baining Guo. Swin transformer: Hierarchical vision transformer using shifted windows. In *ICCV*, pages 10012–10022, 2021.
- [110] Ze Liu, Han Hu, Yutong Lin, Zhuliang Yao, Zhenda Xie, Yixuan Wei, Jia Ning, Yue Cao, Zheng Zhang, Li Dong, Furu Wei, and Baining Guo. Swin transformer v2: Scaling up capacity and resolution. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 12009–12019, 2022.
- [111] Zhuang Liu, Hanzi Mao, Chao-Yuan Wu, Christoph Feichtenhofer, Trevor Darrell, and Saining Xie. A convnet for the 2020s. Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), 2022.
- [112] Ziming Liu, Yixuan Wang, Sachin Vaidya, Fabian Ruehle, James Halverson, Marin Soljačić, Thomas Y Hou, and Max Tegmark. Kan: Kolmogorov-arnold networks. arXiv preprint arXiv:2404.19756, 2024.
- [113] Ilya Loshchilov and Frank Hutter. Decoupled weight decay regularization, 2019.
- [114] Minh-Thang Luong, Hieu Pham, and Christopher D. Manning. Effective approaches to attention-based neural machine translation, 2015.
- [115] Xuezhe Ma, Xiang Kong, Shanghang Zhang, and Eduard Hovy. Macow: Masked convolutional generative flow. In Advances in Neural Information Processing Systems. Curran Associates, Inc., 2019.
- [116] Andrew Maas, Raymond E Daly, Peter T Pham, Dan Huang, Andrew Y Ng, and Christopher Potts. Learning word vectors for sentiment analysis. In *Proceedings*

of the 49th annual meeting of the association for computational linguistics: Human language technologies, pages 142–150, 2011.

- [117] D. Martin, C. Fowlkes, D. Tal, and J. Malik. A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics. In Proc. 8th Int'l Conf. Computer Vision, pages 416–423, 2001.
- [118] Lars Mescheder, Andreas Geiger, and Sebastian Nowozin. Which training methods for GANs do actually converge? In *Proceedings of the 35th International Conference* on Machine Learning, pages 3481–3490. PMLR, 2018.
- [119] Paul Michel, Omer Levy, and Graham Neubig. Are sixteen heads really better than one? In Advances in Neural Information Processing Systems. Curran Associates, Inc., 2019.
- [120] Maria-Elena Nilsback and Andrew Zisserman. Automated flower classification over a large number of classes. In 2008 Sixth Indian Conference on Computer Vision, Graphics and Image Processing, pages 722–729, 2008.
- [121] George Papamakarios, Theo Pavlakou, and Iain Murray. Masked autoregressive flow for density estimation, 2017.
- [122] Gaurav Parmar, Richard Zhang, and Jun-Yan Zhu. On aliased resizing and surprising subtleties in gan evaluation. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), pages 11410–11420, 2022.
- [123] Niki Parmar, Ashish Vaswani, Jakob Uszkoreit, Lukasz Kaiser, Noam Shazeer, Alexander Ku, and Dustin Tran. Image transformer. In *International Conference* on Machine Learning (ICML), 2018.

- [124] Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, Alban Desmaison, Andreas Köpf, Edward Yang, Zach DeVito, Martin Raison, Alykhan Tejani, Sasank Chilamkurthy, Benoit Steiner, Lu Fang, Junjie Bai, and Soumith Chintala. Pytorch: An imperative style, high-performance deep learning library, 2019.
- [125] Jeffrey Pennington, Richard Socher, and Christopher Manning. GloVe: Global vectors for word representation. In Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing (EMNLP), pages 1532–1543, Doha, Qatar, 2014. Association for Computational Linguistics.
- [126] B. T. Polyak and A. B. Juditsky. Acceleration of stochastic approximation by averaging. SIAM Journal on Control and Optimization, 30(4):838–855, 1992.
- [127] Ilija Radosavovic, Raj Prateek Kosaraju, Ross Girshick, Kaiming He, and Piotr Dollar. Designing network design spaces. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), 2020.
- [128] Colin Raffel, Noam Shazeer, Adam Roberts, Katherine Lee, Sharan Narang, Michael Matena, Yanqi Zhou, Wei Li, and Peter J. Liu. Exploring the limits of transfer learning with a unified text-to-text transformer. Journal of Machine Learning Research, 21(140):1–67, 2020.
- [129] Prajit Ramachandran, Niki Parmar, Ashish Vaswani, Irwan Bello, Anselm Levskaya, and Jon Shlens. Stand-alone self-attention in vision models. In Advances in Neural Information Processing Systems. Curran Associates, Inc., 2019.
- [130] Robin Rombach, Andreas Blattmann, Dominik Lorenz, Patrick Esser, and Björn Ommer. High-resolution image synthesis with latent diffusion models. In

Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), pages 10684–10695, 2022.

- [131] Axel Sauer, Kashyap Chitta, Jens Müller, and Andreas Geiger. Projected gans converge faster. In Advances in Neural Information Processing Systems, pages 17480–17492. Curran Associates, Inc., 2021.
- [132] Axel Sauer, Katja Schwarz, and Andreas Geiger. Stylegan-xl: Scaling stylegan to large diverse datasets. In ACM SIGGRAPH 2022 conference proceedings, pages 1–10, 2022.
- [133] Rylan Schaeffer, Brando Miranda, and Sanmi Koyejo. Are emergent abilities of large language models a mirage? In Advances in Neural Information Processing Systems, pages 55565–55581. Curran Associates, Inc., 2023.
- [134] Adam Scherlis, Kshitij Sachan, Adam S. Jermyn, Joe Benton, and Buck Shlegeris.
  Polysemanticity and capacity in neural networks. CoRR, abs/2210.01892, 2022.
- [135] Rico Sennrich, Barry Haddow, and Alexandra Birch. Neural machine translation of rare words with subword units. In *Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 1715– 1725, Berlin, Germany, 2016. Association for Computational Linguistics.
- [136] Jay Shah, Ganesh Bikshandi, Ying Zhang, Vijay Thakkar, Pradeep Ramani, and Tri Dao. Flashattention-3: Fast and accurate attention with asynchrony and lowprecision. arXiv preprint arXiv:2407.08608, 2024.
- [137] Karen Simonyan, Andrea Vedaldi, and Andrew Zisserman. Deep inside convolutional networks: Visualising image classification models and saliency maps, 2014.

- [138] Richard Socher, Alex Perelygin, Jean Wu, Jason Chuang, Christopher D Manning, Andrew Y Ng, and Christopher Potts. Recursive deep models for semantic compositionality over a sentiment treebank. In *Proceedings of the 2013 conference* on empirical methods in natural language processing, pages 1631–1642, 2013.
- [139] Kenneth O. Stanley, David B. D'Ambrosio, and Jason Gauci. A hypercube-based encoding for evolving large-scale neural networks. *Artificial Life*, 15(2):185–212, 2009.
- [140] George Stein, Jesse C. Cresswell, Rasa Hosseinzadeh, Yi Sui, Brendan Leigh Ross, Valentin Villecroze, Zhaoyan Liu, Anthony L. Caterini, Eric Taylor, and Gabriel Loaiza-Ganem. Exposing flaws of generative model evaluation metrics and their unfair treatment of diffusion models. In *Thirty-seventh Conference on Neural Information Processing Systems*, 2023.
- [141] Chen Sun, Abhinav Shrivastava, Saurabh Singh, and Abhinav Gupta. Revisiting unreasonable effectiveness of data in deep learning era. In Proceedings of the IEEE International Conference on Computer Vision (ICCV), 2017.
- [142] Richard Sutton. The bitter lesson, 2019.
- [143] Christian Szegedy, Vincent Vanhoucke, Sergey Ioffe, Jon Shlens, and Zbigniew Wojna. Rethinking the inception architecture for computer vision. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 2016.
- [144] Yuhta Takida, Masaaki Imaizumi, Takashi Shibuya, Chieh-Hsin Lai, Toshimitsu Uesaka, Naoki Murata, and Yuki Mitsufuji. SAN: Inducing metrizability of GAN with discriminative normalized linear layer. In *The Twelfth International Conference on Learning Representations*, 2024.

- [145] Gongbo Tang, Rico Sennrich, and Joakim Nivre. An analysis of attention mechanisms: The case of word sense disambiguation in neural machine translation. In Proceedings of the Third Conference on Machine Translation: Research Papers, pages 26–35, Brussels, Belgium, 2018. Association for Computational Linguistics.
- [146] Jiayan Teng, Wendi Zheng, Ming Ding, Wenyi Hong, Jianqiao Wangni, Zhuoyi Yang, and Jie Tang. Relay diffusion: Unifying diffusion process across resolutions for image synthesis, 2023.
- [147] Antonio Torralba, Rob Fergus, and William T. Freeman. 80 million tiny images: A large data set for nonparametric object and scene recognition. *IEEE Transactions* on Pattern Analysis and Machine Intelligence, 30(11):1958–1970, 2008.
- [148] Hugo Touvron, Andrea Vedaldi, Matthijs Douze, and Herve Jegou. Fixing the traintest resolution discrepancy. In Advances in Neural Information Processing Systems. Curran Associates, Inc., 2019.
- [149] Hugo Touvron, Matthieu Cord, Matthijs Douze, Francisco Massa, Alexandre Sablayrolles, and Herve Jegou. Training data-efficient image transformers & distillation through attention. In *Proceedings of the 38th International Conference* on Machine Learning, pages 10347–10357. PMLR, 2021.
- [150] Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Ł ukasz Kaiser, and Illia Polosukhin. Attention is all you need. In NeurIPS, 2017.
- [151] Ashish Vaswani, Prajit Ramachandran, Aravind Srinivas, Niki Parmar, Blake Hechtman, and Jonathon Shlens. Scaling local self-attention for parameter efficient visual backbones, 2021.

- [152] Mario Viti, Nadiya Shvai, Arcadi Llanza, and Amir Nakib. A 0-shot self-attention mechanism for accelerated diagonal attention. In Proceedings of the Winter Conference on Applications of Computer Vision (WACV), pages 7308–7315, 2025.
- [153] Elena Voita, David Talbot, Fedor Moiseev, Rico Sennrich, and Ivan Titov. Analyzing multi-head self-attention: Specialized heads do the heavy lifting, the rest can be pruned. In Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics, pages 5797–5808, Florence, Italy, 2019. Association for Computational Linguistics.
- [154] Steven Walton. Isomorphism, normalizing flows, and density estimation: Preserving relationships between data, 2022.
- [155] Steven Walton, Ali Hassani, Abulikemu Abuduweili, and Humphrey Shi. Training compact transformers from scratch in 30 minutes with pytorch. *medium.com/pytorch*, 2021.
- [156] Steven Walton, Ali Hassani, Xingqian Xu, Zhangyang Wang, and Humphrey Shi. Efficient image generation with variadic attention heads. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR) Workshops, 2025.
- [157] Steven Walton, Valeriy Klyukin, Maksim Artemev, Denis Derkach, Nikita Orlov, and Humphrey Shi. Distilling normalizing flows. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR) Workshops, 2025.
- [158] Zhendong Wang, Huangjie Zheng, Pengcheng He, Weizhu Chen, and Mingyuan Zhou. Diffusion-GAN: Training GANs with diffusion. In *The Eleventh International Conference on Learning Representations*, 2023.

- [159] Jason Wei, Yi Tay, Rishi Bommasani, Colin Raffel, Barret Zoph, Sebastian Borgeaud, Dani Yogatama, Maarten Bosma, Denny Zhou, Donald Metzler, Ed H. Chi, Tatsunori Hashimoto, Oriol Vinyals, Percy Liang, Jeff Dean, and William Fedus. Emergent abilities of large language models. *Transactions on Machine Learning Research*, 2022. Survey Certification.
- [160] Ross Wightman. Pytorch image models, 2019.
- [161] Ross Wightman, Hugo Touvron, and Hervé Jégou. Resnet strikes back: An improved training procedure in timm, 2021.
- [162] Sanghyun Woo, Shoubhik Debnath, Ronghang Hu, Xinlei Chen, Zhuang Liu, In So Kweon, and Saining Xie. Convnext v2: Co-designing and scaling convnets with masked autoencoders. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), pages 16133–16142, 2023.
- [163] Guangxuan Xiao, Yuandong Tian, Beidi Chen, Song Han, and Mike Lewis. Efficient streaming language models with attention sinks, 2024.
- [164] Han Xiao, Kashif Rasul, and Roland Vollgraf. Fashion-mnist: a novel image dataset for benchmarking machine learning algorithms, 2017.
- [165] Ceyuan Yang, Yujun Shen, Yinghao Xu, and Bolei Zhou. Data-efficient instance generation from instance discrimination. In Advances in Neural Information Processing Systems, pages 9378–9390. Curran Associates, Inc., 2021.
- [166] Fisher Yu, Yinda Zhang, Shuran Song, Ari Seff, and Jianxiong Xiao. Lsun: Construction of a large-scale image dataset using deep learning with humans in the loop. arXiv preprint arXiv:1506.03365, 2015.
- [167] Sangdoo Yun, Dongyoon Han, Seong Joon Oh, Sanghyuk Chun, Junsuk Choe, and Youngjoon Yoo. Cutmix: Regularization strategy to train strong classifiers with
localizable features. In Proceedings of the IEEE/CVF International Conference on Computer Vision, pages 6023–6032, 2019.

- [168] Shuangfei Zhai, Ruixiang Zhang, Preetum Nakkiran, David Berthelot, Jiatao Gu, Huangjie Zheng, Tianrong Chen, Miguel Angel Bautista, Navdeep Jaitly, and Josh Susskind. Normalizing flows are capable generative models, 2024.
- [169] Bowen Zhang, Shuyang Gu, Bo Zhang, Jianmin Bao, Dong Chen, Fang Wen, Yong Wang, and Baining Guo. Styleswin: Transformer-based gan for high-resolution image generation. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), pages 11304–11314, 2022.
- [170] Hongyi Zhang, Moustapha Cisse, Yann N Dauphin, and David Lopez-Paz. mixup: Beyond empirical risk minimization. arXiv preprint arXiv:1710.09412, 2017.
- [171] Han Zhang, Ian Goodfellow, Dimitris Metaxas, and Augustus Odena. Self-attention generative adversarial networks. In Proceedings of the 36th International Conference on Machine Learning, pages 7354–7363. PMLR, 2019.
- [172] Xiang Zhang, Junbo Zhao, and Yann LeCun. Character-level convolutional networks for text classification. arXiv preprint arXiv:1509.01626, 2015.
- [173] Zizhao Zhang, Han Zhang, Long Zhao, Ting Chen, Sercan Ö Arik, and Tomas Pfister. Nested hierarchical transformer: Towards accurate, data-efficient and interpretable visual understanding. In *Proceedings of the AAAI Conference on Artificial Intelligence*, pages 3417–3425, 2022.
- [174] Long Zhao, Zizhao Zhang, Ting Chen, Dimitris Metaxas, and Han Zhang. Improved transformer for high-resolution gans. In Advances in Neural Information Processing Systems, pages 18367–18380. Curran Associates, Inc., 2021.

- [175] Zhengli Zhao, Sameer Singh, Honglak Lee, Zizhao Zhang, Augustus Odena, and Han Zhang. Improved consistency regularization for gans. In Proceedings of the AAAI conference on artificial intelligence, pages 11033–11041, 2021.
- [176] Hongkai Zheng, Weili Nie, Arash Vahdat, and Anima Anandkumar. Fast training of diffusion models with masked transformers. In *Transactions on Machine Learning Research (TMLR)*, 2024.
- [177] Hui Zhong, Zaiyi Chen, Chuan Qin, Zai Huang, Vincent W. Zheng, Tong Xu, and Enhong Chen. Adam revisited: a weighted past gradients perspective. Frontiers of Computer Science, 14(5), 2020.
- [178] Zhun Zhong, Liang Zheng, Guoliang Kang, Shaozi Li, and Yi Yang. Random erasing data augmentation. In Proceedings of the AAAI Conference on Artificial Intelligence, pages 13001–13008, 2020.
- [179] Bolei Zhou, Hang Zhao, Xavier Puig, Sanja Fidler, Adela Barriuso, and Antonio Torralba. Scene parsing through ade20k dataset. In CVPR, 2017.
- [180] Jun-Yan Zhu, Taesung Park, Phillip Isola, and Alexei A Efros. Unpaired image-toimage translation using cycle-consistent adversarial networks. In Computer Vision (ICCV), 2017 IEEE International Conference on, 2017.