Explorations in Qualitative Macroeconomics

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Abstract

In this chapter, we present an approach to qualitative reasoning about macroeconomic systems. The basic element of our scheme is the market model, an entity that adjusts itself to restore equilibrium after perturbation. We define a process that performs market adjustment following perturbation of a parameter. Complex economic theories are represented in terms of multiple markets with their interaction realized by variables of one market being parameters to others. We generalize our reasoning process to cope with multiple-market models. Comparing final to initial values of model variables implements comparative statics. We consider modeling situations of disequilibrium and present a method for generating multi-market dynamics. We illustrate our notions by consideration of Keynesian macroeconomic models.

I. Introduction

In the absence of complete, quantitative knowledge about the structure and behavior of complex economic systems, economists have long relied on qualitative methods of causal ordering and comparative statics to explain and predict economic effects of fiscal and monetary policies (e.g., Schumpeter, 1934; Keynes, 1935; and Samuelson, 1947). One example of such a qualitative prediction would be that a fall in the exchange rate of the U.S. dollar will lead to increased U.S. exports and, thus, to a reduction in the U.S. trade deficit; another would be that rising national debt will lead to higher demand for money and, thus, to higher interest rates and so to lower business investment. Note that these arguments, while concerning economic factors that could be measured in quantitative terms, both rely solely on qualitative values and relationships.

Qualitative reasoning is important in the formulation, interpretation, and evaluation of economic theories. Qualitative reasoning can serve to guide the design of quantitative studies that often require costly, extensive data acquisition and analysis. By first reasoning with a qualitative model, an economist can determine relevant quantitative values, such as marginal sensitivities among particular economic variables, knowledge of which would most likely reduce the uncertainty inherent in qualitative economic prediction. The quantitative values can be determined by consideration of available or acquisition of new economic data. By first reasoning qualitatively, an economist is able to determine those quantitative analyses that are likely to have the greatest impact on our understanding of complex economic systems.

In recent years, we have been exploring application of qualitative modeling and simulation techniques, as developed in the field of artificial intelligence for reasoning about physical systems, to the domain of macroeconomic systems. Our main contributions have been the formalization of a basic market model, the definition of a scheme for representing complex qualitative models in terms of multiple markets, and the specification of qualitative reasoning paradigms based upon our representation, including traditional, comparative statics methodology. Here, we review some of our earlier results (Farley, 1986; Farley and Lin, 1990a, 199b; Lin and Farley, 1991) and begin the exploration of new directions in the modeling and simulation of qualitative economic dynamics.

In the next section, we define our scheme for modeling basic market mechanisms and demonstrate an approach to qualitative reasoning based upon these models. Our reasoning paradigm is a form of forward-directed, qualitative simulation, useful for generating predictions as to the ultimate effects of proposed parameter perturbations. We then discuss the characterization of complex economic systems based upon interactions among multiple markets and extend our single-market representation and simulation scheme to cover multiple-market models, as well.

We then consider certain issues of economic dynamics, i.e., cases where some markets appear not to adjust at all or adjust at differing relative rates. We conclude by contrasting our approach with those of Samuelson (Samuelson, 1947) and others (Lancaster, 1965; Ritschard, 1983), who have previously proposed the solution of qualitative linear systems as a basis for qualitative reasoning about economic systems.

II. Qualitative Modeling

Qualitative modeling has emerged as a powerful methodology for reasoning about the behavior of complex physical systems (Bobrow, 1985; Weld and deKleer, 1991). Several techniques for the qualitative representation and simulation of such systems have been defined and implemented as automated reasoning systems. Qualitative representation in these schemes is realized by applying abstraction operations to elements of a previously determined, quantitative model. Abstraction operations are applied both to the value domains of a model's parameters and variables and to the constraints among those elements that serve to characterize component and overall system behaviors. Reasoning can then proceed in terms of these reduced value sets and simplified relations.

The quantitative value domains of model variables are transformed into finite, ordered sets of landmark values (implicitly including possible positive and negative infinities) and the intervals that lie between them. For example, when representing the temperature of some physical substance, the following qualitative value domain could be used: {absolute-zero (solid) freezing-point (liquid) boiling-point (gas)}. The landmark value of absolute-zero represents the lowest possible value, while the landmarks of freezing-point and boiling-point represent temperatures at which a substance changes physical form and chemical reactive characteristics. The open intervals between landmark values (shown in parentheses) represent temperature ranges over which no relevant, significant behavioral changes occur. Thus, we have transformed the infinite-sized quantitative domain of real temperature values into a qualitative domain having only six symbolic values.

A particularly useful qualitative value domain is the {(-), 0, (+)} domain. It has proved valuable for characterizing direction of change within dynamic systems and for indicating variable's value relative to its normal levels during diagnostic reasoning (Farley, 1989). This domain employs the single landmark value of 0 (indicating no change or normal), with (-) and (+) values representing the infinite intervals below and above 0, respectively. Throughout the remainder of our discussion, we will drop the parentheses around the + and - interval values, understanding that they represent a value range rather than a single, landmark value.

We will extend the {-, 0, +} domain with the unknown value "?". This value represents the ambiguous result that may arise when combining qualitative values from this value domain by standard algebraic operations. For example, the result of adding a + value and a - value is ?. Later, we will use the ? to represent the ambiguity that arises when one way of computing a variable's value in a given model leads to the + value and another leads to the - value.

Relational constraints among qualitative variable values, which serve to characterize the behavior of a system, are represented as simplified abstractions of quantitative, algebraic constraints. One approach represents underlying differential equations in a simplified, difference equation form known as confluences (de Kleer and Brown, 1984). For example, the qualitative change in flow (∂Q) through a pipe is qualitatively proportional to the difference in the changes in pressures at the two ends of the pipe ($\partial P1$ and $\partial P2$), yielding the following confluence: $\partial Q = \partial P1 - \partial P2$.

The approach we adopt is similar to the monotonic increasing (M^+) or decreasing (M^+) functional relationships between pairs of variables used by Kuipers and others (Forbus, 1984; Kuipers, 1986). These monotonic relationships can be supplemented by indications of correspondence between assumption of landmark values. For example, the qualitative constraint $x = M_0+(y)$ indicates that x is a monotonic increasing function of y, with a correspondence of x and y values at the zero landmark. One advantage this representation has is that we can directly represent causal direction, seeing changes to the dependent variable (e.g., x) as caused by changes to the independent variable (e.g., y). To these two basic relationships, we can allow one variable to be equal to (defined to be) the derivative of another.

III. Qualitative Market Models

Economic theories are often stated in terms of causal relationships and equilibrium conditions among sets of relevant variables and parameters that describe basic market components of an economic system. Demand and supply, as inversely related functions of price, are two of the most fundamental concepts in analytic economics. They are used to represent causal ordering relationships among economic variables and to organize the relationships among these variables into manageable subsystems, called markets. Typically, qualitative demand and supply functions are written as follows:

$$D = f_d(\underset{-}{P}, \ldots) \quad \text{and} \quad S = f_s(\underset{+}{P}, \ldots). \qquad (1)$$

The variables D and S represent quantities of a certain commodity that are demanded and supplied, respectively, within a particular market. Variable P refers to the price level. A *variable* in a particular market represents a level that can change due to that market's activity. A *parameter* to a particular market represents a level determined outside the market but which can affect levels of a market's variables through the market's behavior.

The demand and supply functions normally are taken to represent a causal ordering among the variables (Simon and Iwasaki, 1985). The direction of the causal ordering is from the *independent* variables on the right-hand side of the equation to the *dependent* variable on the left-hand side. Changes in the values of the independent variables are seen to be the *causes* of change in the dependent variable's value. The sign, + or -, under a variable on the right-hand side of the equation indicates the positive or negative effect that an increase in the independent variable has on the dependent variable, and vice versa when the independent variable decreases. The sign indicates the slope at points along a graphical curve that could be drawn to represent the relationship between the pairs of variables In our notation, demand and supply functions of (1) can be represented as follows:

$$D = M^{-}(P) \quad \text{and} \quad S = M^{+}(P)\,. \qquad (2)$$

The equilibrium point of a market is identified as the intersection of its demand and supply curves, i.e., the price at which D = S. In economic theory, a *tatonnement adjustment* process (i.e., the law of demand and supply) (Debreu, 1959) is used to explain the stability of market equilibrium. Whenever there is pressure of excess demand (i.e., $D - S > 0$) in the market, the price level P will change in a positive direction to "clear" the market, returning it to equilibrium. As price level rises, demand becomes lower and is accompanied by increasing supply, according to the causal relations presented in (2). These changes, caused by the change in price level, serve to restore equilibrium in the market. Similarly, price level P falls whenever there is excess supply (i.e., $D - S < 0$) in the market, leading to increased demand and lowered supply.

Positively-sloped supply and negatively-sloped demand curves guarantee that excess demand (or supply) can be eventually eliminated by the adjustment of P. The new equilibrium will be maintained until another external disturbance of a market parameter occurs. The adjustment process discussed above can be expressed by the following qualitative equation:

$$\partial P = M_0 + (D-S). \qquad (3)$$

The total derivative ∂P, representing the qualitative derivative or direction of change in price level, has a positive monotonic relation to excess demand, with landmark correspondence at zero. The adjustment relation differs in form from the causal relations discussed above, in that here the derivative, not the level, of P is related to the level of X.

We illustrate our qualitative representation of a market in Figure 1, the basic supply-demand market with an income parameter included. Each qualitative market model is labeled by a **Name**, includes sets of **Market Parameters**, **Market Variables**, and **Causal Relations**, and is completed by several homeostatic control elements, namely an **Equilibrium Variable**, **Equilibrium Expression**, and **Adjustment Variable**.

[Figure 1]

Name is simply a symbol (i.e., string of characters) used to reference a particular market model. **Market Parameters** represent quantity levels that are determined outside a particular market and remain unaffected by behavior of the market. In our example, income Y and wages W are considered parameters. Since a market's parameters do not undergo continuous change from the perspective of a market, we associate with each parameter a single value representing the (cumulative) change in level due to external perturbation from

Name
Supply-Demand

Parameters
I : Income

Variables
D : Demand
S : Supply

Causal Relations
$D = M^{-}(P)$
$D = M^{+}(I)$
$S = M^{+}(P)$

Equilibrium Variable
X : Excess-Demand

Equilibrium Expression
{D} - {S}

Adjustment Variable
P : Price

Figure 1. Basic Supply-Demand Market Model

its initial level. Since a parameter can decrease or increase, remain unchanged, or become ambiguous over the course of reasoning, the change in level aspect of a parameter's value will be taken from the extended {-, 0, +} domain.

Market Variables represent elements that may change during market simulation due to internal influences of market activity, as represented by a market's **Causal Relations**. The value associated with each element of **Market Variables** is a pair, the first representing the variable's change in level due to perturbations up to that point and the second the direction of any ongoing change (Kuipers, 1986). Both aspects of a market variable's value take on values from the extended {-, 0, +} domain defined earlier.

Causal Relations are qualitative functional relationships, expressed in terms of the M^+ and M^- notation. These represent the traditional idea of causal-ordering relationships among a market's parameters and variables. An element of **Causal Relations** represents the effect that a change to a market parameter has upon a market variable or that a change to a market variable may have upon another market variable. The effect of a parameter perturbation will be to perturb the level of the dependent market variable, while a change to a market variable's level or direction of change will effect the corresponding aspect of the dependent market variable.

Now we come to those model elements which give a market its homeostatic property. The **Equilibrium Expression** is defined to be the qualitative difference between selected **Market Variables** and **Market Parameters**. We represent the **Equilibrium Expression** in the form: {Positive-Set}-{Negative-Set}. The *Positive-Set* consists of demand aspects of the market, those variables and parameters influencing the value of the **Equilibrium Expression** in a positive manner; the *Negative-Set* consists of supply elements that inversely influence that value. The **Equilibrium Variable** assumes the current value of the **Equilibrium Expression**, which is equal to excess demand in the market or the difference between positive and negative influences on a market's equilibrium.

The value of the **Equilibrium Variable** is of the same form as an element of **Market Variables.** This value is unambiguous only in those cases when all non-zero elements of the Positive-Set take on the same value (level or direction of change) and all non-zero elements of the Negative-Set have the same, but opposite, value. The **Adjustment Variable** is a special element of the market model. Its direction of change is directly related to the level of the **Equilibrium Variable**, with correspondence at the 0 landmark. For example, if X were the **Equilibrium Variable** and A the **Adjustment Variable**, then we would have the qualitative relation $\partial A = M_0^+(X)$ as an implicit element of the market's model. As such, when a market is perturbed from equilibrium, an adjustment is initiated and maintained until reestablishment of market equilibrium.

The concept of market as represented here is more than simply a set of equations, it constitutes the basic homeostatic element of economic theory. A market always adjusts the level of a particular variable in response to disequilibrium, upon which the levels of other variables of the market causally depend. In a stable model, these dependencies are such that any disturbance to the market, felt through perturbations of its parameters, is brought back to a new point of equilibrium. A market model will be *qualitatively stable* if the **Adjustment Variable** is only inversely related (M^-) to elements of the Positive-Set of the **Equilibrium Expression** and only positively related (M^+) to elements of the Negative-Set.

IV. Qualitative Economic Reasoning

Consideration of change in an economic system as it moves from one position of equilibrium to another is carried on according to a methodology known as *comparative statics* . "This method of comparative statics is but one special application of the more general practice of scientific deduction in which the behavior of a system (possible through time) is defined in terms of a given set of functional equations and initial conditions" (Samuelson, 1947, Chapter II, p.8). This methodology is a form of local perturbation analysis, a technique often applied in the analysis of physical and electrical systems, as well.

Given a market-based econimic model, the basic reasoning paradigm is to (1) perturb the initial system state by altering the value of one or more market parameters, (2) propagate the effects upon market variables according to a specialized form of qualitative simulation until a new equilibrium is reached, and then (3) compare the resultant state (i.e., values of market variables) with the initial state, determining differences between the values associated with market variables in the initial and final equilibrium states.

[Figure 2]

Figure 2 presents a graphical representation of our market simulation method, based upon our qualitative market models, which implements step (2) of this process. Figure 3 presents the sequence of states that would arise in updating the basic supply-demand market when income is perturbed upward. In figure 3, each parameter or variable is followed by its current value; only changes are shown at each state transition. In the initial qualitative state Ste, variables P, D, and S are undisturbed from equilibrium levels. When we perturb the market by increasing income Y, we create a new state St_p. According to the **Causal Relations**, this causes an increase in demand, as shown in state St_1. Upon an initial change to the level of any variable or parameter in the **Equilibrium Expression**, that expression is evaluated to determine the effect upon equilibrium. In our example, this results in a positive level being established for the **Equilibrium Variable** X, representing the newly created, excess demand.

[Figure 3]

State St_1 is not an equilibrium state; thus, the **Adjustment Variable** is affected. This results in a positive direction of change for P in state St_2; whenever excess demand exists, price will be increasing. Following initiation of the adjustment process, **Causal Relations** are

Market Update Process

given a parameter perturbation,
propagate its effects within a market

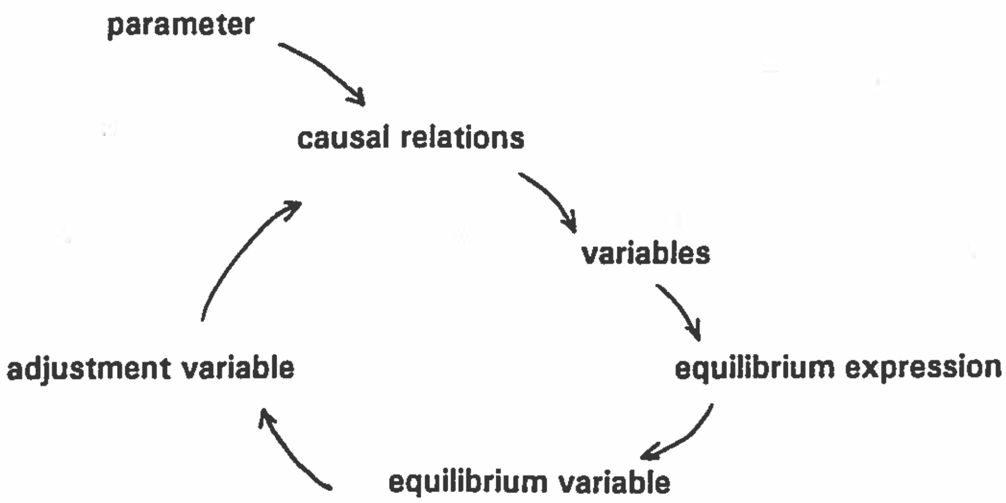

Figure 2. Market Simulation Process

St_e		initial equilibrium
	Y (0)	
	D (0, 0)	
	S (0, 0)	
	X (0, 0)	
	P (0, 0)	
St_p		after income perturbation
	Y (+)	
St_1		after propagating through causal relations
	D (+, 0)	
	X (+, 0)	
St_2		after impact on adjustment variable
	P (0, +)	
St_3		after second propagation through causal relations
	D (+, -)	
	S (0, +)	
	X (+, -)	
St_f		after application of comparative statics
	Y (+)	
	D (+, 0)	
	S (+, 0)	
	X (0, 0)	
	P (+, 0)	

Figure 3. Market Simulation with Comparative Statics

again applied; here, ∂D becomes negative and ∂S positive, as shown in St_3; these changes are in response to the positive direction of change in price P. The **Equilibrium Variable** is again affected, as ∂X becomes negative.

We see that the market model is qualitatively stable, in that the direction of change for X is toward equilibrium (i.e., opposite of the change in level due to market perturbation). State St_f represents the differences in level values for the perturbed parameter and market variables between initial and final equilibrium states, as determined by subsequent comparative statics analysis.

A comparison of final variable levels with original values, as determined by consideration of changes in variable levels and any active directions of change during the simulation in conjunction with possible consultation of the **Equilibrium Expression**, implements the comparative statics method and establishes state St_f. In our example, changes to P and S can be determined solely on the basis of their directions of change during simulation. However, demand D first increases in level due to the demand shock and then decreases due to subsequent price adjustment. The overall change in demand would be qualitatively ambiguous if it were not for the previously determined result that supply increases. By definition of equilibrium, demand equals supply at both equilibria. Thus, D_f is greater than D_e. Our system is able to infer changes in variables involved in the **Equilibrium Expression**, as long as only one variable of the condition is unknown and the evaluation of the expression is not otherwise ambiguous.

V. Multiple Market Models

Complex economic theories, such as those proposed in macroeconomics, can be constructed from multiple instances of simple market models. Applying *ceteris paribus* assumptions, economists reason about one market of the model at a time, propagating results between interrelated markets in a piecemeal fashion. Each market of the model has different variables and parameters representing demand and supply aspects from the domain of the market. The markets interact during qualitative simulation through the activation of connections. A *connection* is a variable from one market that is a parameter for another market of the model. When such a variable changes, in level or in direction of change, we say that the corresponding connection (or connections) involving that variable has become *active*. A sequential visitation of the markets, following active connections and performing individual market adjustments during the visits, yields overall model behavior.

We introduce a global **Economic System** notion, consisting of a **Name, Markets, System Parameters, System Variables,** and **Connections. Markets** is simply the set of names of the component markets that constitute the economic system model. Thus, an economist can create a set of market models from which a variety of economic system models can be built. **System Parameters** is defined to be those elements of **Market Parameters** from the component markets that are not included in the **Market Variables** of any another component market. **System Variables** is simply the union of **Market Variables** from the component markets.

The set of **Connections** represents the interactions between component markets. Each element of **Connections** is a triple, consisting of a source market name, a destination market name, and the element of **Market Variables** in the source market which is an element of **Market Parameters** in the affected market. If we draw a multiple-market **Economic System** model, elements of **Markets** constitute the nodes, while the **Connections** become directed arcs from source to affected market, labelled by the associated variable.

To illustrate this scheme, we consider a model of macroeconomics, in which income - investment - saving relationships in the product market interact with money demand - interest rate relationships in the money market. This demand-side, macroeconomic theory is often called the *IS-LM* model. Qualitative reasoning in terms of the *IS-LM* model has proved useful for explaining and predicting basic effects that government policies have upon the economy. Figure 4 presents qualitative models of the money and product markets. Figure 4 presents our representation of money and product markets; Figure 5 presents our representation of the *IS-LM* model as an **Economic System**. When creating this model, one only need indicate the names of markets to be included in the **Economic System**. Other aspects of the *IS-LM* model are directly computed through reference to the individual market models defined earlier.

[Figure 4]

[Figure 5]

Consideration of the effects that an increase in government spending has upon economic variables under the *IS-LM* model will illustrate the extensions to the single-market simulation method that are now necessary. Figure 6 presents an annotated trace of an extended simulation methodology for our *IS-LM* example involving increased government spending. Here we see that a perturbation in government spending creates disequilibrium in the product market, resulting in an adjustment of income. This adjustment of income subsequently affects the money market, where income is a parameter. We say the connection from the product market to the money market due to income has been *activated.* The money market is therefore perturbed from its equilibrium, resulting in an adjustment of interest rate. As a parameter of the product market, this interest rate adjustment returns by a newly activated connection to influence again the product market, causing a change in investment. As no new connections are activated by this market update, the propagation phase of the qualitative simulation is complete. As can be easily seen, each affected market, being qualitatively stable, is heading for a new equilibrium.

[Figure 6]

After the simulation is completed by processing all activated connections, comparisons can be made between final and original variable values to determine the overall effects of the initial

Name
Money Market

Variables
L : Money Demand
R : Interest Rate

Parameters
M : Money Supply
Y : Income
P : Price level

Causal Relationships
$L = M^+(Y)$
$L = M^-(R)$

Equilibrium Variable
X_m : eXcess demand for money

Equilibrium Expression
{L, P} - {M }

Adjustment Variable
R : Interest Rate

Name
Product Market

Variables
I : Investment
S : Savings
Y : Income

Parameters
T : Taxes
G : Goverment Spending
R : Interest rate

Causal Relationships
$S = M^+(Y)$
$I = M^-(R)$

Equilibrium Variable
X_p : eXcess demand for Products

Equilibrium Expression
{I, G} - {S, T}

Adjustment Variable
Y : Income

Figure 4. Money Market and Product Market Models

Name
IS-LM Model

Markets
Product Market
Money Market

Parameters
T : Taxes
G : Goverment Spending
M : Money Supply
P : Price level

Variables
Y : Income
I : Investment
S : Savings
L : Money Demand
R : Interest Rate

Connections
(Money Market, Product Market, R)
(Product Market, Money Market, Y)
(External, Product Market, G)
(External, Product Market, T)
(External, Money Market, M)
(External, Money Market, P)

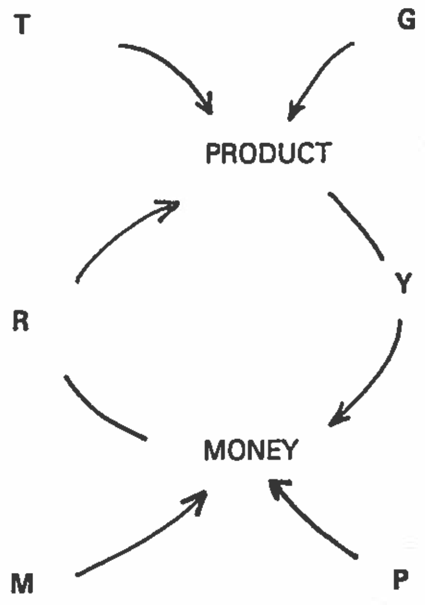

Figure 5. IS-LM Model of Macroeconomics

perturbation, completing the method of comparative statics. In our example, most variables have changed in one direction only. An increase in government spending is shown to result in higher income, savings, and interest rates, while business investment is seen to decline. The demand for money, which increases at first in response to the rise in income returns unchanged to its initial level, to be at equilibrium with the unchanged money supply.

Our example with the *IS-LM* model illustrates one assumption of our methodology. During the qualitative simulation of each market, we assume that application of tatonnement adjustment is sufficient to resolve any ambiguities raised by subsequent, conflicting changes in market parameters. According to this market-clearing view of market adjustment, the first perturbation to reach a market determines the direction of adjustment for the simulation. In our example, the increase in interest rate in the money market (initiated as market adjustment at state St_5) will be sufficient to overcome the continued upward pressure on money demand caused by increasing income generated by ongoing adjustment in the product market. By this assumption, it is not necessary to "go around the cycle" again and again, noting a new upward perturbation of Y in the money market that is followed by yet another increase in interest rate, etc.

Another source of ambiguity in our qualitative simulation paradigm can be illustrated with the addition of the two supply-side markets, labor and output, to the *IS-LM* model, creating our representation of the *General Macroeconomic* model (Keynes, 1935). Figure 7 presents new market definitions, while Figure 8 specifies the *General Macroeconomic* nodel. Notice now that a variable in one market may be a parameter in more than one other market. Thus, more than one connection may become active after completing an update of a particular market. We can characterize different simulation strategies based upon how we select among currently active connections. If we select connections in the order of activation, we produce a *breadth-first* style of simulation. If we always select the most recently active connection,

State	G	I	S	Y	L	R
St_e	(0)	(0,0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)

the initial equilibrium;

product market

	G	I	S	Y	X_p	R
St_p	(+)	(0,0)	(0,0)	(0,0)	(+,0)	(0)
	G is perturbed upward ($G_i > G_e$), creating excess demand in the product market;					
St_1	(+)	(0,0)	(0,0)	(0, +)	(+, 0)	(0)
	the excess demand sets off an upward adjustment of income;					
St_2	(+)	(0,0)	(0,+)	(0, +)	(+, -)	(0)
	this causes an increase in savings and a decrease in excess demand					

money market

	Y	L	R	X_m
St_3	(+)	(0 ,0)	(0, 0)	(0 , 0)
	the increase in income ($Y_i > Y_e$) perturbs the money market,			
St_4	(+)	(+,0)	(0, 0)	(+, 0)
	causing excess demand for money ($L_i > L_e$);			
St_5	(+)	(+,-)	(0, 0)	(+, 0)
	this sets off an upward adjustment of interest rates;			

product market

	G	I	S	Y	X_p	R
St_6	(+)	(0 ,0)	(0, +)	(0, +)	(+, -)	(+)
	The increase in interest rates ($R_i > R_e$) comes back to influence the product market,					
St_7	(+)	(- ,0)	(0, +)	(0, +)	(+, -)	(+)
	causing a decrease in investment ($I_i < I_e$);					

comparative statics

	G	I	S	Y	L	R
St_f	(f, +)	(f, -)	(f, +)	(f, +)	(f, 0)	(f, +)

a new equilibrium is then established for the overall economic system, with comparative static changes indicated.

Figure 6. Qualitative Simulation of IS-LM Model

we get a *depth-first* style of simulation.

[Figure 7] [Figure 8]

Can the order in which we traverse active connections affect the outcome of a qualitative simulation? Unfortunately, the answer is yes, due in part to the simplifying assumption we have made above regarding the market-clearing power of tatonnement adjustment. Our market-clearing assumption means that the first parameter perturbation to reach a market and disturb its equilibrium will determine the direction of that market's adjustment process for the rest of that simulation. Subsequent parameter perturbations, and their effects upon market variables or the equilibrium condition, are dominated by the previously established adjustment process. Thus, if there are two different paths of connections that lead to a particular market from a given perturbation of model parameters and those different paths result in a different direction of change in the **Equilibrium Variable** and therefore in the **Adjustment Variable** of that market, then the traversal order of active connections will make a difference in the outcome.

This situation does occur when reasoning in terms of our *General Macroeconomic* model. If we again perturb government spending upward, there are two possible effects upon wages, corresponding to two different walks over connections between markets. When government spending rises, the product market is thrown out of equilibrium, resulting in an upward adjustment of income. In one simulation ordering, that adjustment perturbs the money market, resulting in a rise in interest rates. This in turn perturbs the labor market, resulting in higher supply of labor and a downward adjustment in wages. In another simulation ordering, the increase in income perturbs the output market, resulting in an upward adjustment of prices. The price increase causes an increase in demand for labor in the labor market, resulting in an adjustment in wages upward. To resolve this ambiguity, we would need to know the relative magnitudes of effects along these two reasoning paths.

We could deal with the issue of multiple outcomes in our system by simulating all possible

Name
Labor Market

Variables
N : Demand for Labor
H : Labor Supply
W : Wage rate

Parameters
P : Price level
R : interest Rate

Causal Relationships
$N = M^{+}(P)$
$N = M^{-}(W)$
$H = M^{+}(W)$
$H = M^{+}(R)$

Equilibrium Variable
X_n : eXcess demand for Labor

Equilibrium Expression
{N} - {H}

Adjustment Variable
W : Wage rate

Labor Market Model

Name
Output Market

Variables
Q : Output
P : Price Level

Parameters
Y : Income
K : Capital Stock
W : Wages

Causal Relationships
$Q = M^{+}(W)$
$Q = M^{+}(P)$
$Q = M^{+}(K)$

Equilibrium Variable
X : eXcess demand for Output

Equilibrium Expression
{Y} - {Q}

Adjustment Variable
P : Price level

Output Market Model

Figure 7. Labor and Output Market Models

Name
General Macroeconomic Model

Markets
Product Market
Money Market
Labor Market
Output Market

Parameters
T : Taxes
G : Goverment Spending
M : Money Supply
K : Capital Stock

Variables
Q : Output
P : Price Level
Y : Income
I : Investment
S : Savings
L : Money Demand
R : Interest rate
N : Labor Demand
H : Labor Supply
W : Wage Rate

Connections
(Money Market, Product Market, R)
(Money Market, Labor Market, R)
(Product Market, Money Market, Y)
(Output Market, Money Market, P)
(Output Market, Labor Market, P)
(Product Market, Output Market, Y)
(Labor Market, Output Market, W)
(External, Output Market, K)
(External, Money Market, M)
(External, Product Market, T)
(External, Product Market, G)

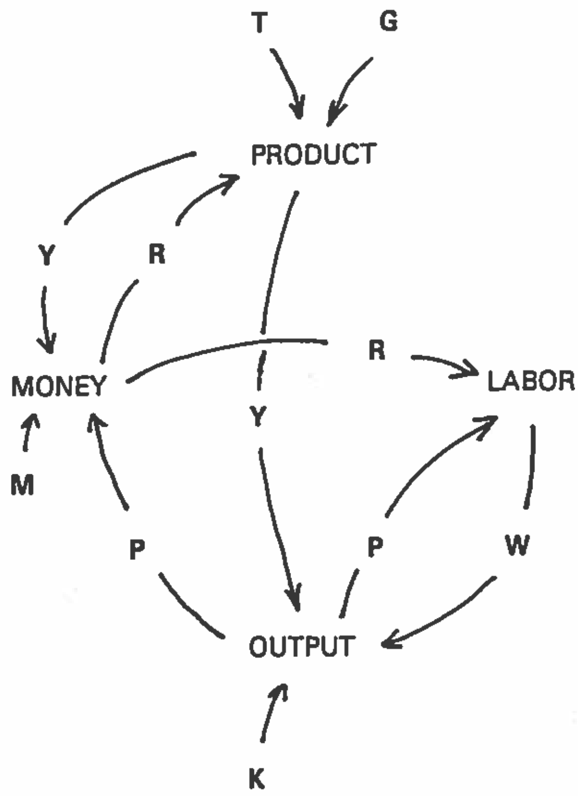

Figure 8. General Macroeconomic Model

propagation orderings of perturbations over active connections and then comparing results from the corresponding comparative statics analyses. If a model variable has the same comparative statics outcome for all simulation orderings, then it is assigned that value. If the comparative statics value produced for a given variable differs with simulation ordering, it is assigned the value "?", indicating that the effect upon that variable for the given perturbation(s) can not be determined (or is ambiguous) with respect to the model.

There is one problem with this approach, i.e., the computational complexity of pursuing all possible orderings can become extreme. In fact, there are 388 different reasoning sequences for the propagation of change (i.e., orderings for the selection of active connections) in our example above, given the change in government spending. Results of a simulation following all paths indicate that output, interest rates, savings, and income all would rise, while investments would decrease, in all possible orderings. The effects of increased government spending on prices, wages, labor supply, labor demand, and demand for money were all ambiguous, increasing for some simulation orderings and decreasing for others. While correctly capturing the propagational ambiguity inherent in our *General Macroeconomic* model, the simulation process proved to be costly even with this relatively simple model.

To overcome computational complexity, we modify our approach to multiple-market simulation in the following manner. Each active connection is marked by a list of previously visited markets leading to its activation. When an active connection is selected for processing, we check to see whether the destination market is already on the list of previously visited markets. If so, we only propagate causal relations of the destination market and do not revisit the equilibrium and adjustment aspects of the market update procedure. This is considered the end of one *partial simulation*. We continue this process until all activated connections have been processed, as in our original method. We now can determine values for variables by computing comparative statics first for each partial simulation and then for the union of their results, comparing partial results to check for consistency or ambiguity and thus determine each variable's overall outcome.

[Figure 9]

Figure 9 presents a textual specification of the simulation method as described above. Figure 10 presents a trace for our example of increased government spending in the *General Macroeconomic* model. As shown, our method requires processing only 14 active connections, while producing the same results as our previous method which processed nearly 2000 connections in following the 388 different, complete simulation sequences.

[Figure 10]

The essential feature of this simulation method is the use of partial simulations, each ending when a market is revisited along a current chain of reasoning or no active connections remain. In our example, the connection chain 1-2-3 forms one partial simulation, where government spending influences income in the product market; this disturbs the money market, increasing money demand and leading to a rising interest rate that finally returns to affect investments negatively in the product market. Another is formed by the chain 1-8-11-14, where government spending influences income in the product market; this results in rising prices in the output market, which in turn lowers the supply of real money in the money market; this produces an upward adjustment of interest rates (this time with an overall decrease in demand for money), resulting in lower investments in the product market. The results of these partial simulations are then combined to produce the ultimate, comparative statics results.

How do we justify our simulation procedure, which does not reconsider adjustment when effects return to a market? We assume the overall model is qualitatively stable, i.e., all markets can reestablish equilibrium through adjustment. If an effect reinforcing the direction

```
Given Economic System E and Perturbation P = (p, d),
                              with p a parameter and d a direction of change.

      Activate the connection for P
      Until [no active connections]
            select active connection C,
                  with destination market M, previous markets PM,
                        and perturbation P;
            perform-market-update(M, PM, P);
      end.


where,

   perform-market-update(M, PM, P)
      If [market M is not an element of PM]
        then
            Update level of parameter p according to d;
            Perform any updates by Causal Relations;
            Determine new value of Equilibrium Variable;
            If market M no longer in equilibrium
               then
                  Update Adjustment Variable;
                  Perform any updates by Causal Relations;
                  Determine direction of change of Equilibrium Variable;
            Activate connections impacted by changes in market;
        else
            Update level of parameter p according to d;
            Perform any updates by Causal Relations;
            Mark state as end of this partial simulation.
```

Figure 9. Partial Simulation Approach to Multi-Market Reasoning

C	Δ	Previous Markets (From)	Destination	Market Update
1	(G, +)	External	Product	Y: (i, 0, +); S: (i, 0, +)
2	(Y, +)	Product (1)	Money	L: (i, +, -); R: (i, 0, +)
3	(R, +)	Product, Money (2)	Product	I: (i,-,0)
4	(R, +)	Product, Money (2)	Labor	H: (i, +, -);W: (i, 0, -); N: (i, 0, +)
5	(W, -)	Product, Money, Labor (4)	Output	Q: (i, +, -); P: (i, 0, -)
6	(P, -)	Product, Money, Labor, Output(5)	Money	
7	(P, -)	Product, Money, Labor, Output (5)	Labor	N: (i, -, +)
8	(Y, +)	Product (1)	Output	P: (i, 0, +); Q: (i, 0, +)
9	(P, +)	Product, Output (8)	Labor	N: (i, +, -); W: (i, 0, +); H: (i, 0, +)
10	(W, +)	Product, Output, Labor (9)	Output	Q: (i, -, +)
11	(P, +)	Product, Output (8)	Money	R: (i, 0, +); L: (i, 0, -)
12	(R, +)	Product, Output, Money (11)	Labor	H: (i, +, -); W: (i, 0, -); N: (i, 0, +)
13	(N, +)	Product, Output, Money, Labor (12)	Output	Q: (i, +, +)
14	(R, +)	Product, Output, Money (11)	Product	I: (i, -, 0)

Comparative Statics: G: (+); I: (-, 0); Y: (+, 0); S: (+, 0); R: (+, 0);
L: (?, 0); N: (?, 0); W: (?, 0); Q: (?, 0); P: (?, 0)

Figure 10. Trace of Partial Simulation Method

of disequilibrium returns to a market, that only would result in greater adjustment in the current direction, which adjustment we assume is sufficient to overcome the existing and additional excess or shortage and need not be revisited. If, on the other hand, an effect that returns to a market were to lessen the disequilibrium, we assume this effect is not powerful enough to switch the sign of the disequilibrium and cause a change in the direction of adjustment. If it were that powerful, then the feedback from this secondary adjustment would only return to the market again, causing another reversal and so on, setting up an infinite, unstable oscillation in qualitatively terms. Thus, it is reasonable to conclude that if a model is qualitatively stable, the initial adjustment direction is dominan when considering overall effects of feedback.

VI. Disequilibrium and Dynamics

In this section, we consider situations of economic system disequilibrium and begin to investigate issues of economic dynamics. Disequilibrium is an important factor in any analysis of economic systems. First, the initial state of a market model may not be in equilibrium. Secondly, the existence of disequilibrium may persist for a significant period of time following market perturbation. The central feature of a disequilibrium model is its rigidity in its price elements, due to lack of full or complete adjustment processes in markets of the model. Indeed, a freely adjusting price mechanism is not a realistic economic consideration for most common economic systems.

If we have markets that refuse to adjust in the near term, how can effects of parameter changes be propagated? Instead of assuming complete price adjustment as basis for determining change, each market can also be affected by quantity "spillover" in closely related markets that remain in disequilibrium. Spillover is persistent excess demand or supply that can itself change in level or direction and thereby affect connected markets. Studies of disequilibrium economics, conducted by Barro and Grossman [1976], Portes [1977], Ito

[1980], Benassy [1982], and Quandt [1988], have developed this notion and are important extensions of Keynes' *General Theory* [1935].

Although the cornerstone of our qualitative reasoning scheme will remain the market-based update approach, now the adjustment process in a market may be incomplete or "retarded" within the time period of interest. Such a fixed-price or quantity-constrained disequilibrium seems to be a more realistic formulation for multiple-market economic simulation. In the following discussion, we extend our method of qualitative simulation to allow reasoning about multiple-market systems from a disequilibrium perspective.

We will refer to markets for which the adjustment process is blocked or retarded for some unspecified reason as *sticky markets*. In market simulation of a sticky market, parameter perturbations are propagated through the **Causal Relations** of the market as before; however, no adjustment process is applied, which process normally would affect the value of the **Adjustment Variable** and so return the market to equilibrium. Other than this change, the simulation method is as before, following active connections until none remain or a loop is detected. A sticky market acts as if it makes no adjustment at all when considered in qualitative terms. Such exaggeration is a standard tool of qualitative analysis (Weld, 1989).

The question we want to ask is what effect does the assumption that a particular market is sticky have upon the behavior of an economic system, given a certain parameter perturbation? Our methodology for answering this question is as follows: Given an indication of a sticky market and a perturbation of a system parameter, we simulate the economic system twice, once with the market behaving normally and once with it acting as a sticky market, changing the market simulation process as described above. We then compare the final states produced by the two simulations and attribute any differences between them as being due to effects of the market being considered to be sticky. This reasoning scheme represents an adaptation of comparative analysis, a methodology discussed by Weld (1988).

To represent disequilibrium models that include spillover effects, we need further modeling elements. To introduce our new elements, we consider our earlier specification of the traditional Keynesian *General Macroeconomic* model. In its initial formulation, we assumed all markets were "competitive", or frictionless. Now, we will assume that the money market is frictionless, as many disequilibrium theorists suggest is the case when restrictive government policies are not in place; this means interest rates can be adjusted freely to clear the money market after perturbation. Any possibility of disequilibrium in the *General Macroeconomic* model is usually seen to arise in the output and labor markets. Experience has shown that it is often the case that adjustments of prices P and wages W are insufficient to clear the output and labor markets, respectively. Recall that in the output market, the equilibrium condition is that income Y (demand) equals output Q (supply); in the labor market, the condition is that labor demand N equals labor supply H.

Traditional analysis of the model suggests there are significant spillover effects between these two supply-side markets when they are sticky. We must be able to represent the notion of spillover in demand or supply in our market models. As such, we introduce the notion of an *excess function* E, such that $E(v_1, v_2) = max(v_1, v_2, 0)$. In accordance with our discussion above, we define the spillover of excess labor demand to be the variable EN = E(N, H); the spillover of excess labor supply is represented as the new market variable EH = E(H, N). Note that, by definition, these two qualitative variables have their value ranges restricted to {0, +} and that at most one of them can have the value + at any given time. We define analogous variables, EY and EQ, to represent uncleared excess in income and output in the output market, respectively.

Now we can use these spillover variables, defined in terms of the excess function, to represent effects of disequilibrium between markets of our *General Macroeconomic* model. For example, the spillover-based interaction between the output and labor markets as discussed by economists can be captured by two causal relations, $H = M^{\cdot}(EY)$ and $N = M^{\cdot}(EQ)$, in the labor market. Likewise, the causal relation $Q = M^{\cdot}(EN)$ is now included in the product market, while savings S and taxes T, both demand components of Y, are causally affected as $M^{\cdot}(EH)$. Now, when a market is perturbed during simulation, but does not adjust, it still may disturb other markets through the impact of its spillover variable that serves as parameter to other markets.

It is clear that persistent disequilibrium may prevail in models with sticky markets. In such situations, equilibrium becomes more an accidental phenomenon than the norm. In discussions of disequilibrium for the *General Macroeconomic* model, where labor and output markets are considered sticky and interrelated through spillover effects as defined above, five macroeconomic states of interest have been identified: E (equilibrium), where $Q = Y$ and $N = H$; C (classical unemployment), where $Q < Y$ and $N < H$; K (Keynesian unemployment), where $Y < Q$ and $N < H$; R (repressed inflation), where $Q < Y$ and $H < N$; and U (underconsumption), where $Y < Q$ and $H < N$. To reason about these disequilibrium regimes, we must be able to assume an initial state other than equilibrium. If we then consider the effects of an increase in government spending from these various initial states, our qualitative reasoning technique generates several results consistent with accepted macroeconomic theory, including the following among E, R, and K:

(1) Given initial state E, an increase in government spending leads to state R.

(2) Given an intial state R, an increase in government spending remains in state R.

(3) Given an intial state E, a decrease in government spending leads to state K.

(4) Given initial state K, an increase in government spending can lead to state E.

When we consider the effect of an increase or decrease in government spending policy, we find that, depending on the initial state, different outcomes are predicted. For Keynesian unemployment, an expansionary policy of increased government spending can be effective in restoring a full employment equilibrium. However, an expansionary policy alone can not change the state of repressed inflation, which remains mired in R. If the model is perturbed from an initial equilibrium by an expansionary demand shift, the model becomes "stuck" in a regime of repressed inflation, demonstrating the ineffectiveness of expansionary policy in such an economic system. In general, our qualitative simulation scheme produces not only an acceptable interpretation of outcomes from disequilibrium theory, but also provides information about regime change due to policy shifts expressed as parameter perturbations (See also Lin and Farley [1991]).

Consideration of disequilibrium situations moves us closer to an examination of more general issues in qualitative economic dynamics. Our recent research efforts have focused on developing a simulation process to capture elements of market interaction dynamics. In our above approach to disequilibrium reasoning, we essentially assumed that sticky markets never adjust, which, when discussing short-term perturbations, seems reasonable. Such an analysis technique, where an effect is pushed to the extreme, has been called "exaggeration" in the qualitative physics literature [Weld, 1990].

One way we can investigate issues of economic dynamics is to soften this exaggerated distinction and allow the modeler to specify the relative rates at which markets of a model are assumed to adjust. When modeling a particular market, one indicates how many "time steps" it takes for the market to return to equilibrium following its displacement from equilibrium; this can be either 0 (i.e., in the same step as it is perturbed) or a positive integer. The simulation process proceeds as in the case without sticky markets, propagating effects throughout the model. However, this is now considered only to be a single "time step". Depending on the rate at which a market adjusts and how many time steps have elapsed since the market began its current adjustment process, the market may or may not return to equilibrium during the current time step. If all markets return to equilibrium at the end of a given time step, the simulation is complete. Otherwise, a new time step is initiated, with continued adjustments in the "active markets", i.e., those not yet at equilibrium, triggering off changes in others as their adjustments are propagated as perturbations to related markets along appropriate connections.

To illustrate the type of trace of dynamic behavior this method can produce, let's again consider the *IS-LM* model. For this example, we assume that the product market takes 2 time units to adjust and the money market 1 time unit to adjust, beyond the time step in which each market is perturbed. Figure 11 presents a trace of market variable values after a complete propagation of effects during each time step. Of key importance is the result that a faster market will establish a new equilibrium only to be perturbed from that equilibrium in the next time step due to the continuing adjustment of a slower market.

[Figure 11]

The results indicated in state St_f indicate ambiguity as to cumulative results for the market variables due to the cyclic, oscillatory behaviors observed. However, if we assume a damping in the level of intermarket perturbations as time proceeds, which is consistent with overall model stability assumptions, comparative static comparisons between initial and final levels for the market variables become clear, as shown in state $St_{f'}$. The damping assumption guarantees that comparative statics results between initial and final states determined by our dynamic simulation approach are consistent with those produced by our earlier methodology based on perturbation analysis.

Figure 12 presents the qualitative curves followed by the two market equilibrium and adjustment variables of the *IS-LM* model under the above assumptions as to market latencies and damping of successive market impacts. As with more traditional, dynamical systems analyses, we can see that delays in market adjustment processes create oscillatory behaviors even when considered at this qualitative level of representation.

The system, initially in equilibrium, is perturbed by an increase in government spending.

	I	S	Y	X_p	L	R	X_m
St_1	(0,-)	(0,+)	(0,+)	(+,-)	(+,-)	(0,+)	(+,-)

The markets respond to the perturbation, beginning adjustment processes;

	I	S	Y	X_p	L	R	X_m
St_2	(-,0)	(+,+)	(+,+)	(+,-)	(0,0)	(+,0)	(0,0)

the money market completes its adjustment, but the product market lags behind;

	I	S	Y	X_p	L	R	X_m
St_3	(-,-)	(+,0)	(+,0)	(0,0)	(+,-)	(+,+)	(+,-)

the product market completes its adjustment, but in the process perturbs the money market;

	I	S	Y	X_p	L	R	X_m
St_4	(-,0)	(+,-)	(+,-)	(-,+)	(0,0)	(+,0)	(0,0)

the money market completes its adjustment, but perturbs the product market;

	I	S	Y	X_p	L	R	X_m
St_5	(-,+)	(?,-)	(?,-)	(-,+)	(0,-)	(+,-)	(-,+)

the product market continues its adjustment, perturbing the money market;

	I	S	Y	X_p	L	R	X_m
St_f	(?,0)	(?,0)	(?,0)	(0,0)	(0,0)	(?,0)	(0,0)

finally, both markets complete their adjustment processes and the simulation is complete.

	I	S	Y	X_p	L	R	X_m
$St_{f'}$	(-,0)	(+,0)	(+,0)	(0,0)	(0,0)	(+,0)	(0,0)

By assuming a damping in perturbations, we are able to disambiguate other wise undertain results.

Figure 11. Trace of Economic System Dynamics

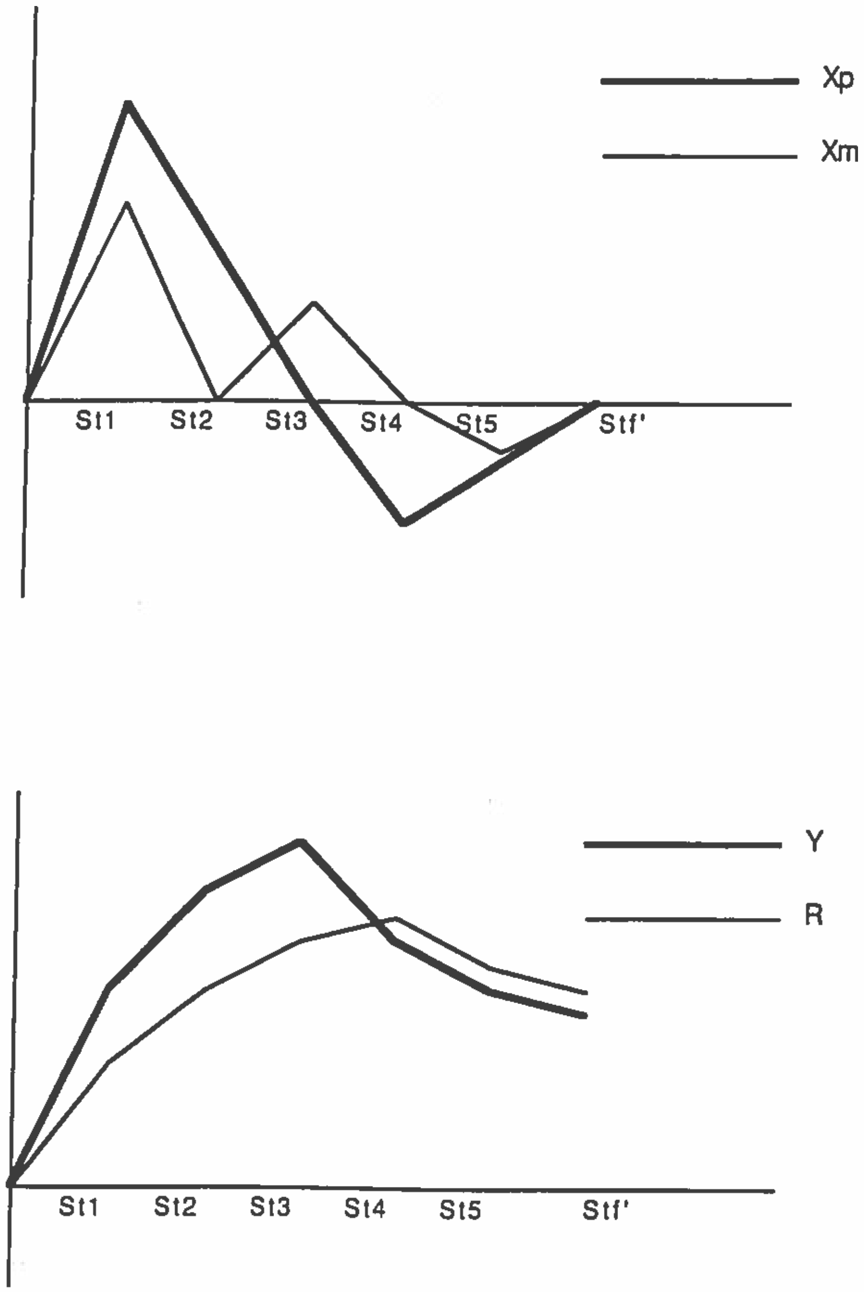

Figure 12. Graphs of Equilibrium and Adjustment Variables

[Figure 12]

VII. Conclusion

We have demonstrated how representation and simulation techniques originally discussed in the context of qualitative physics can be adapted and extended to realize methods of economic reasoning and comparative statics (Iwasaki and Simon ,1986a). The work reported here makes the transition from a totally equation-based approach, as initially discussed by Samuelson (Samuelson, 1947) and developed by others (Lancaster, 1965; Ritschard, 1983), to one reflecting the teleological notions of tatonnement adjustment and causal relations.

Under traditional qualitative approaches, a model among a set of economic variables and parameters was represented as a two-dimensional matrix (or array) A of signs: - , 0, +. The value A(i,j) represented the sign of the direction of influence that a change in variable or parameter j would have on the value of variable i. A value of 0 represented independence of the two variables. Techniques for manipulating qualitative or signed matrices were developed by economic theorists to deal with issues of solvability and stability of economic systems. Matrix decompositions and directed graphs are typically implemented as computerized algorithms to perform analyses of qualitative matrix models (Allingham and Morishima, 1973; Quirk, 1981).

Given Samuelson's definition of qualitative calculus for economic analysis, qualitative solvability and stability became powerful theoretical properties. However, conditions for these properties to hold are very strict and unlikely to be fulfilled, except for some very small systems. Meaningful applications of such qualitatively well-determined systems are almost never to be found in the real world. Not until recently did anyone investigate the computability issue for solving a large and complex qualitative system (Ritschard, 1983). Using weaker properties than stability and solvability, qualitative models can be used to provide partial answers when searching for unambiguous predictions of a large model.

Our market-based approach more directly captures classical economic reasoning, generating traces of market activities which yield natural explanations of predicted effects. Our definition of qualitative stability and assumptions regarding the course of qualitative simulation have been aimed at attempting to make manageable a complex reasoning problem. The topic of representing and simulating important aspects of economic dynamics within a qualitative framework remains an area that is rich for future research efforts.

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