Minimum-Time Multidrop Broadcast

**Arthur Farley, Andrzej Pelc,
and Andrzej Proskurowski**

**CIS-TR-95-05
February 1995**

Department of Computer and Information Science
University of Oregon

Minimum-Time Multidrop Broadcast

Arthur M. Farley * Andrzej Pelc †
Andrzej Proskurowski *‡

Abstract

The multidrop communication model assumes that a message originated by a sender is sent along a path in a network and is communicated to each site along that path. In the presence of several concurrent senders, we require that the transmission paths be vertex-disjoint. The time analysis of such communication includes both start-up time and drop-off time factors. We determine the minimum time required to broadcast a message under this communication model in several classes of graphs.

*Computer Science Department, University of Oregon, Eugene, OR 97403, USA.
†Département d'Informatique, Université du Québec à Hull, Hull, Québec J8X 3X7, Canada. Research supported in part by NSERC grant OGP 0008136.
‡Research supported in part by NSF grant NSF-CCR-9213431

1 Introduction

One of the basic information dissemination tasks in a communication network is that of broadcasting: one site originates a message to be sent to all other sites in the network. Depending on the technology of the network, different models of information dissemination are used. The *telephone* model assumes that only two sites connected by a direct link can communicate in a time unit, precluding any other communication involving the two sites (although other sites may communicate in the same manner concurrently). The *line* communication model assumes a similar exclusive pair-wise communication where communicating sites can be connected by a succession of adjacent links, and no link is used in two simultaneous transmissions. In this paper, we consider a multidrop version of this line communication model, wherein all sites along the path followed by a message become informed of the message.

We model the topological aspects of a communication network by a simple, undirected graph, $G = (V, E)$, in which vertices V correspond to network sites and edges E correspond to communication links between them. We consider a *multidrop* communication model. Communication takes place in rounds, where a round corresponds to a collection of vertex-disjoint paths in the graph. In this model, a site initiates a call that communicates a message to all sites on a path starting with the calling site. Several such calls can occur in the same round if they do not interfere by using the same vertex (or edge).

The time required by a multidrop call from one vertex to x other vertices is $s+dx$, where s is the transmission start-up time and d is the message drop-off rate. We simplify this expression to the form $1 + cx$, where c represents the relative message drop-off rate, i.e, d/s. The time to complete a round is the maximum time over all calls in that round. The time of a multidrop broadcast scheme is the sum of durations of all rounds in the scheme. Thus, it is equal to $r + ct$, where r is the number of rounds (called the *start-up term*) and t is the sum of maximum number of drop-offs in every round (ct is called the *drop-off term*).

For a given network topology and broadcast originator, what is the minimum possible time for a multidrop broadcast? One limiting factor is the eccentricity of the originating vertex in the corresponding graph. The *eccentricity* of a vertex v in a graph G is the maximum distance from v to any other vertex of G.

Lemma 1: In any graph G, the drop-off term from a vertex v with eccentricity e is at least ce, under any multidrop broadcast scheme.

Proof: Vertices along at least one path of length e must be informed sequentially, possibly over several rounds. ■

Lemma 2: In any graph G, the drop-off term from a vertex v with eccentricity e, for which there are at least two vertices at distance e from v, is at least $c(e+1)$, under any multidrop broadcast scheme.

Proof: Vertices along at most one path of length e can be informed sequentially. ■

Lemma 3: If there exists a multidrop broadcast from a vertex v of eccentricity e in time $r + ce$, then no minimum-time multidrop broadcast from this vertex can involve more than r rounds.

Proof: It follows from Lemma 1 that any broadcast from v with $r' > r$ rounds would require at least $r' + ce > r + ce$ time. ■

Hromkovič *et al.* [1, 2] initiated the formal study of multidrop broadcasting. They adopt a simpler timing model whereby each call requires one unit of time, regardless of the number of vertices along the path of a call. This measure simply counts the number of rounds to complete a multidrop broadcast. With this model of time cost, the existence of a Hamiltonian path from a given source assures a broadcast requiring one time unit.

Our model of time more closely approximates characteristics of the MasPar *xnetc* communication process and wormhole routing in more general networks. The relative drop-off rate c is quite small on the MasPar processor, i.e., approximately .005 for 8-bit data transfers (cf. [3]). In wormhole routing the relative drop-off rate varies with implementation and technology, see Ni and McKinley [4].

For different values of the relative drop-off rate c (as determined by network technology), schemes with differing numbers of rounds may yield the overall minimum time for the same network topology. Thus, we will have to explore optimum r-round broadcast (minimum time over all r-round broadcasts) for several values of r. By Lemma 3, we need not consider schemes with number of rounds greater than the minimum number of rounds necessary to realize eccentricity in the drop-off term.

The goal of this paper is to extend research on the multidrop model by using our more realistic timing model and exploring schemes leading to minimum-time broadcast in several classes of graphs. Specifically, we will study multidrop broadcasting in complete graphs, paths, cycles, trees, and grids.

2 Complete graph K_n

In the case of the complete graph, the task of covering it by sets of disjoint paths becomes an exercise in number theory when paths of different length are considered. Therefore, for this topology only, we will assume that the broadcasting process taking place in a complete graph with n vertices is achieved by covering the graph with sets of paths of *uniform* length x. Since the technology of a network often determines the parameter c, our initial goal is to find values of x allowing most efficient broadcasting for a specific value of c. To do this, we determine ranges of values of c for which a given value of x is optimal.

We will assume that the number of vertices n is large enough to be approximated by x^r, where $r \approx \log_x n$ is the number of rounds in a broadcast. The time of an r-round broadcast is thus $r(1+c(x-1)) = (\log_x n)(1+c(x-1))$. This time is to be compared with the time when x vertices are called in one round, $r(1+cx) = (\log_{x+1} n)(1+cx)$. It is the intersection of these two functions of x that determines the optimum uniform length of a calling path for a given value of network parameter c. This can be represented as the range of values of c for which the given calling path length is time-optimal. In other words, given c, we want to find a value of $x > 1$ for which $c_x < c \leq c_{x-1}$. The lower limit of this interval, c_x, satisfies the equation

$$(\log_x n)(1 + c_x(x-1)) = (\log_{x+1} n)(1 + c_x x)$$

and thus

$$c_x = \frac{1 - \log_{x+1} x}{1 - x + x \log_{x+1} x}.$$

In Table 1 we show some representative values of c_x.

x	2	3	4	6	8	12	22	38	63	100
c_x	1.41	0.55	0.31	0.15	0.01	0.05	0.02	0.01	0.005	0.00275

Table 1: Length of the call paths and the corresponding optimal value of the relative drop-off rate.

In the next three sections we will consider different subclasses of trees: paths, binomial trees and complete binary trees. We will exploit the constraint on a multidrop broadcast imposed by the degree of the originator and by the maximum degree in the tree.

Lemma 4: The minimum number of rounds required to complete a multidrop broadcast in a tree is not less than the greater of:

1. the degree of the originating vertex;
2. one less than the maximum vertex degree in the tree.

Proof:

1. The vertex originating the broadcast constitutes a separator (or is a leaf) of the tree; thus, it has to send messages into its subtrees in different rounds.
2. Let v be a vertex of maximum degree Δ. If v is the originator, we are done by part 1. Otherwise, in one round at most two neighbors of v can be informed and informing the remaining neighbors requires at least $\Delta - 2$ additional rounds. ∎

3 Path P_n

We now consider multidrop broadcasting in paths of n vertices, P_n. There are two cases to consider, one when the originator is an end-vertex and the other when the originator is an interior vertex of the path.

If the originator is an end-vertex of the path, then one round of duration $1 + c(n-1) = 1 + ce$ suffices. By Lemma 1, it is also the minimum time broadcast.

When the originator is an interior vertex of the path, any broadcast requires at least two rounds, by Lemma 4. The "obvious" broadcast involving two calls from the originator, one in each direction, requires $2 + c(n-1)$ time units. This scheme is not optimal, however. A better broadcast scheme would increase the degree of parallellism among calls. The following scheme proves to be optimal. The originator calls its neighbor in the direction of a furthest vertex. In the second round, the broadcast is completed with the originator and its informed neighbor calling their respective subpaths. With the exception of the originator being the middle vertex of P_{2k+1}, if the originator has eccentricity e then our scheme takes $1 + c + 1 + c(e-1) = 2 + ce$. Since any broadcast requires at least two rounds, this is a minimum time broadcast, by Lemmas 1 and 4. For the exceptional originator above, a minimum time broadcast takes at least $2 + c(e+1)$ time, by Lemma 2. We can state the above remarks as the following theorem.

Theorem 1: The minimum time for multidrop broadcast in a path of n vertices is $1+c(n-1)$ when the originator is one of the end-vertices of the path and $2 + ce$ for the originator in the interior of the path (e is the eccentricity of the originator), with one exception. When n is odd and the originator is in the middle of the path, this minimum time is $2 + c(e+1) = 2 + c\lceil \frac{n}{2} \rceil$. ∎

4 Binomial Trees

In trees other than paths, the number of rounds will increase with maximum degree of a vertex, as each new branch implies the necessity of another round of calls. In this section, we consider multidrop broadcasting in a binomial tree. The *binomial tree* of rank 0 is the single vertex tree. The binomial tree of rank $i > 0$ is the rooted tree obtained from the binomial tree of rank $i-1$ by addition of a principal subtree to the root vertex that is a (copy of) the binomial tree of rank $i-1$. As such, a rank i binomial tree has depth i, has 2^i vertices, and has i principal subtrees from the root vertex, which are the binomial trees of rank $i-1, \ldots, 0$.

The binomial tree of rank $i > 1$, T_i, has two vertices of degree i and two vertices of degree $i-1$. These vertices induce a path P_4 in T_i. These vertices can be viewed as roots of four disjoint subtrees isomorphic to the binomial tree of rank $i-2$. This view allows us to determine the number of rounds necessary to complete a minimum time multidrop broadcast in T_i. We will refer to the root of a binomial tree and the root of its largest subtree (the two degree i vertices) as *broadcast centers* of the tree.

By Lemma 4, a minimum time multidrop broadcast in the binomial tree of rank i requires at least i rounds, if the broadcast originates in a broadcast center. The binomial tree T_i is the *minimum time broadcast tree* with $n = 2^i$ vertices, in which $i = \log n$ rounds of length one calls suffice to complete a broadcast from broadcast centers (see, for instance, [5]).

Consider a multidrop broadcast in the binomial tree of rank $i = \log n$. If the broadcast originates in a broadcast center, then in the first round the other broadcast center is called. During each successive round, an informed vertex calls the root of its largest uninformed subtree. By this scheme, the time required by the broadcast is $\log n + c \log n = \log n + ce$, since the eccentricity of the root is $e = \log n$. This is optimal by Lemmas 1, 3 and 4.

If the broadcast originates in the subtree rooted in a degree $i-1$ vertex (the root of the second largest subtree of a broadcast center, isomorphic to T_{i-2}) then the minimum time can be achieved by calling the other degree $i-1$ vertex in the first round, thereby informing all four vertices of high degrees. In the subsequent $i-2$ rounds, these four informed vertices broadcast the message in the binomial trees of rank $i-2$ of which they are roots, for a total time of $\log n - 1 + ce$, where e is the originator's eccentricity. Again, this is optimal by Lemmas 1, 3 and 4.

If the broadcast originator is not a broadcast center and is not in a subtree rooted by a vertex of degree $i-1$, then not all four of the high degree vertices can be informed in the first round. Calling three of these vertices in the first round by routing the call to the furthest degree $i-1$ vertex, v, allows broadcasting to be completed in $\log n - 1$ rounds while requiring $\log n - 1 + c(e+1)$ time. This is the optimal among schemes with $\log n - 1$ rounds. Indeed, every such scheme must inform vertex v in the first round, requiring drop-off term $c(e-i+2)$; in the second round all degree $i-2$ vertices must be informed, requiring drop-off term $2c$. Finally, the remaining $i-3$ rounds add a drop-off term of at least $c(i-3)$, for a total drop-off term of at least $c(e+1)$.

An alternative scheme is for the originator to call the further broadcast center in the first round. The subsequent rounds mimic the optimal broadcast above with both broadcast centers informed. This requires at most $\log n + ce$ time, where e is the eccentricity of the originator, and thus is an optimal

$\log n$-round scheme. It follows from Lemmas 1, 3 and 4 that one of these schemes is always optimal: the former for $c < 1$ and the latter for $c \geq 1$.

Theorem 2: The minimum time for multidrop broadcast in a binomial tree of rank i with $n = 2^i$ vertices, from any vertex of eccentricity e, is less than $\log n + c(e+1)$. If the broadcast originates in one of the two broadcast centers of the tree, then the minimum time is $\log n + ce$. If it begins in one of the two subtrees of rank $i-2$, then the minimum time is $(\log n - 1) + ce$. For other vertices, the minimum time is $(\log n - 1) + c(e+1)$ for $c < 1$, and $\log n + ce$ for $c \geq 1$. ■

5 Complete binary tree B_h

Consider multidrop broadcasting from the root of a complete binary tree B_h with h levels (*i.e.*, $n = 2^h - 1$ nodes). We will show that the minimum number of rounds necessary to complete a broadcast is h and exhibit a simple scheme for a minimum-time h-round broadcast.

Lemma 5: The minimum number of rounds necessary to complete a multidrop broadcast from the root of B_h is h, for $h > 1$.

Proof: (by induction on h) The inductive hypothesis is obviously true for $h = 2$. Assume that, for a smallest number of levels $h > 2$, it is possible to broadcast in B_h in fewer than h rounds. In any multidrop broadcast, the root (and any other vertices) of one of the principal subtrees can only be called in round 2 or later. Thus, a broadcast from the root of this subtree would have to be completed in fewer than $h-1$ rounds. This contradicts the assumption of minimality of h. ■

Consider the following broadcast scheme (labeling the levels of B_h by $i = 1, \ldots, h$). Assume that at the beginning of round i, $1 < i < h$, all vertices on level $i-1$ are informed, and so is every other vertex on level i. These 2^{i-1} vertices make calls to half of the level $i+1$ vertices, where calls from level $i-1$ vertices drop-off the message to the uninformed vertices of level i. This maintains the invariant. The calls in rounds 1 and h follow from the above by the obvious constraints of the tree. By simple induction, the scheme completes the broadcast in h rounds. The time required by this scheme is $t_h = h + c(2h-2)$.

Theorem 3: The minimum time for multidrop broadcast in a complete binary tree B_h from its root is $h + c(2h - 2)$, for $h > 1$.

Proof: We prove by induction on h that the above described scheme is optimal. This is obvious for $h = 2$. Assume that the scheme defines a minimum-time broadcast in B_h and consider broadcasting from the root of B_{h+1}. The new level adds at least one round. A $2c$ increment in the drop-off term is required. The two edges from the root of B_{h+1} must be called in different rounds prior to completing the broadcast in the second principal subtree, called (at the earliest) in round 2. ∎

We now analyze the case of a general originator v at level $i \geq 2$. If $h = 2$, the minimum time is clearly $1 + 2c$. Suppose $h > 2$. When $i = 2$, a call to the root in the first round achieves the same time as an optimal broadcasting from the root, $h + c(2h - 2)$, easily seen to be minimum. For $i > 2$, the originator v calls a vertex at level 3 in the other principal subtree, thereby emulating the results of two first rounds of broadcasting from the root. This gives the total time $(h-1) + c(2h + i - 4)$. To see that this time is optimal, let w be the root of the principal subtree T_w to which v does not belong. First notice that the number of rounds cannot be decreased: otherwise broadcasting in T_w from its root w could be completed in less than $h-1$ rounds, contradicting Lemma 5. Next observe that the minimum drop-off term is at least that of broadcasting in T_w from w, plus ci (the drop-off term required to reach w), resulting in a total of $c(2h + i - 4)$.

Theorem 4: The minimum time for multidrop broadcast in a complete binary tree B_h, for $h > 2$, is $h + c(2h - 2)$ from a vertex at level 2, and $(h - 1) + c(2h + i - 4)$ from a vertex at level $i \geq 3$. ∎

6 Cycle C_n

We will now consider broadcasting in the cycle of n vertices, C_n. In such a cycle, the eccentricity of any vertex is $e = \lfloor \frac{n}{2} \rfloor$.

Since C_n has a Hamiltonian path starting at any vertex, there exists a one-round broadcast that takes $1 + c(n-1)$ time. However, we can increase the degree of parallellism with a two-round broadcast. A call in the first round to i vertices, $0 < i < n$, leaves the necessity of a call to $\lceil \frac{(n-i-1)}{2} \rceil$ vertices, for the total time of at least $2 + c(i + \lceil\frac{(n-i-1)}{2}\rceil)$. This is minimized by $i = 1$ and achieved by the following scheme. The originator calls a neighbor in the first round, and the two informed vertices complete the broadcast by calling approximately the same number of vertices in the second round. This broadcast takes $2+c\lceil\frac{n}{2}\rceil$ time units. In a cycle with even number n of vertices, $\frac{n}{2} = e$. For a cycle with odd number n of vertices we have $e + 1 = \lceil\frac{n}{2}\rceil$. As there are two vertices at distance e from any vertex, $c(e+1)$ is the minimum drop-off term, by Lemma 2. By Lemma 1, we need not consider broadcast schemes with more rounds. Hence, the optimal scheme is the better one of the two above.

Theorem 5: The minimum time for multidrop broadcast in a cycle of n vertices is

$$\begin{array}{rl} 1 + c(n-1), & \text{for } c < \frac{1}{\lfloor\frac{n}{2}-1\rfloor} \\ 2 + c\lceil\frac{n}{2}\rceil, & \text{otherwise.} \end{array}$$ ■

7 Grid $M_{p\times q}$

Consider the grid $M_{p\times q}$ with p rows and q columns. Since there is a Hamiltonian path starting at any vertex of the grid (except for the 3×3 case), the corresponding broadcast in the grid requires $1 + c(pq - 1)$ time. Obviously, $pq - 1$ is usually much greater than the eccentricity of any originating vertex, which is of the order of $p + q$. We will consider three locations of the originator: at a corner, along a side, and at an interior point of the grid.

Corner vertex originator

When the originator is a corner vertex, its eccentricity is $e = p + q - 2$. A two-round broadcast involving the calls along the first row of the grid and then to all columns requires time $2 + c(p + q - 2) = 2 + ce$. Therefore, there is no better two-round scheme (although there are other optimal two-round broadcasts). By Lemma 3, we need not consider schemes with more rounds.

Theorem 6: The minimum time for multidrop broadcast in a $p \times q$ grid from a corner vertex is

$$\begin{array}{rl} 1 + c(pq - 1), & \text{for } c < ((p-1)(q-1))^{-1} \\ 2 + c(p + q - 2), & \text{otherwise.} \ \blacksquare \end{array}$$

Side vertex originator

Next we consider the originator at a non-corner vertex on the side of the grid. Without loss of generality we may assume that it is situated on the side of length q. Let a denote the distance of the originator from the farthest end of its row. Thus, the eccentricity of the originator is $e = a + p - 1$ and $a \geq (q-1)/2$.

First we show an optimal three-round scheme. In the first two rounds the originator's row is informed following the optimal two-round broadcast scheme in a path, described in Section 3. In the third round, all informed vertices call their columns, in parallel. This scheme takes time $3 + ce$, except when the originator is in the middle of its row, in which case it takes time $3 + c(e+1)$. Since, in the latter case, there are two vertices at distance e from the originator, our scheme realizes the minimum time, by Lemma 2. Thus our scheme is optimal among three-round schemes, and there is no need to consider schemes with more than 3 rounds.

We now consider two-round schemes. The following scheme has a drop-off term at most 50% greater than the theoretical lower bound of $2 + ce$. If $p \leq a$, then in the first round the originator informs all vertices in its column and in the neighboring column on the side of the farthest vertex. In the second round all informed vertices call in parallel appropriate parts of their rows (see Figure 1). The total time is $2 + c(2p + a - 2)$, except when q is odd and $a = (q-1)/2$, in which case it is $2 + c(2p + a - 1)$.

If $p > a$, then in the first round the originator informs all vertices in its column. In the second round, each informed vertex calls the appropriate parts of its row and an adjacent row. The details of the call can be described by defining cycles of length $2q$ formed by adjacent pairs of rows, starting with the top most row. The informed vertices call their cycle segments in the clockwise direction (see Figure 2) for the total time of $2 + c(p + 2a - 1)$.

If p is odd, the last row is not covered by a cycle and the calling scheme will be somewhat different. During the first round, we extend the call to include

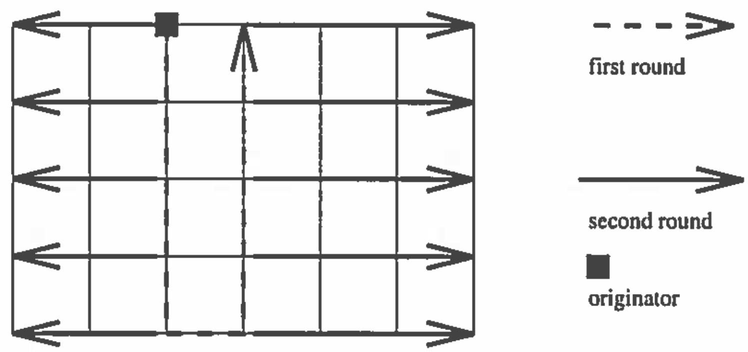

Figure 1

another vertex in the last row. These two vertices inform the remaining vertices of the last row in the second round, while other rows are informed as before (see Figure 3). The total time in this case is $2 + c(p + 2a)$. The worst-case ratio of the drop-off term to the theoretical minimum is reached when $p \approx a$ and equals 1.5 in this case. This comparison is made against the lower bound that follows from Lemma 1, $l_1 = 2 + ce$.

The following argument yields a better lower bound value for the grid. Suppose that in the first round the originator calls $i - 1$ vertices. In the second round all informed vertices might participate as transmitters in the completion of the broadcasting process. Let $j-1$ be the length of the longest call in the second round. Then the total time is $2 + c(i + j - 2)$, which under the constraint $ij \geq pq$ yields $l_2 = 2 + c(2\sqrt{pq} - 2)$. While not guaranteed to be tight, the lower bounds l_1 and l_2 are estimates of a minimum two-round broadcast time in a $p \times q$ grid. The problem of determining a tighter lower bound on the minimum time two-round broadcast remains open, as is the question whether a better two-round scheme exists in general.

Theorem 7: Let t_i, for $i = 1, 2, 3$, be the minimum time for i-round multidrop broadcast in a $p \times q$ grid from a non-corner vertex on the side of length q. Let a be the distance from the originator to the further corner of its side. Then

$t_1 = 1 + c(pq - 1)$;

$$t_2 \leq \begin{cases} 2+c(2p+a-1) & \text{if } p \leq a \\ 2+c(p+2a) & \text{otherwise;} \end{cases}$$

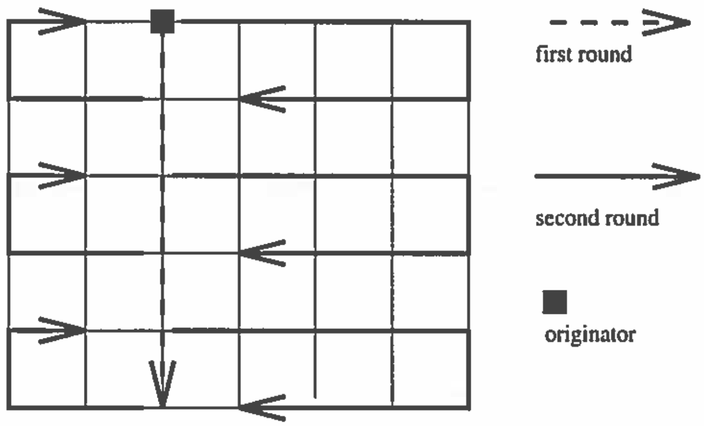

Figure 2

$$t_3 = \begin{cases} 3 + c(a+p) & \text{if the originator is in the middle of its side} \\ 3 + c(a+p-1) & \text{otherwise.} \blacksquare \end{cases}$$

The above theorem supports the intuition that for very small values of c one round is best, for intermediate values of c two rounds are best and for large values of c, three rounds lead to the shortest time.

Interior vertex originator

Finally, consider the originator in the interior of the grid. Without loss of generality, assume that $p \leq q$, *i.e.*, columns are not longer than rows. Let a denote, as before, the distance of the originator from the farthest end of its row and b the distance of the originator from the farthest end of its column; hence, the eccentricity is $e = a + b$.

We start by showing an optimal four-round broadcast. First make two rounds of length one calls to inform the three vertices with which the originator forms a square in the direction of a farthest vertex (at distance e) from the originator. Each of the four informed vertices is a corner vertex of a subgrid with two-round minimum drop-off solutions (*cf.* Theorem 6). The time required by this scheme is $4 + ce$ for the originator with exactly one vertex at distance e, it is $4 + c(e+1)$ for the originator with exactly two vertices at distance e, and it is $4 + c(e+2)$ for the originator with four vertices at

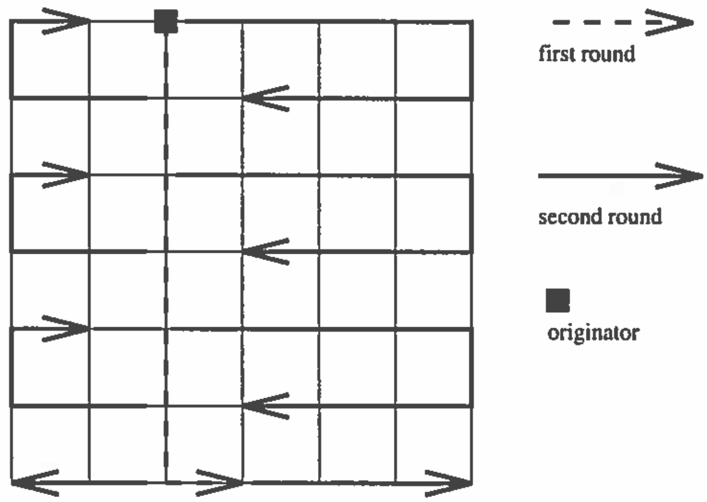

Figure 3

distance e (the latter happens if and only if the originator is in the center of an odd-by-odd grid). By Lemmas 1 and 2, in the first two cases the time is optimal. The following Lemma indicates that the third case involves the minimum drop-off term, as well. Thus, we need not consider schemes with more than four rounds.

Lemma 5: The minimum drop-off term in an optimal multidrop broadcast from the middle vertex in a $p \times q$ grid (p and q odd) is $c(e+2) = c(1+(p+q)/2)$.

Proof: Without loss of generality, the first call results in two informed vertices, one at distance e and the other at distance $e+1$ from two uninformed vertices. By Lemma 2, these uninformed vertices cannot become informed in less than $c(e+1)$ additional drop-off term, for the total drop-off term of at least $c(e+2)$, as postulated. ∎

An efficient three-round broadcast is realized by a small modification to the above scheme. Instead of informing the vertices of the square in two one-call rounds, the originator calls all three other vertices in the first round. The total time is $3+c(e+1)$, $3+c(e+2)$ and $3+c(e+3)$ in the three above cases, respectively.

We now turn attention to two-round schemes. Our calling scheme for a side vertex originator can be modified according to the position of the originator in the interior of the grid (see Figure 4). As before, it takes time at most $2 + c(2p + a - 1)$. Now the lower bounds are $2 + c(2\sqrt{pq} - 2)$ (as before) and $2 + c(a + b)$. It can be shown that the drop-off term we achieve is always at most 60% larger than that of the larger of these bounds.

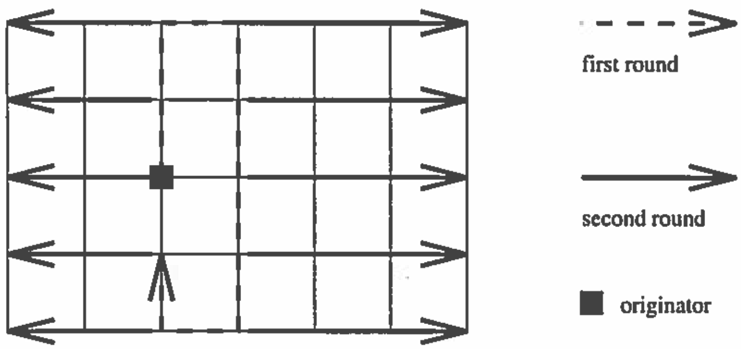

Figure 4

Theorem 8: Let t_i, for $i = 1, 2, 3, 4$, be the minimum time for i-round multidrop broadcast from an interior vertex of a $p \times q$ grid, with $p \leq q$. Then

$t_1 = 1 + c(pq - 1)$

$t_2 \leq 2 + c(2p + a - 1)$

$$\left. \begin{array}{c} 3 + ce \leq t_3 \leq 3 + c(e+1) \\ t_4 = 4 + ce \end{array} \right\} \quad \text{if the originator is neither in the middle of a row nor of a column,}$$

$$\left. \begin{array}{c} 3 + c(e+1) \leq t_3 \leq 3 + c(e+2) \\ t_4 = 4 + c(e+1) \end{array} \right\} \quad \text{if the originator is in the middle of a row (column) but not both,}$$

$$\left. \begin{array}{c} 3 + c(e+1) \leq t_3 \leq 3 + c(e+3) \\ t_4 = 4 + c(e+2) \end{array} \right\} \quad \text{if the originator is in the center of the grid.}$$ ∎

Note that, for some interior vertex positions of the originator, the eccentricity lower bound on the drop-off term can be achieved in three-round broadcasts. This happens when a two-edge call in the first round results in three informed vertices that can complete two-round broadcasts in their respective subareas of the grid in the remaining $2 + c(e-2)$ time. This time can be achieved when the originator vertex is not more than 1/3 in from either side of the grid. (Note that in the limiting case of a side vertex, the three-round broadcast has been shown to meet this lower bound in Theorem 7.)

8 Conclusions

We have presented minimum-time, multidrop broadcast algorithms for complete graphs, paths, cycles and binomial and binary trees. We estimated this time for grids. In each case, we proposed a scheme working in optimal time or corresponding to the given upper bound. Finding an optimal scheme for an arbitrary originator in a grid remains an open problem.

In our model, we have required simultaneous calls to be vertex-disjoint. This avoids the issue of what message is dropped-off when more than one call passes through a vertex. In broadcasting, since all messages are the same, this issue is moot as long as multiple reception does not cause an error. Adopting a model only requiring edge-disjoint calls can have a significant impact on the performance of a network. This is especially true in trees, where the originator no longer would be a communication separator. Consider, for instance, the star with $n+1$ vertices, *i.e.*, $K_{1,n}$. In our original model, any broadcast from the center would require n rounds of length one calls. However, allowing calls "crossing" at the center (*i.e.*, not vertex-disjoint) the broadcast can be completed in $\log n$ rounds with calls of length at most two.

A natural line of future research is to investigate optimal multidrop broadcast schemes and their running time for other important architectures, such as hypercubes, cube-connected cycles or tori. For example, for the d-dimensional hypercube H_d, the standard broadcasting scheme involving d rounds of length one calls in all dimensions, takes time $d + ce$, where $e = d$ is the eccentricity of any vertex. Thus, schemes of more than d rounds need not be considered. A simple and efficient i-round multidrop broadcast ($1 \leq i \leq d$) can be obtained as follows. Let $d_1, d_2, ..., d_i$ be integers differing by at most 1, whose sum is d. H_d can be represented as the product $H_{d_1} \times ... \times H_{d_i}$. In the j-th round, for $j \leq i$, every informed vertex calls all vertices in its copy of H_{d_j} via a hamiltonian path. This scheme takes time $i + c(2^{d_1} + ... + 2^{d_i} - i)$. It is easy to show that the drop-off term is always at most twice the minimum possible, and that it is minimum if i divides d. If c is sufficiently small, such schemes with fewer rounds can be optimal.

Acknowledgements: Thanks are due to K. Diks and J. Kratochvíl for helpful discussions.

References

[1] J.Hromkovič, R. Klasing and E.A. Stöhr, Gossiping in vertex-disjoint paths mode in interconnection networks, *Proceedings of 19th Workshop on Graph-Theoretic Concepts in Computer Science* Springer Verlag *LNCS* 790, 288-300;

[2] R. Feldmann, J. Hromkovič, S. Madhavapeddy, B. Monien and P. Mysliwietz, Optimal algorithms for dissemination of information in generalized communication modes, Proceedings of Parallel Architectures and Languages Europe, *PARLE '92*, Springer Verlag *LNCS* 605, 115-130;

[3] C. Gavoille, Evaluation of the MasPar Performances, RT-92-01, ENS Lyon (1992).

[4] L.M. Ni and P.K. McKinley, A survey of wormhole routing techniques in direct networks, *Computer* (Feb. 1993), 62-76;

[5] A. Proskurowski, Minimum broadcast trees, *IEEE Transactions on Computers C-30*, 5(1981), 363-366;