Qualitative Reasoning in Economics

Arthur M. Farley
and
Kuan-Pin Lin

CIS-TR-88-03
March 15, 1988

DEPARTMENT OF COMPUTER AND INFORMATION SCIENCE
UNIVERSITY OF OREGON

Given Economic System Model **ES**
and Perturbation **P**, as in Figure 4:

General Mutiple-Market Simulation Procedure

0. Establish initial equilibrium state.

1. For each Market **M** of **ES** having **p** in **Parameters,**
perform-market-update (M, P).

2. Until all connection variable changes have been propagated
perform-market-update (M, P),
where **P** is a perturbation of a connection
variable that is parameter to market **M.**

3. If all markets in **ES** are stable
then create final state
else report instability.

Figure 8. Multiple Market Simulation Procedure

Qualitative Reasoning in Economics

by

Arthur M. Farley
Department of Computer and Information Science
University of Oregon
Eugene, OR 97403

and

Kuan-Pin Lin
Department of Economics
Portland State University
Portland, OR 97207

Abstract

In this paper, we present a scheme for the qualitative representation and simulation of economic theories. We define an associated paradigm of reasoning in accordance with the traditional method of comparative statics. The primitive element of our representation scheme is the simple, demand-supply, market model. More complex economic theories are represented as multiple-market models, with interactions between markets realized by variables of one market model being parameters of another. We define an extension of our single-market simulation process, making it possible to reason about such complex, multiple-market models. We illustrate our notions through consideration of the Keynesian IS-LM macroeconomic model.

Length: Text < 3000; Figures (when reduced and reformatted) = 1000.

Topic: Commonsense Reasoning (qualitative reasoning)

I. Introduction

In the absence of complete quantitative knowledge about the structure and behavior of complex economic systems, economists have long relied on qualitative techniques of causal ordering and comparative statics to explain the economic effects of government fiscal and monetary policies. Examples of such explanations include the supply-side argument that a reduction in taxes for the rich will lead to increased investments and eventual, trickle-down prosperity for the poor. Other examples include the argument that a fall in the value of the United States dollar will lead to increased U.S. exports and, thus, a reduction in international trade deficits or that an increasing national debt will lead to higher demand for money and, thus, higher interest rates.

Economic theories are most often expressed in terms of qualitative abstractions of highly complex, quantitative systems. Quantitative methods are called upon mainly to better specify order-of-magnitude relations among variables. The qualitative models provide a framework for the selection and determination of relevant quantitative elements, e.g., coefficients of marginal dependencies among qualitatively related variables. Qualitative reasoning elements have been prevalent in the classic writings on economics (e.g., Keynes [1935], Samuelson [1947], and Schumpeter [1934]).

In this paper, we discuss the adaptation of qualitative modeling and simulation techniques developed in artificial intelligence to the domain of market-based, economic systems. This research represents a refinement and extension of our earlier work [Farley, 1986]. Our representation scheme borrows elements from qualitative modeling proposals for physical systems. Our primary contribution lies in the formalization of qualitative market models and of a simulation paradigm suited to market-based economic reasoning, including techniques for realization and application of the comparative statics methodology.

II. Qualitative Modeling in Economics

Qualitative modeling has emerged as a powerful technique for reasoning about mechanical or electrical devices (Bobrow, 1985). Qualitative abstraction is applied both to the value domains of a model's parameters and variables and to the forms of constraints that describe component and system behaviors. The quantitative value domains of model variables are transformed into finite, ordered sets of landmark values (including positive and negative infinity) and the intervals that lie between them. A particularly important qualitative value domain, the {-, 0, +} domain, has proven useful for characterizing direction of change within a dynamic system and for indicating reference to normality during diagnostic reasoning (Farley, 1987). This domain employs the single landmark value of 0 (indicating no change or normal level), with negative (-) and positive (+) values corresponding to the intervals below and above the landmark, respectively.

The constraints between qualitative variable values that characterize the behavior of a system are represented as simplified abstractions of quantitative, algebraic constraints. One approach represents differential equations in a simplified difference equation form known as confluences (de Kleer and Brown [1984]). The approach that we will use is based upon increasing (M^-) and decreasing (M^+) monotonic functional relationships between pairs of variables (Forbus [1984], Kuiper's [1984]). These monotonic, functional relationships can be supplemented by indications of landmark correspondences. For example, the qualitative constraint $x = M_0^+(y)$ indicates that x is a monotonic increasing function of y, with value correspondence at the zero landmark. By adding algebraic constraints, including that one variable is the derivative of another, to these monotonic functional abstractions, Kuipers [1984, 1986] defines a methodology for reasoning about the structure and behavior of physical systems based upon incomplete, qualitative knowledge .

In economics, most theories are stated in terms of causal relationships and equilibrium conditions among sets of relevant variables and parameters that describe market components of an economic system. The investigation of change in an economic system, as it moves from one position of equilibrium to another, is carried on according to a methodology known as *comparative statics* . "This method of comparative statics is but one special application of the more general practice of scientific deduction in which the behavior of a system (possible through time) is defined in terms of a given set of functional equations and initial conditions" (Samuelson [1947], Chapter II, p.8). The technique is an instance of perturbation analysis, often applied in the analysis of physical systems, as well.

Demand and supply functions are the most basic concepts of analytic economics. They are used not only to represent the causal orderings among economic variables, but also to organize the qualitative functional relationships existing among them. They are typically written as

$$D = f_d(\underset{-}{P}, \underset{+}{Y}, \ldots) \quad \text{and} \quad S = f_s(\underset{+}{P}, \underset{-}{W}, \ldots). \qquad (0)$$

D and S represent quantities of a certain commodity that are demanded and supplied, respectively, within a particular market. Variable P and parameters Y and W refer to the price level for market consumption, the income of market consumers, and the price level of productive resources (e.g., wage rate for skilled labor), respectively. A *variable* of a particular market represents a level that can change by that market's behavior; a *parameter* of a market represents a level determined outside the market but which can effect variable levels through market behavior.

The demand and supply functions normally are taken to represent a causal ordering among the variables. The direction of the causal ordering is from the *independent* variables on the right-hand side of the equation to the *dependent* variable on the left-hand side. Changes in the values of the independent variables are seen to *cause* changes in the dependent variable's value. The sign, + or -, under a variable on the right-hand side of the equation indicates the positive or negative effect that an increase in the independent variable has on the dependent variable. The sign also indicates the slope of a graphical curve that could be drawn to represent the relationship between the variables.

Using the notation of Kuipers [1984, 1986], the demand and supply curves can be represented as follows:

$$D = M^{-}(P) \text{ and } S = M^{+}(P)\,. \qquad (1)$$

Recall that the qualitative functional relationships indicate that demand (supply) is monotonically decreasing (increasing) with respect to price level. Thus, if price is increased, demand is caused to decrease and supply to increase. Considering the other parameters in (0), we can write $D = M^{+}(Y)$ and $S = M^{-}(W)$, completing representation of our knowledge regarding D and S.

The market equilibrium of a single commodity market is identified as the intersection of its demand and supply curves, at the price where D = S. In economic literature, the *tatonnement adjustment* process (the law of demand and supply) [Debreu, 1959] is used to explain the stability of market equilibrium. Whenever there is pressure of excess demand (D-S > 0) in the market, the price level P will change in a positive direction to clear the market, returning it to equilibrium. Demand is thereby caused to become lower and is accompanied by increasing supply, according to the equations of (1). Similarly, price level P falls due to excess supply (D-S < 0) in the market. Positively-sloped supply and negatively-sloped demand guarantee that excess demand (or supply) is eventually eliminated with adjustment of P. The new equilibrium will be maintained until another external disturbance of a model parameter occurs. Qualitatively, the adjustment rule can be expressed by the following equation:

$$\partial P = M_0{}^{+}(X). \qquad (2)$$

X represents excess demand and is equal to D-S. ∂P, the direction of adjustment (or change) in price level, has a positive monotonic relation to excess demand X, with landmark correspondence at zero. Qualitative equations (1) and (2), together with the equilibrium condition that D = S (i.e., X = 0), form the basic model of demand-supply market mechanisms in economics.

We formalize the qualitative representation of the basic demand-supply market in Figure 1. Every qualitative market model will consist of a **Name,** sets of **Variables, Parameters, Definitions,** and **Causal Relations,** together with an **Equilibrium Variable** and **Adjustment.** As discussed earlier, **Variables** represent market levels that may change during market simulation. The value space associated with each variable is a pair, the first component representing its level and the second its direction of change (Kuipers, 1986). The first normally will take on symbolic values corresponding to the initial, any intermediate, and final levels; the second will take on values from the {-, 0, +}

domain defined earlier. For example, suppose P has the value (P_e, +) at some point during a simulation; this means that P is at level P_e and is increasing. **Parameters** represent levels that are determined outside the particular market under study and, therefore, remain constant during simulation of that market. Since a parameter is constant, the value associated with any parameter is simply a single symbol representing its level.

There are two sets of relations that hold among model values. **Definitions** represent algebraic relationships that hold by definition at all times. **Causal Relations** are qualitative relationships, expressed in terms of the M^+ and M^- notation of Kuipers; these represent the traditional, causal ordering relationships among market parameters and variables, as discussed above. They correspond to the notion that a change in the independent **Variable** or **Parameter** causes a change, in the indicated direction, in the dependent **Variable**.

The **Equilibrium Variable**, always defined to be the difference between certain other market **Variables** and **Parameters**, is equal to 0 when the market is in equilibrium. The **Equilibrium Variable** differs from other market variables in that its current level is also drawn from the {-, 0, +} domain. The **Adjustment** is expressed as a causal relation between the direction of change of a particular market variable and the level of the **Equilibrium Variable**. It reflects the basic homeostatic nature of supply-demand markets; when a market is not in equilibrium, an adjustment is initiated in some market variable, which, by **Causal Relations**, leads to reestablishment of equilibrium.

III. Qualitative Economic Reasoning

We will first demonstrate the method of comparative statics for the basic demand-supply market. The basic reasoning paradigm of comparative statics is to perturb the initial system state by altering the value of a market parameter, propagate the effects upon market variables by a specialized form of qualitative simulation until a new equilibrium is reached, and then compare the resultant state with the initial state. Let us assume that the market is in equilibrium at values P_e, S_e, D_e, corresponding to point E in the diagram of Figure 2. We then perturb the market by increasing Y, which causes a demand shock shift to curve D'. Economists traditionally use the vertical axis for price (here, the independent variable) and horizontal axis for quantity (the dependent variable) when representing demand-supply curves. At the original equilibrium price level P_e, there is excess demand ($D'-S > 0$) after the change in demand, corresponding to point F. Price then increases, through market adjustment, toward the new equilibrium point G, with quantity supplied moving along the supply curve S from point E and quantity demanded moving along the new demand curve D' from point F, respectively. The results of the demand shock, represented by point G, are a higher price and a larger quantity supplied and demanded.

We will now represent this reasoning in the form of a qualitative simulation, an annotated trace of which is presented in Figure 3. In the initial qualitative state St_e, variables P, D, and S are constant at equilibrium levels Pe, De, and Se, respectively. When we perturb the market by increasing income Y, we create a new state St_p. By reference to the **Causal Relations**, this causes in a positive increase in demand to D', as shown in state St_1. Upon change to the value of a variable, relevant elements of **Definitions** are applied immediately; this results in a positive X, representing the newly created, excess demand. State St_1, corresponding to point F of Figure 2, is not an equilibrium state. The **Adjustment** is thus applied, resulting in a positive ∂P in state St_2. Following initiation of the adjustment, **Causal Relationships** are again applied; in this case, ∂D becomes negative and ∂S positive, as shown in St_3. Through application of the definition of X, ∂X now becomes negative. We see that the market is stable, as the direction of change for X (∂X) is toward equilibrium. St_f represents the subsequent equilibrium state, with new landmark values for P, D, and S, corresponding to point G of Figure 2. Figure 4 presents the general, single-market simulation method illustrated by this example.

A comparison of variable levels with original values, determined by consideration of the directions of change during the simulation and any relevant elements of **Definitions**, completes the comparative statics method. Changes in P and S can be determined solely by consideration of their direction of change during simulation. However, demand D first increases by the demand shock and then decreases due to subsequent price adjustment. The overall change in demand would be qualitatively ambiguous if it were not for the fact that supply only increases due to price adjustment. By definition, demand equals supply at both equilibria, thus D_f is greater than D_e.

IV. Multiple Market Models

Complex economic theories, such as those proposed in the study of macroeconomics, can be constructed from instances of the above, simple market model. By applying *ceteris paribus* assumptions, economists reason about one element of the model at a time, propagating interactions through related elements in a piecemeal fashion. Each element of the model is typically an instance of a demand-supply market model; these interact during qualitative simulation through shared variables and parameters to create the overall model behavior.

Each market of a complex economic system can be modeled according to the scheme described above. When modeling a more complex system, we add an **Economic System** component, consisting of a **Name** and listings of **Markets**, **System Parameters**, **System Variables**, and **Connections**. **Markets** is simply the set of names of the markets constituting the system model. **System Parameters** is defined to be elements of **Parameters** from component markets that are not included in the **Variables** of another market. **System Variables** is the union of **Variables** from the component markets. **Connections** represents the interactions between component markets; each element of **Connections** is a triple, consisting of a source market, an affected market, and an element of **Variables** of the source market which is an element of **Parameters** of the affected market. If drawn in graphic form, elements of **Markets** would constitute the nodes; the **Connections** are arcs from source to affected markets, labelled by the associated variable.

We now explore the Keynesian model of macroeconomics in which income-investment-saving relationships from the product market interact with income-interest rate elements of the money market. This demand-side, macroeconomic theory is often called the *IS-LM model*. Qualitative reasoning in terms of the IS-LM model has proved useful for explaining and predicting basic effects of government policy upon the economy. Figures 5 and 6 present qualitative models of the money and product markets, respectively. Figure 7 presents the combined Economic System representation of the IS-LM Model.

Our qualitative simulation method must be extended to capture reasoning about market interactions. An analysis of the effects that an increase in government spending may have upon economic variables (according to the IS-LM model) will illustrate the changes in qualitative simulation that are necessary. Figure 8 presents the extended simulation method. It uses the single-market, update procedure of Figure 4 as a subprocess, reflecting *ceteris paribus* assumptions underlying the reasoning paradigm. Figure 9 presents a trace of the extended simulation method for our case of interest. During qualitative simulation of the market mechanism, the application of *tatonnement adjustment* eliminates ambiguity in the direction of changes in demand and supply. According to our market clearing view of market adjustment, the increase in interest rates in the money market (state St_5) will be sufficient to overcome the continued upward pressure on money demand caused by increasing income generated in the product market.

Comparisons can be made between final and original variable values to determine the overall effects of the initial perturbation, completing the method of comparative statics. In this case, most variables changed in one direction only, simplifying comparisons. An increase in government spending results in higher income, savings, and interest rate, while business investment declines. The demand for money, which increases in response to the initial rise in income, returns to its initial level to be at equilibrium with the unchanged money supply. Such reasoning about qualitative values may involve direct comparison of values as well as consideration of directions of change and algebraic relations known to hold between various variable values at equilibrium. Technology for qualitative arithmetic reasoning is being developed independently of our research (Simmons, 1986).

V. Conclusion

In this paper, we show how representation and simulation techniques from qualitative physics can be adapted and extended to realize the classical method of economic reasoning known as comparative statics (Iwasaki and Simon [1986a]). Recent research in artificial intelligence, such as that reported by Farley [1986] and Bourgine and Raiman [1986a], has begun to explore the topic of formalizing qualitative reasoning in economics. The work reported here makes the transition from a totally equation-based model to one reflecting the teleological notions of adjustment and causal relation, as used in traditional economic reasoning. We also define a new method of qualitative simulation that allows us to reason about the effects of perturbations in equilibrium-based models. Finally, we improve upon the approach of Bourgine and Raiman [1986a] by better reflecing the market basis of economic modeling and economic reasoning.

The main contribution of Bourgine and Raiman [1986a, 1986b] is their discussion of the use of order-of-magnitude information as a means for removing ambiguity during the process of qualitative reasoning. Our two-market, macroeconomic model has been constructed so as to be free of ambiguity at the qualitative level. Starting from a disequilibrium in either market, the model will always generate a definite simulation solution. For a complex model, involving more variables and denser causal relations, there may be multiple interpretations of change to variables and parameters and, thus, ambiguities in finding a simulation solution. Consider the following, slightly modified IS-LM model, in which investment behavior now depends on income as well as interest rate in the product market. The corresponding model, in equational form, is

$$I(\underset{+}{Y},\ \underset{-}{R}) + G = S(\underset{+}{Y}) + T.$$

This would add $I = M^{+}(Y)$ to the causal relations of our product market model. Given the same adjustment processes for the product and money markets, when there is an increase in government spending, income level is adjusted upward due to excess demand in the product market. Because both investment and savings are now related positively to income, the directional change of excess demand is unknown. Ignoring effects upon interest rates from the money market for a moment, we must assume an order-of-magnitude condition stating that the marginal propensity to save with increasing income is greater than the marginal propensity to invest (i.e., $\partial I / \partial Y < \partial S / \partial Y$), if we are to restore equilibrium. An income increase in the product market will result in higher interest rates in the money market, which, in turn, contributes to a reduction in demand pressure in the product market by curtailing investments. Our order-of-magnitude condition then could be relaxed slightly, but must still be present in some form if we are to conclude simulation at a new equilibrium for the product market.

Name
Basic Supply-Demand Market

Variables
D : demand
S : supply
P : price level

Parameters
Y: income
W: labor cost (wages)

Definitions
$X = D - S$

Causal Relationships
$D = M^{-}(P)$
$D = M^{+}(Y)$
$S = M^{+}(P)$
$S = M^{-}(W)$

Equilibrium Variable
X : excess demand

Adjustment
$\partial P = M_0^{+}(X)$

Figure 1. Qualitative Model of Basic Market Economy

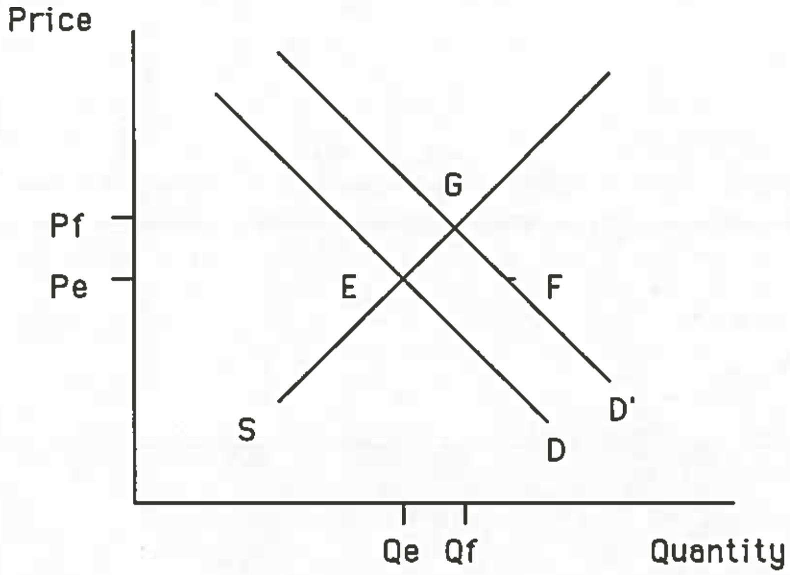

Figure 2. Graphic Representation of Basic Market

State	D	S	P	X	Y
St_e	$(D_e, 0)$	$(S_e, 0)$	$(P_e, 0)$	(0, 0)	Y_e

the initial equilibrium;

St_p	$(D_e, 0)$	$(S_e, 0)$	$(P_e, 0)$	(0, 0)	Y'

perturb Y upward;

St_1	(D', 0)	$(S_e, 0)$	$(P_e, 0)$	(+, 0)	Y'

this causes a change in D to $D' > D_e$, which creates excess demand by definition;

St_2	(D', 0)	$(S_e, 0)$	$(P_e, +)$	(+, 0)	Y'

this produces an upward adjustment of P ($\partial P > 0$);

St_3	(D', -)	$(S_e, +)$	$(P_e, +)$	(+, -)	Y'

this causes change in D and S ($\partial D<0$ $\partial S > 0$), which moves X toward equilibrium;

St_f	$(D_f, 0)$	$(S_f, 0)$	$(P_f, 0)$	(+, 0)	Y'

this results in equilibrium being reestablished.

Comparative Statics

The increase in Y to Y' results in $P_f > P_e$, from upward adjustment;
this causes $S_f > S_e$ and $D_f > D_e$ (as $S_f = D_f$), though $D_f < D'$.

Figure 3. Qualitative Simulation of Basic Supply-Demand Market

Given Market Model **M** and
Perturbation **P** = (**p**, **d**),
where **p** is a parameter
and **d** is a direction of change (+ or -).

Qualitative Market Simulation Procedure

0. establish initial equilibrium state.
1. perform-market-update (M, P).
2. if market is stable
then create final state
else report instability;

with,

perform-market-update (M, P)

1. Update value of parameter p according to d,
followed by Definitions;
2. Perform updates by Causal Relations,
followed by Definitions;
3. Perform updates by Adjustment,
followed by Definitions;
4. perform updates by Causal Relations,
followed by Definitions.

Figure 4. Single Market Simulation Procedure

Name
Money Market

Variables
L : Money Demand
R : interest Rate

Parameters
M : Money Supply
Y : Income
P : Price level

Definitions
$X_m = L - (M / P)$

Causal Relationships
$L = M^+(Y)$
$L = M^-(R)$

Equilibrium Variable
X_m : eXcess demand for money

Adjustment
$\partial R = M_0^+(X_m)$

Figure 5 . Keynesian Money Market Model

Name
Product Market

Variables
I : Investment
S : Savings
Y : Income

Parameters
T : Taxes
G : Goverment Spending
R : Interest rate

Definitions
$Xp = I + G - (S + T)$

Causal Relationships
$S = M^{+}(Y)$
$I = M^{-}(R)$

Equilibrium Variable
X_p : eXcess demand for Products

Adjustment
$\partial Y = M_0^{+}(X_p)$

Figure 6. Keynesian Product Market Model

Name
IS-LM Model

Markets
Product Market
Money Market

Parameters
T : Taxes
G : Goverment Spending
M : Money Supply
P : Price level

Variables
Y : Income
I : Investment
S : Savings
L : Money Demand
R : Interest rate

Connections
(Money Market, Product Market, R)
(Product Market, Money Market, Y)

Figure 7. The Keynesian IS-LM Model of Macroeconomics

State	G	I	S	Y	L	R	IS-LM Model
St_e	G_e	$(I_e, 0)$	$(S_e, 0)$	$(Y_e, 0)$	$(L_e, 0)$	$(R_e, 0)$	
	the initial equilibrium;						

	product market	G	I	S	Y	X_p	R
St_p		G'	$(I_e, 0)$	$(S_e, 0)$	$(Y_e, 0)$	(+ , 0)	R_e
	G is perturbed upward ($G' > G_e$), creating excess demand in the product market;						
St_1		G'	$(I_e, 0)$	$(S_e, 0)$	$(Y_e, +)$	(+, 0)	R_e
	the excess demand sets off an upward adjustment of income;						
St_2		G'	$(I_e, 0)$	$(S_e, +)$	$(Y_e, +)$	(+, -)	R_e
	this causes an increase in savings and a decrease in excess demand;						

	money market	Y	L	R	X_m
St_3		Y'	$(L_e, 0)$	$(R_e, 0)$	(0 , 0)
	the increase in income ($Y' > Y_e$) perturbs the money market,				
St_4		Y'	(L', 0)	$(R_e, 0)$	(+ , 0)
	causing excess demand for money ($L' > L_e$);				
St_5		Y'	(L', -)	$(R_e, +)$	(+ , -)
	this sets off an upward adjustment of interest rates;				

	product market	G	I	S	Y	X_p	R
St_6		G'	$(I_e, 0)$	$(S_e, +)$	$(Y_e, +)$	(+, -)	R'
	the increase in interest rates ($R' > R_e$) comes back to influence the product market,						
St_7		G'	(I', 0)	$(S_e, +)$	$(Y_e, +)$	(+, -)	R'
	causing a decrease in investment ($I' < I_e$), which adds to the decline in excess demand;						

	G	I	S	Y	L	R	IS-LM Model
St_f	G'	$(I_f, 0)$	$(S_f, 0)$	$(Y_f, 0)$	$(L_f, 0)$	$(R_f, 0)$	
	a new equilibrium is then established for the overall economic system.						

Figure 9. Qualitative Simulation of IS-LM Macroeconomic Model