Color Defective Vision and Computer Graphic Displays

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Abstract

The fundamental spectral sensitivity functions of the human visual system define a color space which can be employed to design better color user interfaces. In particular, this color space makes it possible to accomodate individuals with color deficient vision. In order to screen potential users of computer graphics systems, traditional color vision tests, such as the Farnsworth-Munsell 100 hues test, can be implemented using a digitally controlled color television monitor, and these tests can be extended in ways that improve the specificity of their diagnoses. To assist in the design of computer graphic displays, a picture of the world as seen by color deficient observers can be synthesized and guidelines can be given for the selection of colors to be presented to color deficient observers.

1 Introduction

The CIE XYZ matching functions form the basis for color reproduction work in computer graphics. There is, however, a more fundamental color space that is based on the spectral sensitivity functions that are actually found in the human visual system. This space has important implications for color work in computer graphics. With some assumptions about the nature of color defective vision, the transformation between CIE XYZ space and this fundamental space can be derived.

Armed with this deeper understanding of human color vision, it is possible to begin to accomodate color defective users of computer graphic systems. Standard color vision tests can be implemented on a digitally controlled color television monitor and these tests can easily be extended. A picture can be synthesized which attempts to show people with normal color vision how the world appears to a color defective person. Most importantly, specific guidelines can be established for the design of computer graphics displays that will accomodate almost all color deficient users.

2 Principles of Color Science

Color science is based on the results of a set of color matching experiments. In these experiments, a subject matches a series of spectral light sources with the light from three overlapping light sources R, G, and B. The amount of each light source necessary to achieve a match with the spectral light is recorded, and the resulting matching functions $\overline{r}(\lambda)$, $\overline{g}(\lambda)$, and $\overline{b}(\lambda)$ can be used to compute the amount of light necessary to match an arbitrary

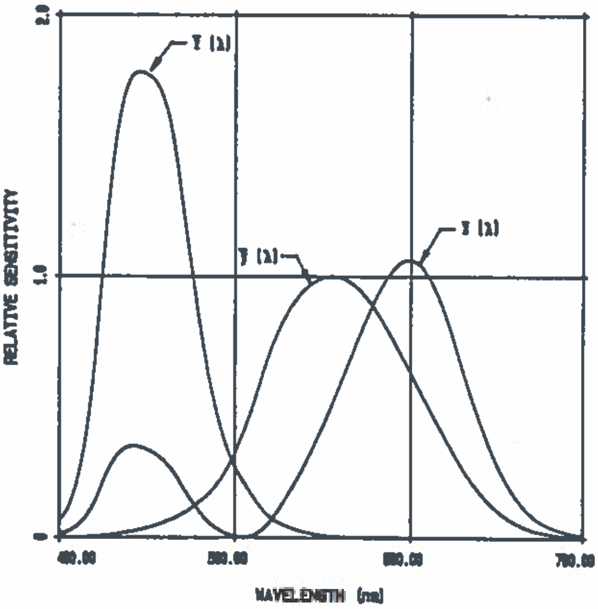


Figure 1: CIE XYZ matching functions.

spectral energy distribution $E(\lambda)$

$$\begin{aligned} R &= \int E(\lambda)\overline{r}(\lambda)\,d\lambda \\ G &= \int E(\lambda)\overline{g}(\lambda)\,d\lambda \\ B &= \int E(\lambda)\overline{b}(\lambda)\,d\lambda \end{aligned} \tag{1}$$

The mean data of GUIL31 and WRIG28 have been adopted by the CIE as a standard set of matching functions and will therefore be used in this paper. However, due to the technology available at the time, these data have limitations and the more recent work of STIL55 may be better for careful vision research (ESTE79).

The 1931 CIE XYZ color matching functions $\overline{g}(\lambda)$, $\overline{y}(\lambda)$, and $\overline{z}(\lambda)$ (Figure 1) are a linear transform of the GUIL31 and WRIG28 color matching data. This transformation is valid because color matching obeys the laws of proportionality and scaling. XYZ tristimulus

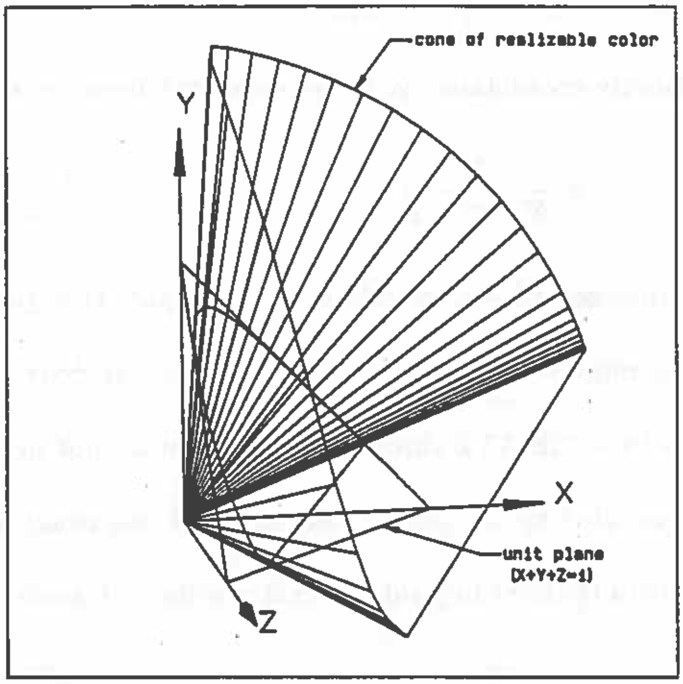


Figure 2: Cone of realizable color and unit plane in CIE XYZ space.

values can be computed through the expressions

$$\begin{aligned} X &= \int E(\lambda)\bar{x}(\lambda)\,d\lambda \\ Y &= \int E(\lambda)\bar{y}(\lambda)\,d\lambda \\ Z &= \int E(\lambda)\bar{z}(\lambda)\,d\lambda \end{aligned} \tag{2}$$

Even though these tristimulus values no longer refer to the amount of three real light sources, it is still true that two different spectral energy distributions with the same tristimulus values will match in color. This property is important for color reproduction work.

The XYZ tristimulus values can be thought of as the endpoints of vectors extending out from the origin of a three dimensional space. Vectors for spectral colors must pass through the locus formed by the matching functions, and these vectors must lie on the cone shown in Figure 2. Since this cone is convex, the vector that results from the integration performed in Equation 1 must lie inside of the cone. The direction of each vector can be specified by the point where the vector pierces the plane $X+Y+Z=1$ in the space (Figure 2). An orthographic projection of this plane onto the XY plane is referred to as a chromaticity diagram. Chromaticity coordinates (x, y) are computed from the expressions

$$x = \frac{X}{X+Y+Z} \qquad\qquad y = \frac{Y}{X+Y+Z} \tag{3}$$

The CIE has proposed a linear transform of XYZ space that yields a color space which has a perceptually uniform chromaticity diagram. This was done because color difference loci plotted on the 1931 CIE XYZ chromaticity diagram are not circular The uniform chromaticity diagram adopted by the CIE in 1960 improved this situation A further refinement was suggested in 1976 (CIE78) to yield the final non-linear transformation

$$u = \frac{4X}{X+15Y+3Z} \qquad\qquad v = \frac{9Y}{X+15Y+3Z} \tag{4}$$

where u and v are referred to as the 1976 CIE Uniform Chromaticity Scale (UCS) coordinates.

3 The Fundamental Spectral Sensitivity Functions

The fundamental human spectral sensitivity functions characterize the physical response of the visual system to individual wavelengths of light. From a physiological standpoint these functions are a product of the transmission characteristics of the ocular medium and the spectral sensitivity of the receptors in the eye. The current evidence is that there are three functions with broad spectral sensitivities lying in the short, medium, and long wavelength regions of the spectrum (MOLL82). From a psychophysical standpoint, the fundamental spectral sensitivity functions should be a linear combination of the color matching functions described in the preceeding section. Using certain assumptions about the nature of color defective vision, a linear transform of the color matching functions to the fundamental spectral sensitivity functions can be derived from the results of color matching experiments on people with color deficient vision.

A color normal person has three fundamental spectral sensitivity functions and is therefore called a trichromat. The fundamental spectral sensitivity functions are designated $\overline{s}(\lambda)$, $\overline{m}(\lambda)$, and $\overline{l}(\lambda)$ for short, medium, and long wavelength respectively. The SML tristimulus values are computed from the expressions

$$\begin{aligned} S &= \int E(\lambda)\overline{s}(\lambda)\,d\lambda \\ M &= \int E(\lambda)\overline{m}(\lambda)\,d\lambda \\ L &= \int E(\lambda)\overline{l}(\lambda)\,d\lambda \end{aligned} \tag{5}$$

Trichromatic chromaticity coordinates for SML space are found from the relations

$$s = \frac{S}{S+M+L} \qquad m = \frac{M}{S+M+L} \tag{6}$$

They represent the point of intersection between a color vector and the unit plane $S+M+L=1$ in SML space.

A dichromat is a person who has only two of the three fundamental spectral sensitivity functions. Even though a complete lack of color discrimination can only occur in individuals with a single type of receptor, it is common practice to refer to dichromats as being color blind. These people are categorized as protanopes, deuteranopes, or tritanopes depending on whether the long, medium, or short wavelength receptor respectively is missing. Since either S, M, or L is zero, the color space of a dichromat collapses to one of the trichromat's coordinate planes and the chromaticity diagram of a dichromat becomes one edge of the trichromat's unit plane. In the case of a protanope as shown in Figure 3(a), the color space becomes the SM plane and the chromaticity diagram becomes the unit line $S+M=1$. Planes in SML space that pass through the L axis collapse to lines of constant protanopic chromaticity on the SM plane. The SML tristimulus values that constitute these planes

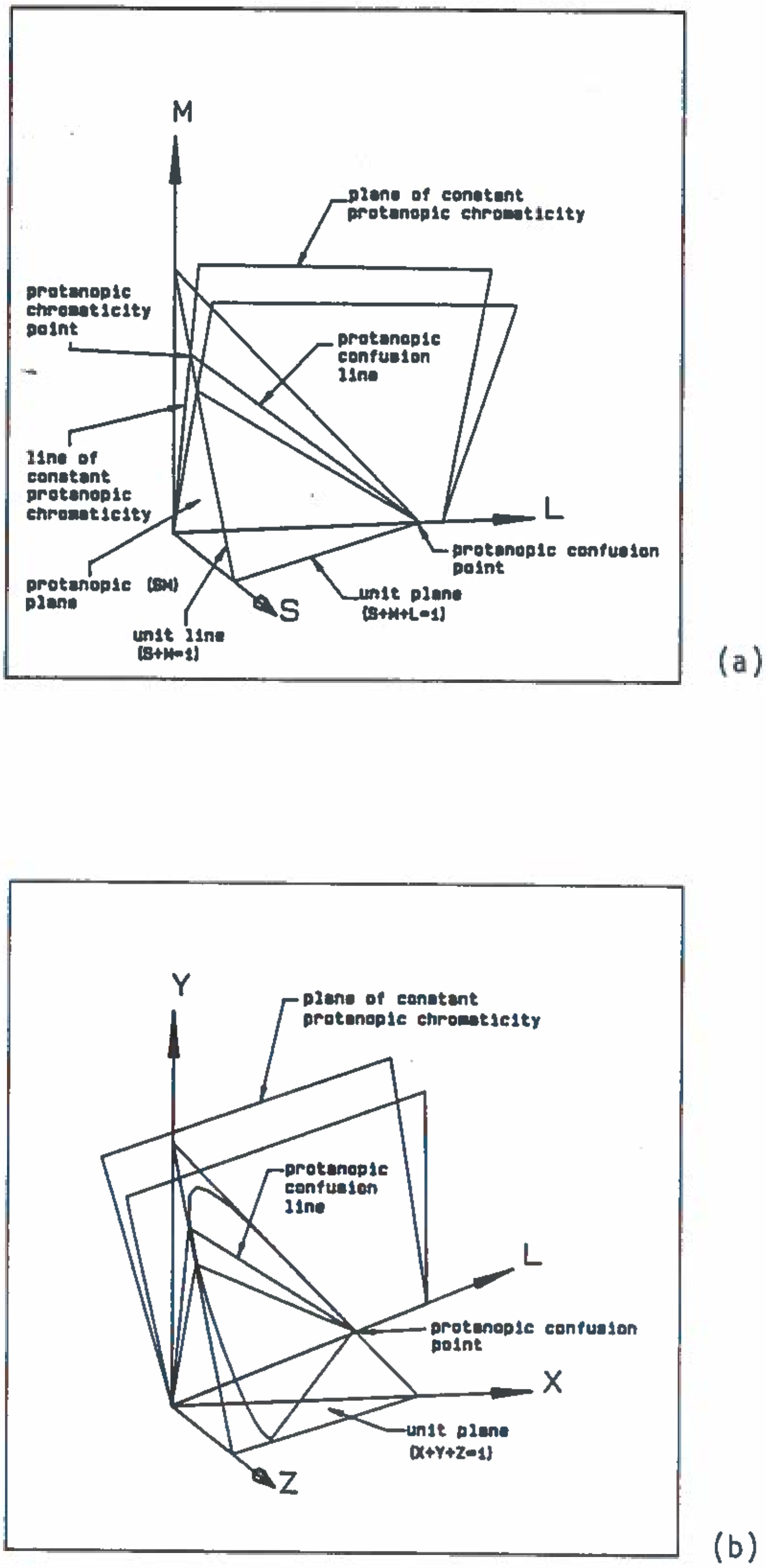


Figure 3: Intersection of the planes of constant protanopic chromaticity defines the position of the L axis in (a) SML and (b) CIE XYZ space.

therefore have constant protanopic chromaticity. The lines of intersection between these planes and the unit plane in SML space are called protanopic confusion lines. Protanopes have difficulty distinguishing between colors that have trichromatic chromaticities that lie along one of these lines. The protanopic confusion lines all intersect at the protanopic confusion point. This is the point where the axis of the missing L fundamental pierces the trichromat's unit plane. A similar analysis holds for deuteranopes and tritanopes as illustrated in Figures 4(a) and 5(a).

Assuming a linear transform from SML space to CIE XYZ space, the planes of constant protanopic chromaticity from Figure 3(a) transform as planes and define new protanopic confusion lines and a new protanopic confusion point on the unit plane in CIE XYZ space. As shown in Figure 3(b), the CIE XYZ chromaticity coordinates of the protanopic confusion point defines the position of the L axis in CIE XYZ space. Color matching experiments performed on protanopes yield chromaticity confusion lines in CIE XYZ space that are consistent with this prediction. When planes of constant deuteranopic and tritanopic chromaticity are transformed from SML space to CIE XYZ space, similar results are obtained as can be seen in Figures 4(b) and 5(b) respectively.

Given the chromaticity coordinates of the dichromatic confusion points in CIE XYZ space, the tranformation from CIE XYZ space to SML space can be derived. The basic form for this expression is

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x_t & x_d & x_p \\ y_t & y_d & y_p \\ 1 - x_t - y_t & 1 - x_d - y_d & 1 - x_p - y_p \end{bmatrix} \begin{bmatrix} k_t S \\ k_d M \\ k_p L \end{bmatrix} \tag{7}$$

where x_p, y_p, x_d, y_d, x_t, and y_t are the confusion points and k_p, k_d, and k_t are normalization factors. From the wide array of proposals for the confusion points (PITT35, PITT44,

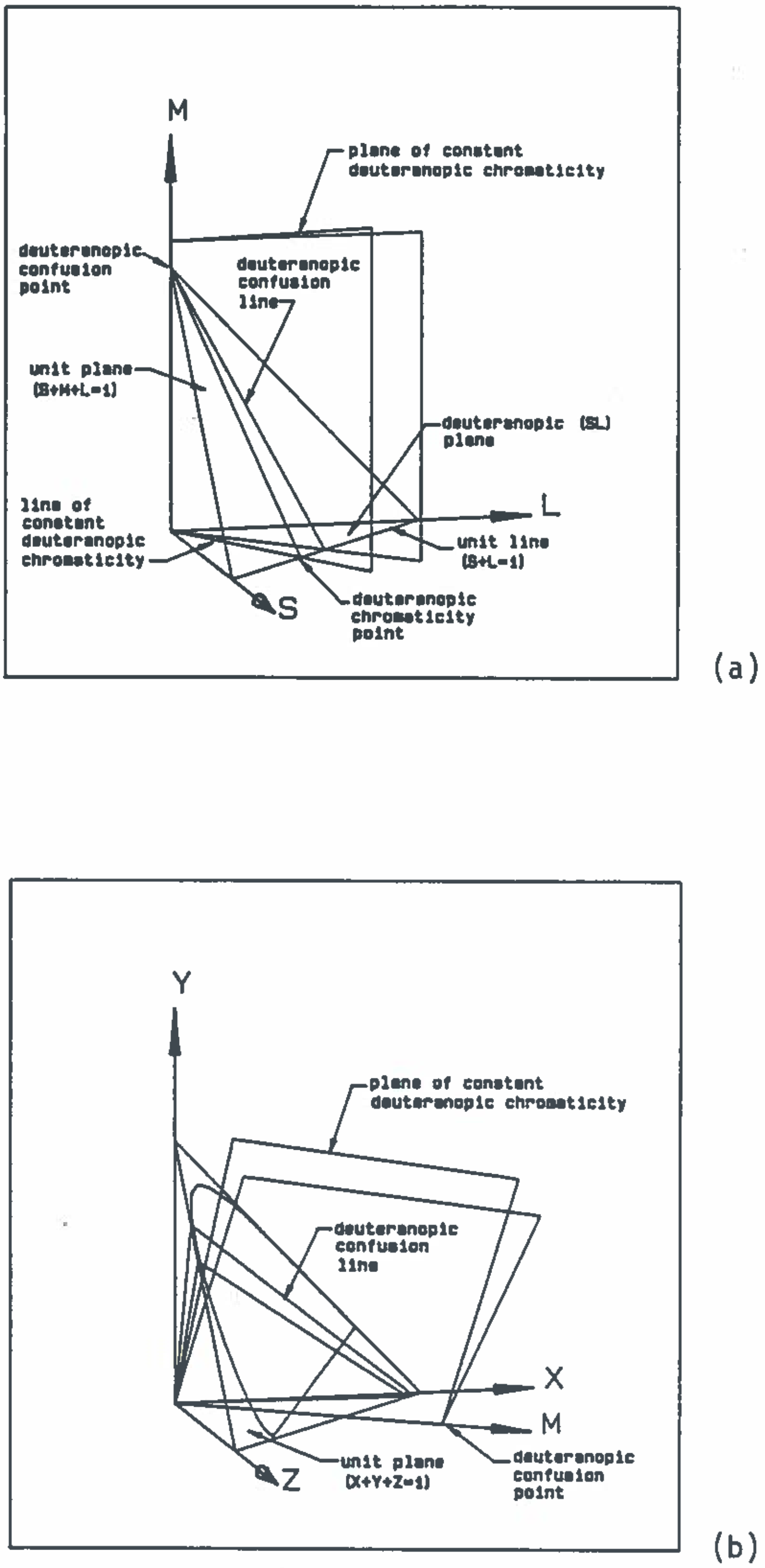


Figure 4: Intersection of the planes of constant deuteranopic chromaticity defines the position of the M axis in (a) SML and (b) CIE XYZ space.

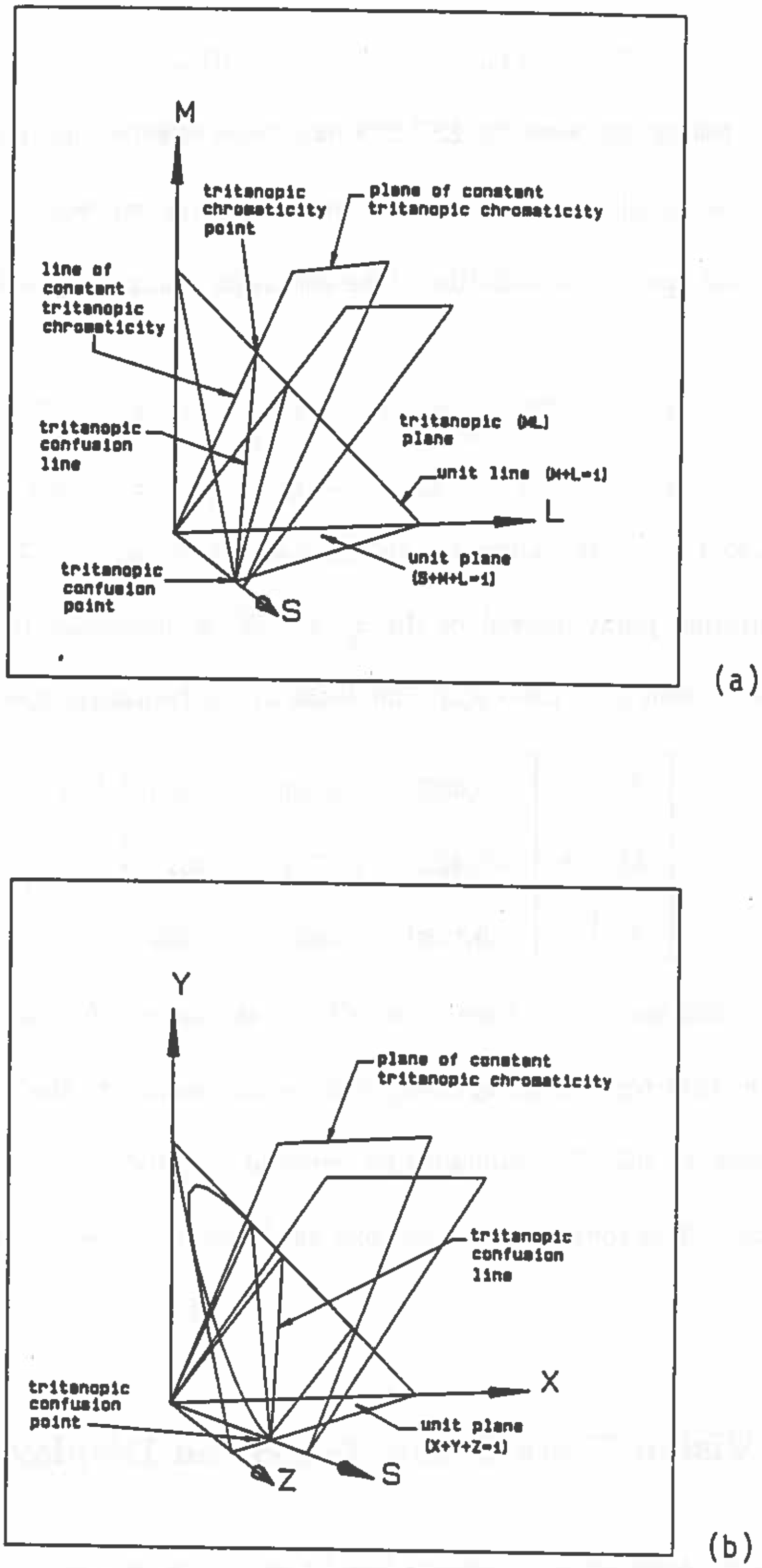


Figure 5: Intersection of the planes of constant tritanopic chromaticity defines the position of the S axis in (a) SML and (b) CIE XYZ space.

JUDD44, JUDD49, JUDD50, WRIG52, THOM53, YUST53, JUDD66, NIME70, VOS70, WALR74), the points suggested by ESTE79 have been selected because this was the only study that considered all three loci simultaneously with the express intention of defining a set of fundamental spectral sensitivities. The confusion points proposed by ESTE79 have the values

$$\begin{aligned} x_p &= .735 & x_d &= 1.14 & x_t &= .171 \\ y_p &= .265 & y_d &= -.14 & y_t &= -.003 \end{aligned} \tag{8}$$

(In order to avoid negative values for the fundamentals, $x_p = .735$ was used for the protanopic confusion point instead of the $x_p = .73$ recommended in ESTE79. y_p was found from the relation $y_p = 1.0 - x_p$.) This leads to the transformation

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} 0.0000 & 0.0000 & 0.5609 \\ -0.4227 & 1.1723 & 0.0911 \\ 0.1150 & 0.9364 & -0.0203 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{9}$$

where the normalization factors have been selected so that the fundamental spectral sensitivity functions that result from applying this transformation to the CIE XYZ matching functions all peak at 1.0. The fundamental spectral sensitivity functions that result are shown in Figure 6. The cone of realizable color that this produces in SML space is shown in Figure 7.

4 Color Vision Tests Using Television Displays

Color vision tests can be an important tool for screening the users of a computer graphics system. Standard tests, such as the Farnsworth-Munsell 100 hues test, can be easily implemented on a digitally controlled color television monitor. The flexibility of this hardware also makes it possible to extend the standard tests and to eventually devise new color

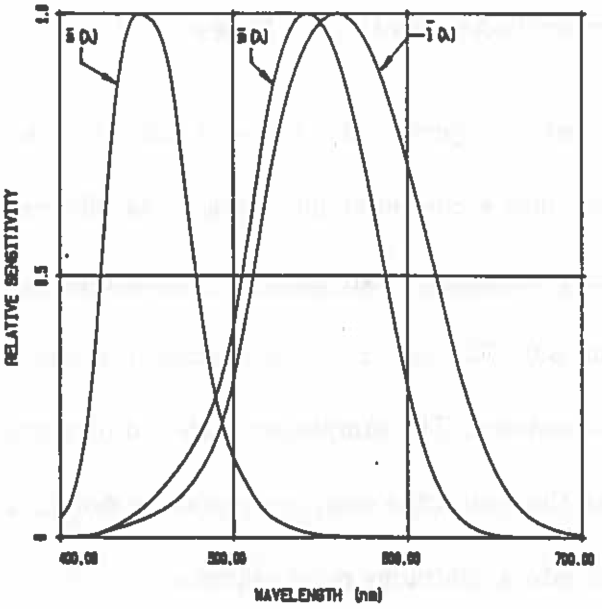


Figure 6: Fundamental spectral sensitivity functions.

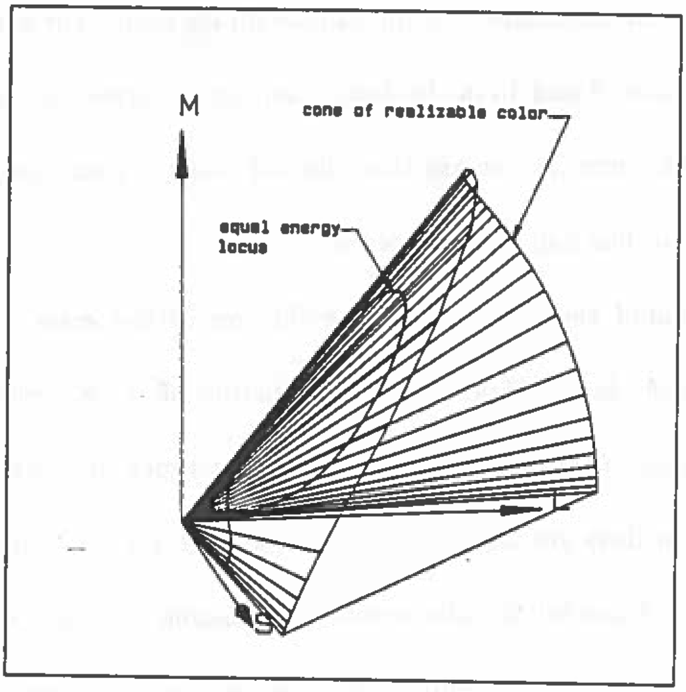


Figure 7: Equal energy spectrum locus and cone of realizable color in SML space.

vision tests.

4.1 The Farnsworth-Munsell 100 Hues Test

In the 100 hues test a subject's color vision is judged by how well they can rearrange a set of color samples into a continous hue circuit. 85 different Munsell hues (out of the 100 that were originally considered) are used for the samples and each sample has Munsell value 6.0 and chroma 6.0. The hue circuit is divided into four quadrants with one tray of 21 samples for each quadrant. The samples at each end of a tray are fixed and the subject is given the tray with the rest of the samples in random order. The subject is instructed to organize the samples into a continous color sequence.

Each of the color samples is assigned a number from 1 to 85 depending on its position in the hue circuit. When the test is complete, an error score is computed for each sample by adding together the differences between the sample number and the number of the sample on its left and the number of the sample on its right. For example, if sample 11 is surrounded by samples 8 and 13 in the final ordering, the error for sample 11 will be (11-8) + (13-11) = 5. The error scores are then plotted using a polar coordinate system where angle corresponds to hue and radius to error.

The theory behind the test and the significance of the error scores can be seen by referring to Figure 8. In this figure the chromaticities of the 85 color samples are plotted along with the confusion lines for the three different types of dichromats. The points at which the confusion lines are tangent to the hue circuit are the places where each type of dichromat could be expected to make errors in arranging the color samples. LAKO69 has suggested that samples 14 to 24 and 57 to 72 for protanopes, samples 12 to 22 and 52 to 64 for deuteranopes, and samples 80 to 9 and 42 to 54 for tritanopes are the regions where

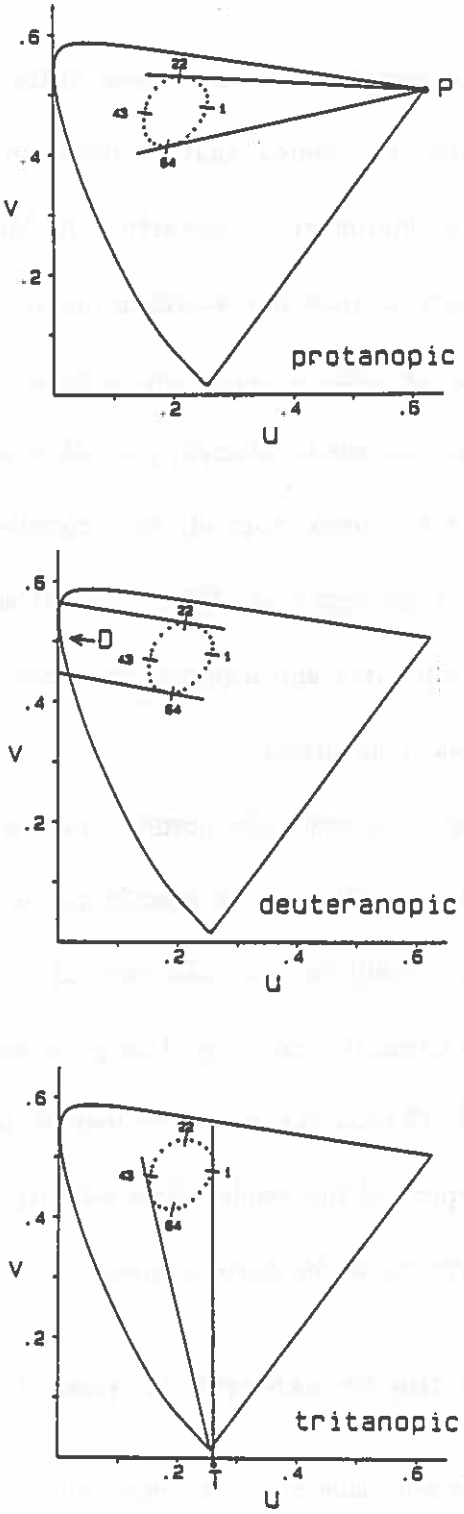


Figure 8: Protanopic, deuteranopic, and tritanopic confusion lines and the chromaticities of the Farnsworth-Munsell 100 hues test.

the most errors should be expected.

Using computer graphics hardware a video version of the 100 hues test can be created. Figure 9 shows the trays (correctly ordered) that are displayed one at a time by the program. The color for each sample is determined by converting the Munsell notation for the sample to the RGB values necessary to drive the television monitor (COWA83, MEYE80). The subjects rearrange the tray by using a tablet and a stylus to point at two samples that should be switched. As each sample is selected, a small arrow appears below it and then disappears after the samples have been swapped. After completing a tray, a subject specifies termination and proceeds to the next tray. When the test is complete the error score for each cap is automatically computed and a plot of the error is made on the display using the traditional 100 hues polar plot format.

The program was tested on several color normal subjects and on several subjects who were known to be color deficient although the specific nature of their defect was unknown. Figure 10 is a representative result for two color normal subjects. Although the error is quite low around the entire circle, it is not zero. This gives some indication of the difficulty of the task. Figures 11 and 12 show the results for four of the color defective subjects. In Figure 11 the pattern is typical of the results obtained with protanopes and in Figure 12 the results are consistent with those for deuteranopes.

4.2 An Extension to the Farnsworth-Munsell 100 Hues Test

The flexibility of a color television monitor connected to a digital frame store makes it possible to extend the 100 hues test in a way that provides a more positive identification of dichromatism. The points at which the confusion lines become tangent to the hue circle on a chromaticity diagram identifies the color samples in the 100 hues test that each type

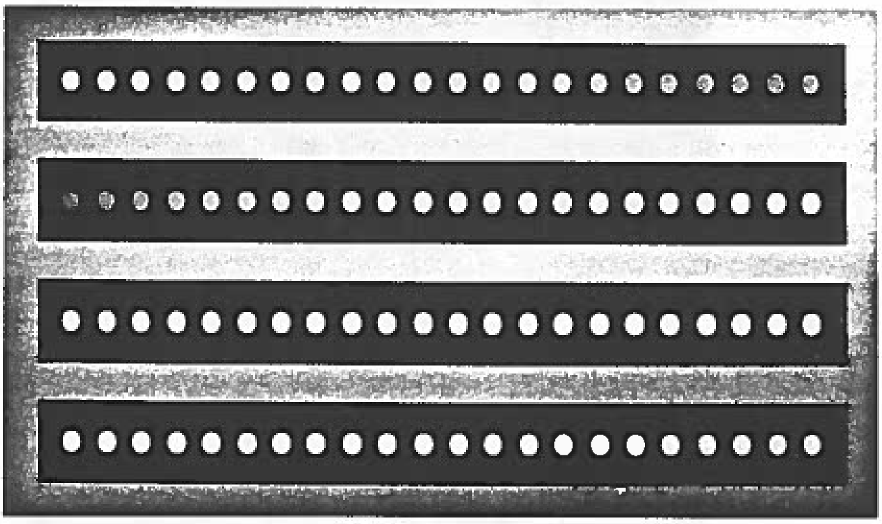


Figure 9: The Farnsworth-Munsell 100 hue color trays.

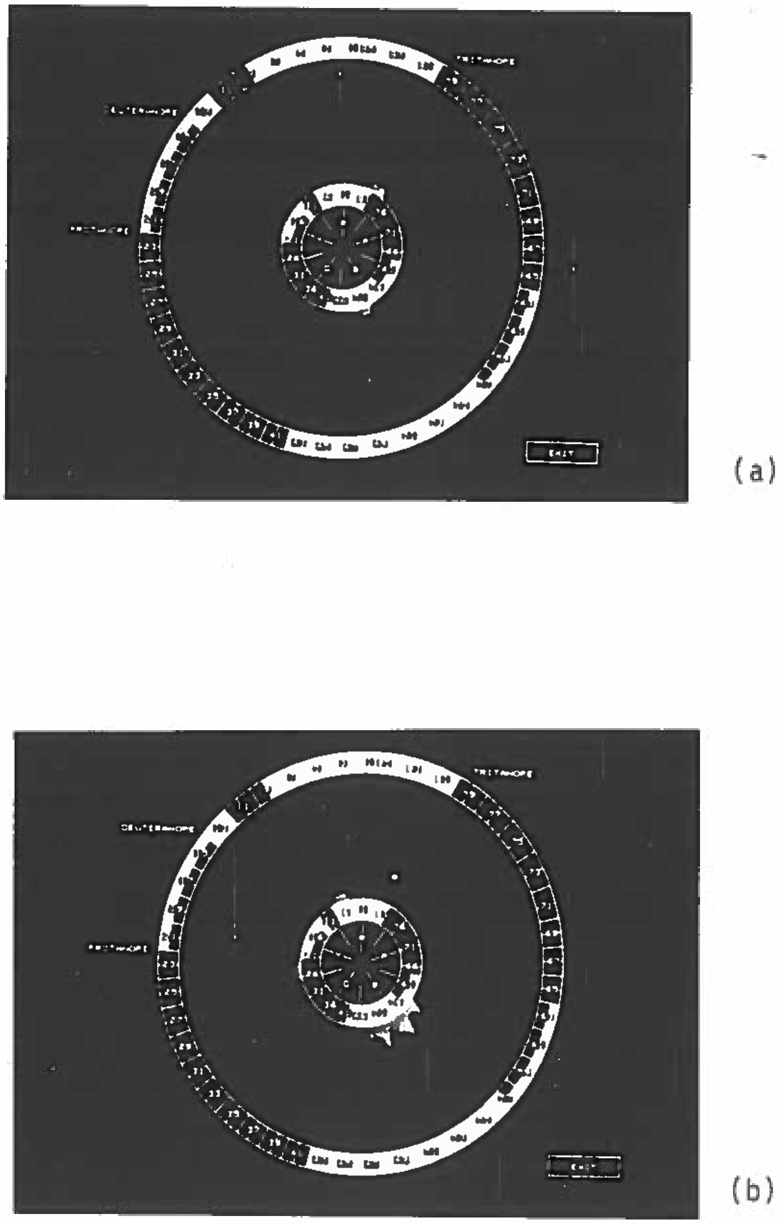
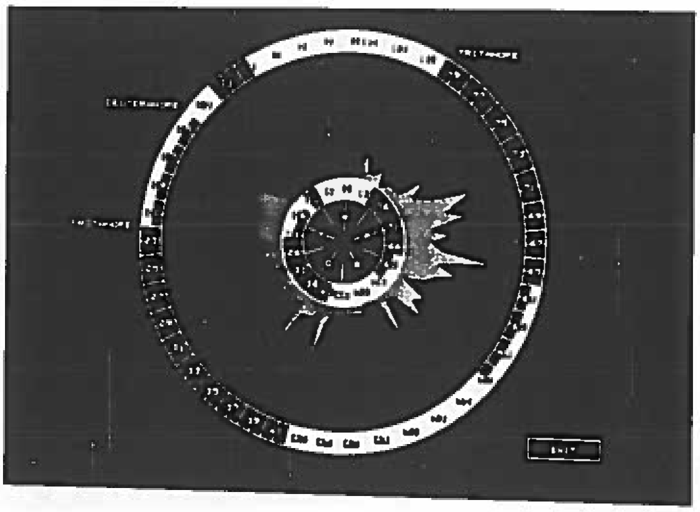
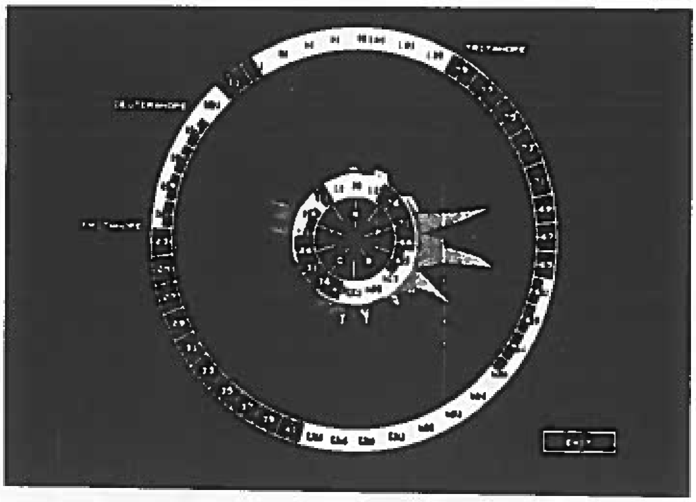


Figure 10: Farnsworth-Munsell 100 hue error plots for two color normal subjects.



(a)



(b)

Figure 11: Farnsworth-Munsell 100 hue error plots for two protanopic subjects.

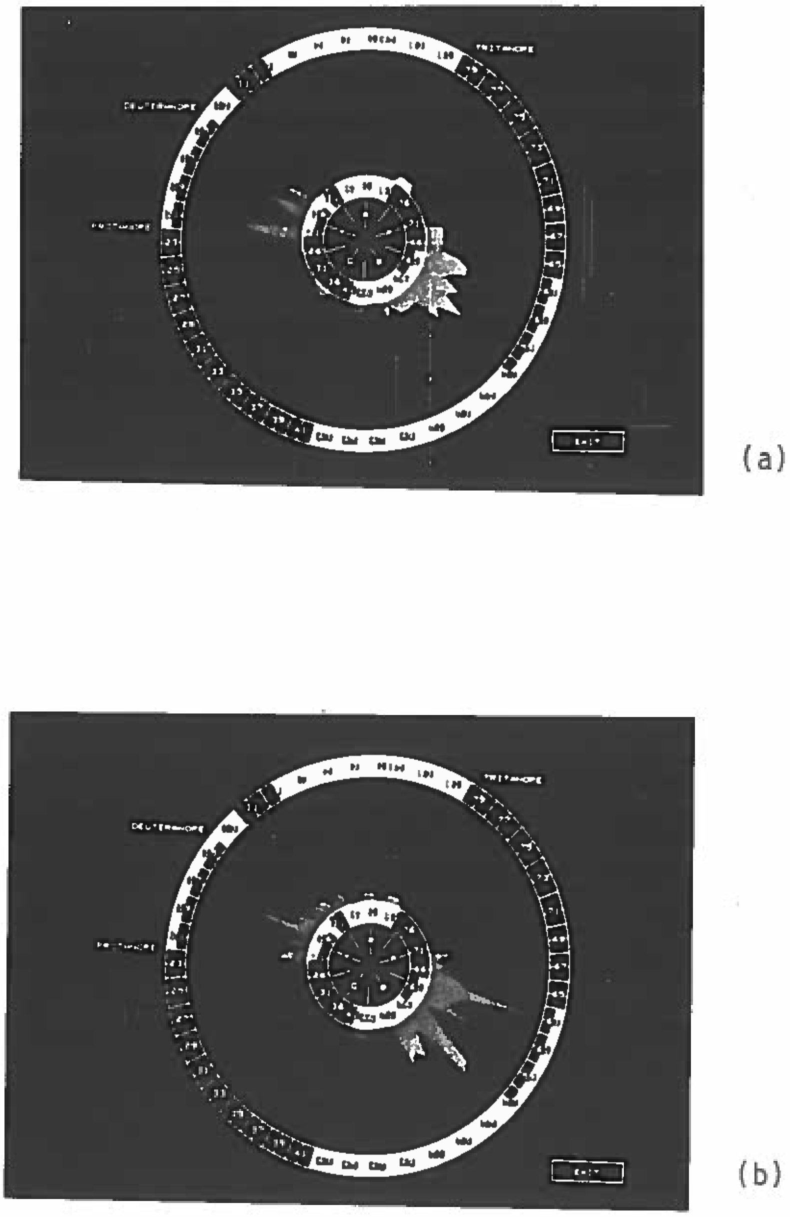


Figure 12: Farnsworth-Munsell 100 hue error plots for two deuteranopic subjects.

of dichromat will have trouble arranging in proper order. Unfortunately only two or three color samples at the point of tangency come close to lying on the same confusion line and the points of tangency for two types of dichromacy are close to one another. This makes it difficult to differentiate between two of the three types of dichromacy from the error scores. If each tray was composed of color samples with chromaticities that all fell along a single confusion line, no ambiguity would exist. In the past, these specific color scales would have been difficult to produce using the color reproduction technologies available when the 100 hues test was invented, but they are now easy to create using a digitally controlled color television monitor.

The extended color vision test consisted of 9 trays of 15 color samples, with each tray to be arranged in a manner similar to the way the trays in the 100 hues test were arranged. There were three trays for each type of dichromacy. Two of the three trays consisted of colors that lay on the confusion lines tangent to the hue circle of the 100 hues test. The center color in each of these trays was the same as the color sample from the 100 hues test at the point of tangency. These points of tangency were found to be color samples 21 and 66 for protanopes, 20 and 62 for deuteranopes, and 6 and 50 for tritanopes. The remaining tray for each type of dichromacy was located on the confusion line through the Munsell color with chroma 0 and value 6.0, and it was centered on the color. The distance covered on the 1976 CIE UCS chromaticity diagram by an entire tray and the distance from color sample to color sample within a tray were held constant in order to keep the difficulty of organizing the trays constant. Table 1 gives the chromaticities of the endpoints of each tray. Figure 13 shows plots of the chromaticities that are spanned by each tray.

This extension to the Farnsworth-Munsell 100 hues test was given to several people with normal color vision and to the protanopes and deuteranopes whose results from the

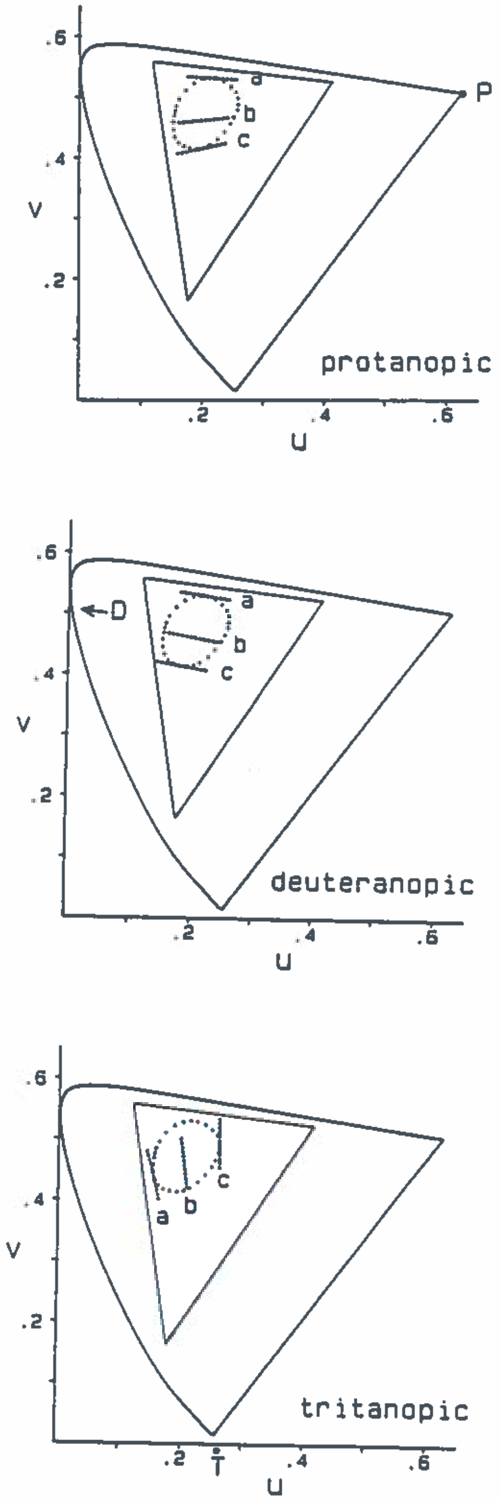


Figure 13: Chromaticities of color trays used to test for protanopic, deuteranopic, and tritanopic color vision in the new color vision test.

	begin		end	
	u	v	u	v
protanope	0.175	0.533	0.255	0.528
	0.161	0.457	0.241	0.465
	0.159	0.406	0.237	0.423
deuteranope	0.179	0.536	0.258	0.524
	0.161	0.468	0.240	0.454
	0.143	0.421	0.221	0.407
tritanope	0.258	0.534	0.259	0.454
	0.196	0.501	0.206	0.421
	0.142	0.480	0.161	0.402

Table 1: Endpoints on the 1976 CIE UCS diagram of the color scales used in the extension to the Farnsworth-Munsell 100 hues test.

	normal		protanope		deuteranope	
	(a)	(b)	(a)	(b)	(a)	(b)
protanopic trays	0	4	168	200	76	80
deuteranopic trays	4	12	72	84	360	336
tritanopic trays	8	8	4	0	4	8

Table 2: Error scores on each type of tray from the extension to the Farnsworth-Munsell 100 hues test for subjects whose error scores on the original Farnsworth-Munsell 100 hues test are shown in Figures 10, 11, and 12.

100 hues test are shown in Figures 11 and 12. The trays were presented to the subjects in random order and the scores were computed using the same algorithm as was used for the 100 hues test. A total error score for each type of dichromat was obtained by summing the errors for the appropriate three trays. The results are shown in Table 2. As can be seen, the results are clearer than those obtained from the Farnsworth-Munsell 100 hues test.

5 Synthesis of a Dichromat's View of the World

Trying to synthesize a picture of the world as it appears to dichromats is a problem which touches on a basic philosophical issue. Even though it is possible to create a reproduction that two people with normal color vision accept as a match to an original scene, it is impossible to determine whether the colors as perceived by each individual are the same.

Similarly, it is possible to state which colors from a trichromat's color space a dichromat confuses, but it is impossible to know what colors they see instead. However, a few individuals have been discovered who are protanopes or deuteranopes in one eye and trichromats in the other eye. The weight of the evidence from research on these people (JUDD48, SLOA48, GRAH58) is that the hue circuit for normal trichromats becomes a hue line for protanopes and deuteranopes with blue (approximately 575 nm) and yellow (approximately 470 nm) as the endpoints of the line and a neutral gray at the midpoint.

Given the chromaticity confusion points and the preceeding information on the color perceptions of protanopes and deuteranopes, it is possible to take a full color picture and manipulate it so that a trichromat can see how the image might appear to a dichromat. In the article where he first described the 100 hues test (FARN43), Farnsworth proposed one of the first uniform chromaticity diagrams and showed how it would collapse to nearly a line for each of the three types of dichromat. This defines a "major axis" on the uniform chromaticity diagram and in the case of protanopes and deuteranopes these lines are very close to the line that runs through the 470 nm and 575 nm endpoints of the hue line determined by experiments on people with one dichromatic and one trichromatic eye. If it is assumed that these "major axes" represent the colors that are actually seen by dichromats, a full color image can be easily mapped into the color space of a dichromat.

Dichromatic versions of the image in the upper left hand corner of Figure 14 were produced using this approach. The RGB values for each pixel were first transformed into CIE XYZ space and the chromaticity and luminance were determined. Next the confusion line that passes through the chromaticity point of each pixel for each type of dichromat was found. The point of intersection between this confusion line and the major axis for each type of dichromat represents the color as it would appear to that type of dichromat

Figure 14: Dichromatic versions of a full color image.

(see Figure 15). The major axes were defined as the lines through 473 nm and 574 nm for protanopes, the line through 477 nm and 578 nm for deuteranopes, and the line through 490 nm and 610 nm for tritanopes. Since the exact path of each of these lines through the center of the chromaticity diagram is unknown, it was decided for consistency to make them all pass through D6500 white. This new chromaticity and the original luminance were then transformed back into *RGB* space. If the new color fell outside the monitor gamut, it was adjusted by either holding its dominant wavelength constant and reducing its purity, or by holding its chromaticity constant and adjusting its luminance. In some cases both types of adjustment were required. The resulting pictures for each type of dichromat are shown in Figure 14.

Because the colors were adjusted by moving them along the chromaticity confusion lines, the new pictures should appear the same as the originals to that particular type of dichromat. In fact when the color television version of Figure 14 was shown to the protanopes and deuteranopes whose 100 hues test results are shown in Figures 11 and 12, they each selected the correct quarter of the screen (upper right for protanope and lower left for deuteranope) as being the most similar to the full color version in the upper left corner. They all said that the blouse and hair were the only things that appeared slightly different. This is not surprising since these colors were quite saturated in the original and required some adjustment in the deuteranopic and protanopic versions.

6 Color Selection for Color Deficient Users

Display design for color deficient users is an important consideration in computer graphics. Approximately 8 percent of the male population and less than 1 percent of the female population suffer from some form of anomalous color vision. Complete dichromatism afflicts

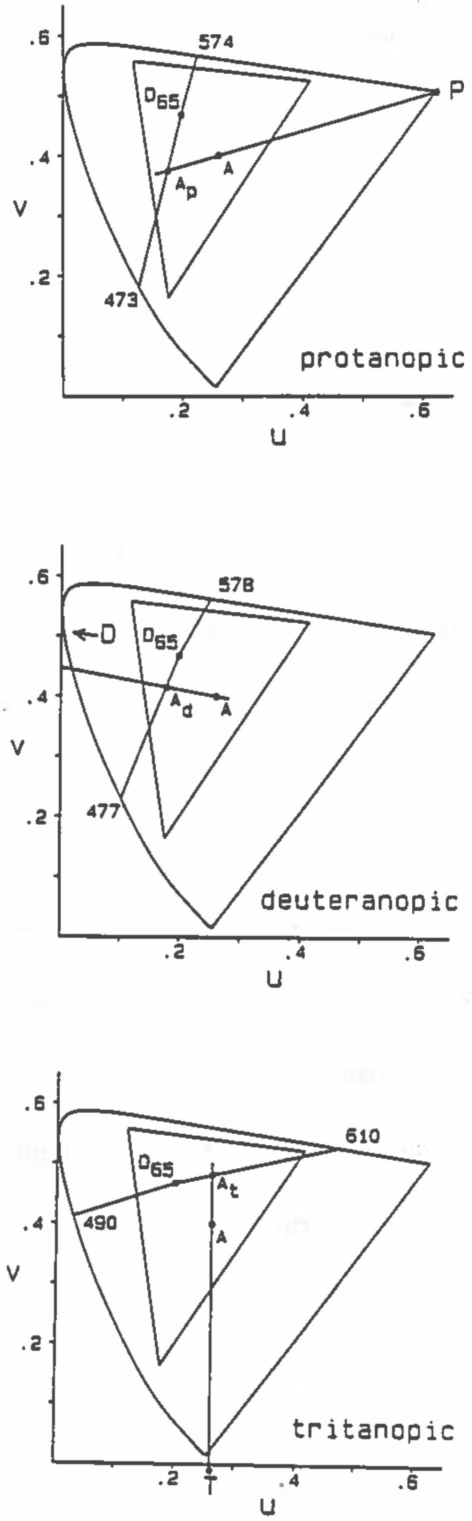


Figure 15: Axes of colors actually seen by dichromats and adjustments make to a single chromaticity point in order to create a dichromatic version of an image.

approximately 2 percent of the male population and a fraction of one percent of the female population (HURV81). The Farnsworth-Munsell 100 hues test and the extensions that were made to it in Section 4.2 can be employed to identify those users that may have trouble interpreting the output of a computer graphics system. These tests and the dichromatic view of the world described in Section 5.0 also provide guidelines for designing displays that are unambigous to color deficient users.

Colors which appear different to a dichromat and are thus to be preferred in display design for dichromats are those which at least differ in luminance and preferably do not lie on the same confusion line in the chromaticity diagram. The best such color scales would be those that are more or less orthogonal to the confusion lines. The colors used to produce the dichromatic version of the full color picture in Figure 14 have such a property (see Figure 15). They also correspond to the colors actually seen by a dichromat. Further confirmation that color scales orthogonal to the confusion lines are to be preferred is given by the results for the new color vision test in Table 2. The protanope and deuteranope both had little trouble organizing the tritanopic color trays because the tritanopic confusion lines are roughly perpendicular to the protanopic and deuteranopic confusion lines.

Although there are three distinct forms of dichromacy and anomalous variations of each type, it is still possible to design a single display that will accomodate virtually all color defective users. Tritanopia occurrs in only .002 percent of the male population and .001 percent of the female population (HURV81). Thus almost all people with color defective vision suffer from some form of protanopia or deuteranopia. Because the confusion lines are roughly parallel for protanopes and deuteranopes, the preferred color scales for both types of color deficiency have the same chromaticity loci as can be seen in Figure 15. (This is reflected in the fact that the protanopic and deuteranopic versions of the picture in

	begin		end	
	u	v	u	v
parallel	0.177	0.402	0.219	0.535
perpendicular	0.266	0.454	0.129	0.482

Table 3: Endpoints on the 1976 UCS diagram of the color scales used in Figure 16.

Figure 14 are so similar.) Finally, the fact that dichromatic vision is a more restrictive form of defective color vision than anomalous trichromatic vision means that a display designed for dichromats will also work for anomalous trichromats. Given the above facts it is possible to select a single set of colors that can be distinguished by virtually all color defective users.

A qualitative test involving the display of engineering data was performed in order to check this result. Two color scales were chosen that had the 1976 UCS endpoints given in Table 3. All colors in the scales had the same Munsell value and the difference in chromaticity between colors in each color scale was a constant distance on the 1976 UCS diagram. One of the scales was chosen so as to lie roughly parallel to the protanopic and deuteranopic confusion lines and the other scale was selected to be approximately perpendicular. The result of using these color scales to display the displacement of a propeller blade is shown in Figure 16 along with dichromatic versions of these pictures. As can be seen, the color scale which lies roughly perpendicular to the protanopic and deuteranopic confusion lines allows people with these types of color deficiency to see the results of the analysis more clearly than does the color scale which lies parallel to their confusion lines.

7 Summary and Conclusions

The fundamental spectral sensitivity functions define an important color space for workers in the field of computer graphics. Color reproduction work can be accomplished by the use of the CIE XYZ matching functions alone. This is only possible, however, because the

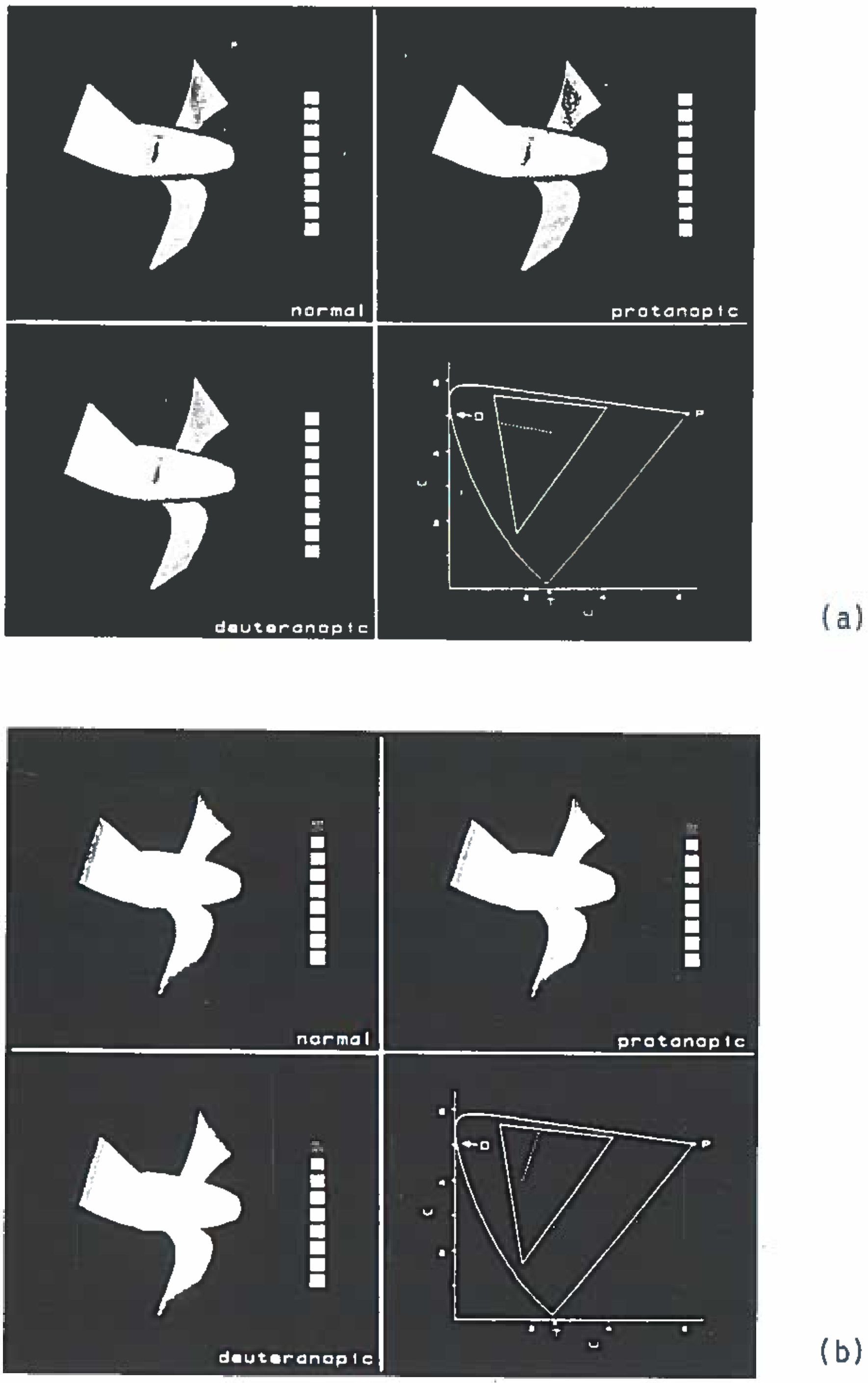


Figure 16: Color scale used to display displacement with chromaticity loci (a) parallel and (b) perpendicular to protanopic and deuteranopic confusion line.

CIE XYZ matching functions are a linear transform of the fundamental spectral sensitivity functions. This transform can be derived if the chromaticity coordinates of the dichromatic confusion points are known. Once specified, the fundamental spectral sensitivity functions provide insights into the nature of human color vision and guidance for the design of displays for color deficient users.

Color vision tests for screening the users of a computer graphics system are easy to implement on a digitally controlled color television monitor. Traditional tests, such as the Farnsworth-Munsell 100 hues test, can be successfully reproduced and more efficiently administered. Extensions to this test can also be created. Colors aligned along known confusion loci are easy to reproduce and can be used as the samples in this test. The results allow a clearer identification of the specific types of dichromatic vision.

An image which attempts to show a person with normal color vision how the world appears to a dichromat can also be produced. This is accomplished by first finding the confusion line for this type of dichromat that passes through the chromaticity of each pixel in the picture. These confusion lines are then followed until they intersect the line which represents the colors actually seen by the dichromat.

Most importantly, computer graphic images can now be synthesized to accomodate color defective users. The two dominant types of dichromacy are protanopic and deuteranopic vision. These two types of color deficiency have almost parallel confusion lines. In addition, dichromatic vision is a more restrictive form of color vision than anomalous trichromacy. This leads to the recommendation that color scales with chromaticity loci that are perpendicular to the protanopic and deuteranopic confusion lines are the best color scales to use for people with color deficient vision.

The work described in this paper represents a first step in using the power of digitally controlled color television displays to identify color defective individuals and to create effective user interfaces for them. Future CRT based color vision tests must be able to differentiate between dichromats and anomalous trichromats. Once the specific nature of the color deficiency has been identified by the use of these tests, colors can be selected which take full advantage of whatever color discrimination abilities the individual has. This will move us one step closer to the day when all user interfaces are custom tailored to the needs of the individual.

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