MATHEMATICS, COMPUTER SCIENCE, AND WEAVING

by

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A THESIS

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The fields of mathematics, computer science, and weaving are interlaced in many ways. This work gives an overview of past and present connections between the three fields and explores new connections. For original research, this work proposes a new way of representing fabric with a graph data structure and discusses algorithms relevant to weaving that work for this graph. An additional section discusses the mathematics involved in color-and-weave, when the visual design or motif differs from the structural design of a fabric. A summary at the end notes some open questions.

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CHAPTER I

INTRODUCTION

This paper is the result of a two year search for connections--real or imagined--between the fields of mathematics, computer science, and weaving. The relationship between these three fields has a long and interesting history. The art of weaving cloth and the study of mathematics are ancient, but only recently have both fields contributed to the beginning of a new field of study which is now called computer science. Some people argue that the first computer was a Jacquard loom used to weave cloth. Computer science has its roots in both mathematics and weaving. During the past century, the three fields have contributed to each other in a variety of ways, but the singular concept that unifies these three fields is the idea of binary representation of information.

My search for connections was by no means exhaustive. Chapter II discusses some of the major historical connections between weaving and computer science. Chapter III focuses on the mathematics implicit in weaving and discusses some 20^{th} Century research in the field of mathematics that explicitly classifies fabrics and the process of weaving. Chapter IV introduces my research into new connections between mathematics, computer science and weaving. In particular, my research includes a new representation of fabric and some formal discussion of the relationship between a fabric's visual motif and structure. I also include a section in Chapter V to discuss the possible future connections between the three fields. The open questions at the end suggest still more connections between the three fields.

Chapter I provides an introduction to the process of weaving and the structure of woven fabric, but by no means attempts to cover the entire field of weaving. Many readers may have little or no familiarity with weaving, so I present the basics necessary to understand the significance of the research in Chapters II, III, and IV as it affects the field of weaving. I hint at the vastness of the field and the variety that exists to show how important a formal classification of fabrics is to those who work in or around the field of weaving. I also present the field of weaving as it was from a weaver's perspective, before I present the academic study in the later chapters, to show how naturally computer science and mathematics apply to the field of weaving. I include a discussion of the design process that weavers use to create new fabrics, and emphasize in later chapters the relevance of the contributions from mathematics and computer science to this design process.

A major difficulty in presenting the weaver's approach is the variety of terms that weavers use to describe parts of fabrics and the mechanisms they use to weave fabric. I will establish conventions for this work, but they may not necessarily reflect any standard usages (if a standard exists) in the field of weaving. A glossary in Appendix A provides definitions for the italicized words. Appendix B provides a list of background references and some helpful sources for anyone interested in pursuing further research on any connection between mathematics, computer science, and weaving.

The Basics of Weaving

Weaving is the process of constructing cloth by interlacing a set (*the weft*) of horizontal parallel *weft strands* at right angles with a set (*the warp*) of vertical parallel *warp strands*. A *strand* could be a string or thread, a strip of paper, or a length of any material, including metal wire. The weaver inserts each weft strand as a row across the warp such that the weft strand runs over or under each warp strand. The sequence of "overs" and "unders" determines the *structure* of the weave. For instance, the sequence of "over one, under one" repeated in one row and "under one, over one" repeated in the next row defines a *plain weave* or *tabby* fabric (see Figure 1), which has the highest level of interlacement of the warp and weft.

The weaver's primary instrument for weaving is traditionally the *loom*. Looms come in all shapes and sizes and a variety of levels of complexity, but they all share a common purpose: a loom secures and tensions the warp. A simple loom is a frame of two beams with warp strands strung between them in parallel (see Figure 2). The weaver manually guides the weft over or under the warp strands one by one. That job would be easier if the weaver could raise up all the strands under which the weft strand runs and then place the whole weft strand at once between the set of raised warp strands and the set of unraised warp strands. When the weaver lowers the raised subset of the warp, the warp strands bend the weft strand into place so that the weft strand assumes the desired sequence of "overs" and "unders."

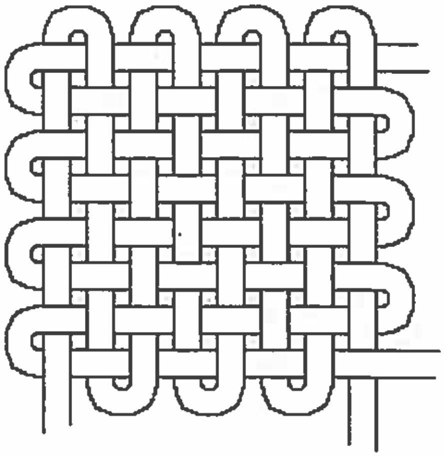


Figure 1. Plain Weave. The warp strands are vertical, the weft strands are horizontal.

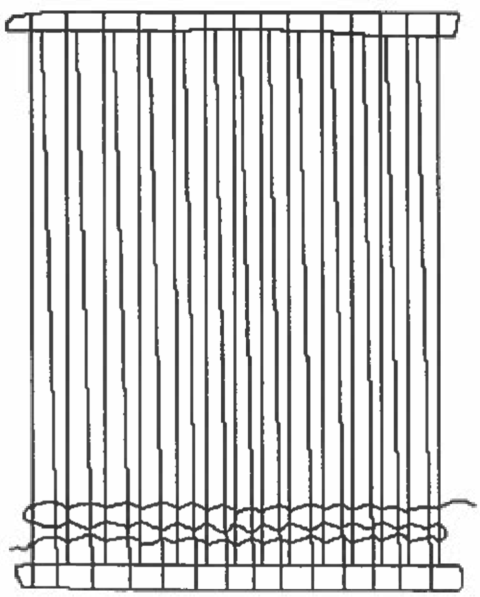


Figure 2. Simple Loom. The warp strands run from beam to beam, and the weft strands run over and under the warp strands (in a plain weave).

A more complex loom provides devices that create *sheds*. A shed is the V-shaped opening created when one set of warp strands is pulled away from the plane of the fabric. A shed-creating device will partition the warp strands into two or more sets, allowing the weaver to create two or more unique rows of pattern. For plain weave (in which every other row has the same pattern), the weaver could insert a flat stick into the warp (see Figure 3) such that it runs under every even-numbered warp strand and over the odd-numbered warp strands. Each time the weaver wants to weave a row of "over one, under one" she raises up the stick on its edge, raising all the warp strings that it passes under, and then she passes the weft strand through the shed. When she turns the stick down to its flat position, the weft presses over the warp strands that were under the stick and under those that were over the stick, creating the desired pattern. To weave the next row, however, the weaver would have to pick up the odd-numbered warp strands by hand or create another device that could pick them up.

In general, such warp-manipulating devices are called *harnesses* or *shafts* (see Figure 4). Many looms have a set of harnesses which partition the warp strands so that the weaver can create each uniquely-patterned row of the weave by raising one or more harnesses, inserting the weft strand, and lowering the harnesses. Each harness is a frame with a series of *heddles* (segments of string or wire which have a loop in the middle) that stretch between the top and bottom of the frame. Traditionally, each warp strand passes through a heddle on only one harness. Some looms use *long-eyed heddles* (heddles with long loops) and allow for a warp strand to pass through heddles on more than one

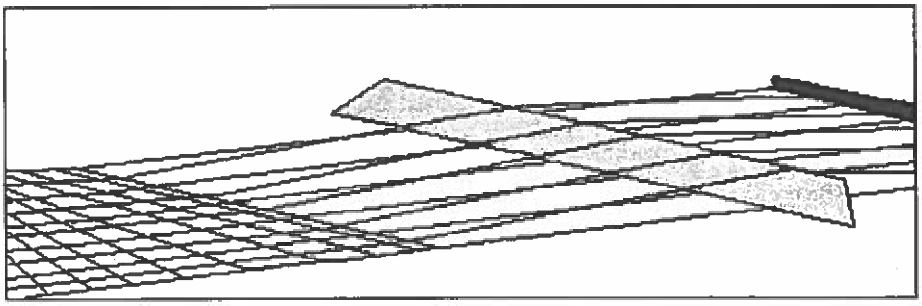


Figure 3. Using a Flat Stick as a Harness.

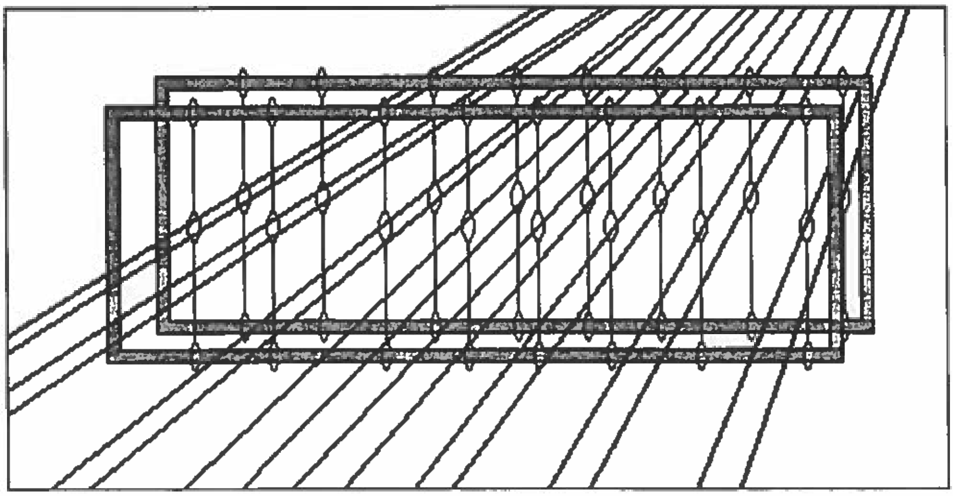


Figure 4. Harnesses. Each warp strand runs through a heddle on either the first (front) or second (back) harness. If we number the warp strands starting from 1 at the left, then the odd-numbered strands are on the first harness and the even numbered strands are on the second harness. When a harness frame moves, all the heddles move, taking the warp strands on that harness with it. These harnesses are set up to make plain weave.

harness, which lets the weaver raise that warp strand by raising any of the harnesses through which that warp strand is threaded. The assignment of warp strands to harnesses is called the *threading*. Once the weaver threads the loom, the threading is usually permanent until the cloth is finished.

Most common looms have four, eight, or sixteen harnesses (or shafts). A greater number of harnesses means that a greater number of uniquely-patterned rows are possible (without manipulation of individual warp strands by hand). As the number of harnesses increases, the freedom of design increases. Physical limitations on most looms, however, determine the number of harnesses that may be raised at any particular time. For instance, on some looms a lever controls each harness and the weaver must simultaneously pull down on the corresponding levers to raise the appropriate harnesses; the weaver may find it difficult to raise three or more harnesses at once, which diminishes the number of unique rows that can occur in one cloth.

A *floor loom* (see Figure 5) frees the weaver's hands by controlling the harnesses with *treadles* (foot pedals). A weaver can tie up one harness to each treadle, but here the same problem occurs if the weaver cannot depress more than a certain number of treadles simultaneously. In addition, most looms do not have room for more than twelve treadles. To make the weaving process easier, the treadles have a tie-up system in which each treadle may be "tied up" to one or more harnesses (indirectly) and each harness may be "tied up" to one or more treadles (indirectly). The assignment of harnesses to the treadles is called the *tie-up*. Each treadle or combination of treadles may correspond to a

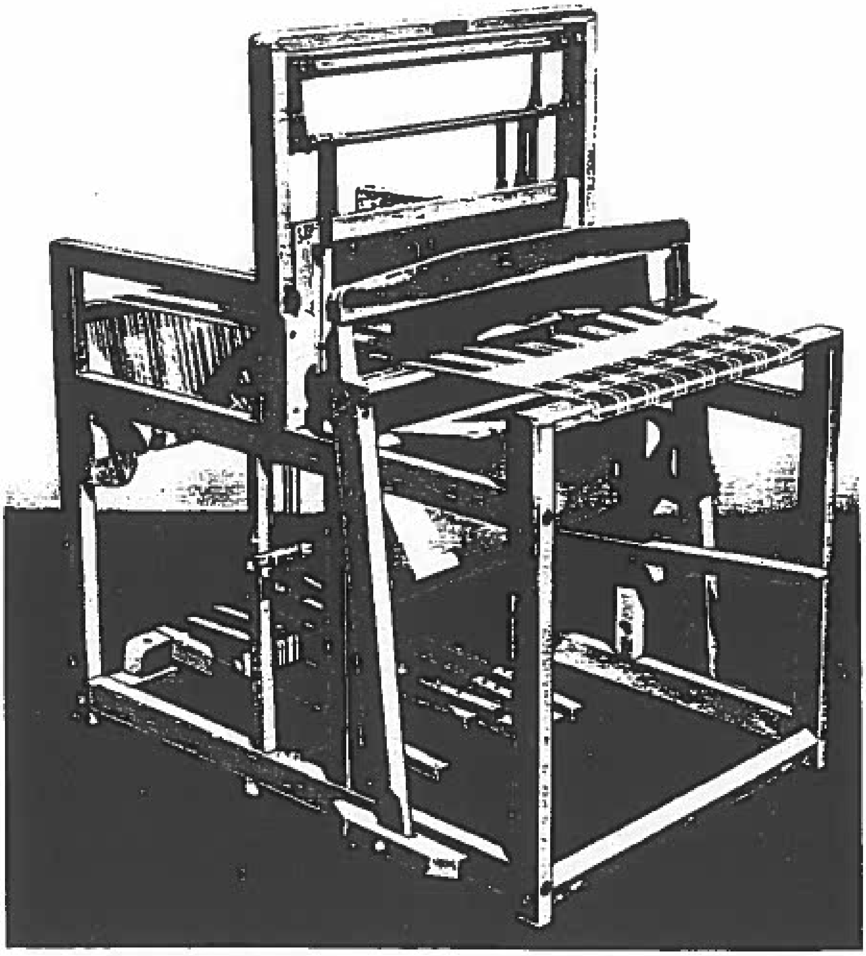


Figure 5. Floor Loom. Adapted from *The Joy of Handweaving* (Tod, 1964, p. 146).

unique shed or uniquely-patterned row in the cloth. The process of weaving then becomes a sequence of pressing the appropriate treadles and inserting weft strands. The loom winds the woven cloth onto a roll at the front of the loom and feeds the unwoven warp from the roll at the back of the loom when the weaver advances the rolls. When done, the weaver releases the warp from the loom and adds finishing touches to the cloth.

The Design Process

The weaver takes into consideration several factors when designing a fabric. She selects the fabric materials based on their physical properties, such as elasticity, texture, and fiber content; visual properties, such as color, thickness, and luster, and; practical properties, such as durability, wash-and-wear qualities, cost, and availability. The weaver also selects the *sett* (number of strands per inch in the warp and weft) of the fabric. The most important design decision is the selection of the *weave* or *structure*. "Structure is the fundamental element in weaving: good cloths are made by careful consideration of fibre, colour, texture and sett, but the underlying weave (affecting and affected by all of these elements) is the basic formation of the character of any cloth" (Sutton, 1982, p. 7).

Weave Structures

Weave structures traditionally fit into categories and families of weaves. Madelyn van der Hoogt (1993) classifies weaves into three main categories: *simple weaves*, *weaves with compound sets of elements*, and *compound weaves*, based on Irene Emery's descriptions (1980). Van der Hoogt organizes the simple weaves into families. The simplest family of weave structures is plain weave (as seen in Figure 1). Basket weave is a variation of plain weave, with the sequence "over two, under two." Other variations of plain weave usually change the colors of the warp or weft strands rather than the structure of the weave.

Twills make up a large family of flexible, sturdy weaves. Twills have *floats*, warp strands or weft strands the pass over more than one (or under more than one) strand before changing to the back of the fabric (or face of the fabric). Simple twills generally appear to have diagonals and follow a sequence in which a row always has the same pattern as the previous row shifted to the right or left by some number of warp strands. Fancier or more complex twills may not appear to have any diagonal at all and some can undulate like waves.

The next family, *satins*, is often associated with a quality of being smooth and shiny on one side and dull on the other side. As in twills, each row of a satin is a copy of the previous row shifted to the left or the right, but satins break up the twill's diagonal lines by following a simple rule: take the number of harnesses used in the weave and divide it into two unequal and *relatively prime* parts, then use one of these numbers as the constant number of warp strands to shift a row to get the next row. (Note: four and six harness weaves cannot be perfect satins because of the requirement that the parts be relatively prime.)

Other weaves, such as mock lenos, lace weaves, and honeycombs, have structures that allow the fabric to deform in such a way that leaves spaces between strands or creates raised and sunken sections in the fabric. Often the fabrics contain sections of consecutive rows which are nearly the same pattern except for a strand or two that vary between the rows. When this occurs, strands may more freely slide around in the fabric; when controlled, these misalignments can create interesting effects as above.

Overshot weaves belong to the category of weaves with compound elements because they have one warp and two wefts (usually a patterned foreground weft and plain weave background weft). The patterned foreground comes from rows with long which make the pattern easier to see. Rows of tabby or plain weave alternate between the pattern rows to make the fabric structure strong. The abundance of floats, especially long ones, can cause the fabric to be less functional or durable if the floats can easily snag on things, but the floats can also add to the aesthetic appeal of a cloth.

The compound weaves have two sets of warp and weft that form two layers in parts or all of a fabric. The fabric may alternate between one or the other layer on the face of the fabric, or it may have one layer on the face and one layer on the back. The layers may also have occasional stitches between them to join the layers into a single unit of fabric that holds together.

The weaver's task is to choose an appropriate weave structure for the cloth. She may use a traditional weave, make variations on those weaves, or combine and connect traditional weaves in new ways to create unique weave structures. Weavers often experiment on the loom, by creating the sequence of rows dynamically instead of following a preset order. A weaver may also make samples of fabrics to get a feel for the weave structure's qualities, or find an interesting fabric and attempt to replicate or adapt its weave structure. The weaver may also wish to restrict the weave by certain structural requirements, such as the necessity for a weave to hold together in a single layer or to have a bounded float length in the warp or weft (or both). Through the ages, weavers have developed ways of representing and storing information about the cloth on paper; many weavers design new fabrics off the loom, working primarily with pen and paper.

Notation

In order to store or communicate weave structures and other cloth characteristics, weavers needed to develop consistent notation for representing cloth on paper (usually graph paper). A variety of notations are used in the world (see Figure 6), but perhaps the most common representation of weave structure is a two-dimensional grid with black and white squares, in the lower right corner of Figure 6. This common style of notation can be used to supply other information about the weave, including information on how to thread the loom and weave the cloth. Throughout this work, a *draft* (see Figure 7) refers to the collection of the information about the finished weave and how to set up the loom to construct that weave. The draft has four parts, the *threading, tie-up,* and *treadling* diagrams, and the *drawdown*, which appear on the page in relation to a set of axes and occupy quadrants.

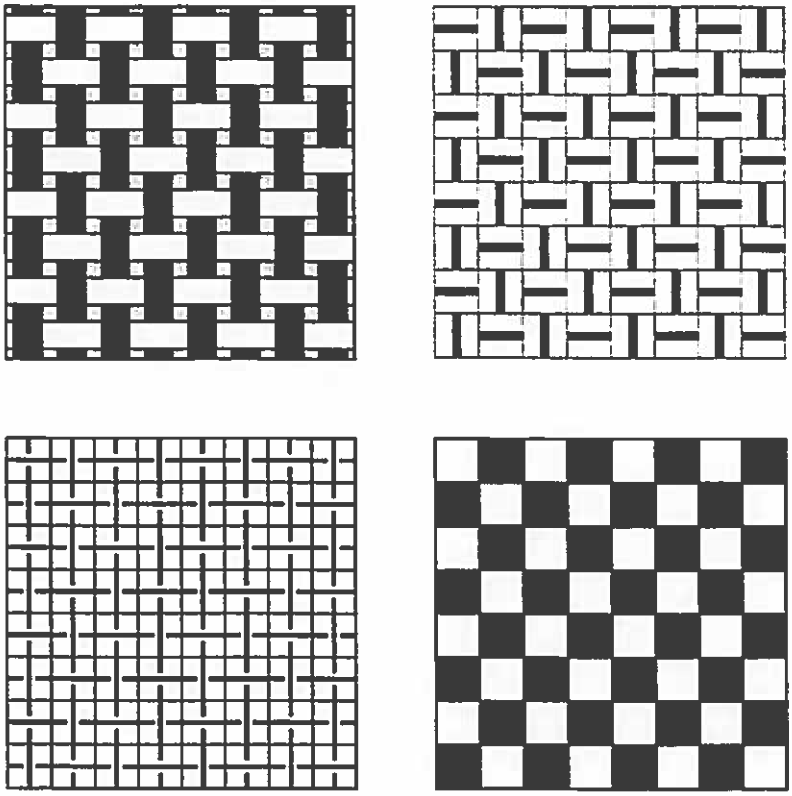


Figure 6. Fabric Notations. The upper left drawing closely approximates the actual fabric. The other three are representations of the fabric structure using notations that some weavers use.

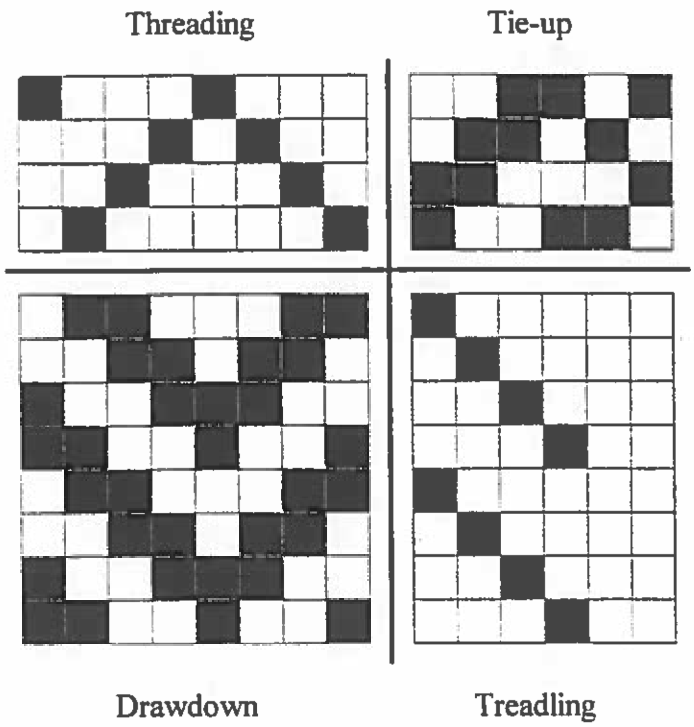


Figure 7. Typical Draft. Any arrangement that reflects the draft about one or both of the axes works as a draft.

The threading is the part of the draft that indicates the assignment of warp strands to the harnesses; it usually appears at the top or bottom of the draft. Each row represents a harness and each column represents a warp strand. Usually, the bottom row of the threading corresponds to the first harness. A black square appearing in a given row and column indicates that the warp strand corresponding to that column threads through the harness corresponding to that row (refer to Figure 8).

The tie-up is the part of the draft that indicates how the harnesses are used in combination to create sheds (and which treadles correspond to those sheds); it usually appears in one of the four corners of the draft in line with the threading and treadling. Each row corresponds to a harness and each column corresponds to a treadle. Each column contains black squares that each correspond to a harness that will raise when the weaver depresses the treadle that corresponds to that column. A standard tie-up (refer to Figure 9) for a four-harness loom contains all six combinations that have two harnesses up and two harnesses down.

The treadling is the part of the draft that indicates the sequence in which the weaver presses the treadles; it usually appears on the left or right side of the draft. Rows are read from bottom to top. Each column corresponds to a treadle, and each row corresponds to a step in the treadling sequence. Treadlings usually have only one black square per row. The weaver executes a step corresponding to a row of fabric by pressing down the treadle corresponding to the black square in that row and then inserting a weft strand into the shed.

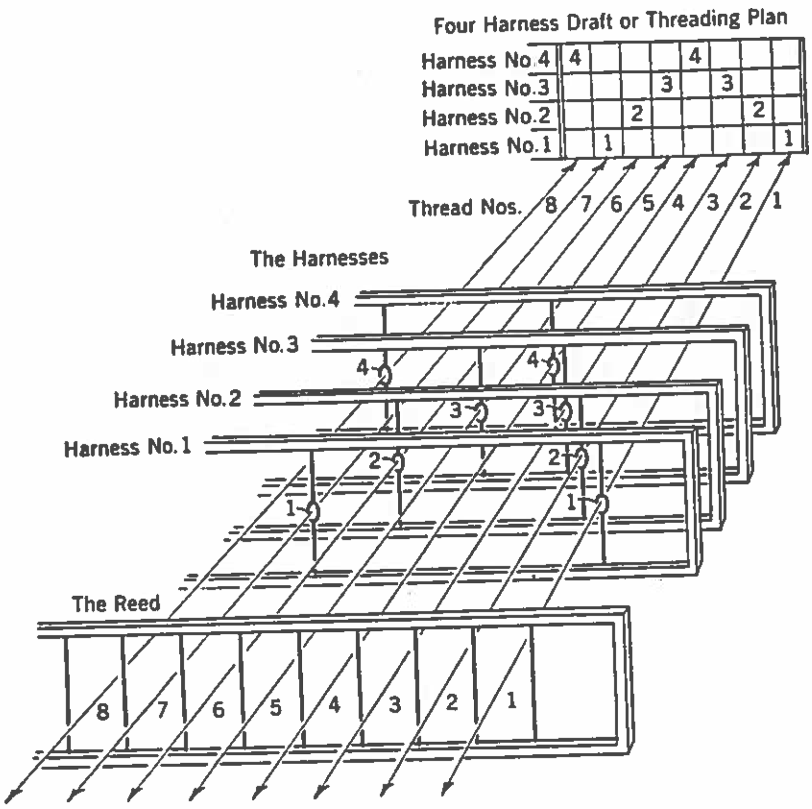


Figure 8. The Threading Diagram. Adapted from *The Joy of Hand Weaving* (Tod, 1964, p. 148).

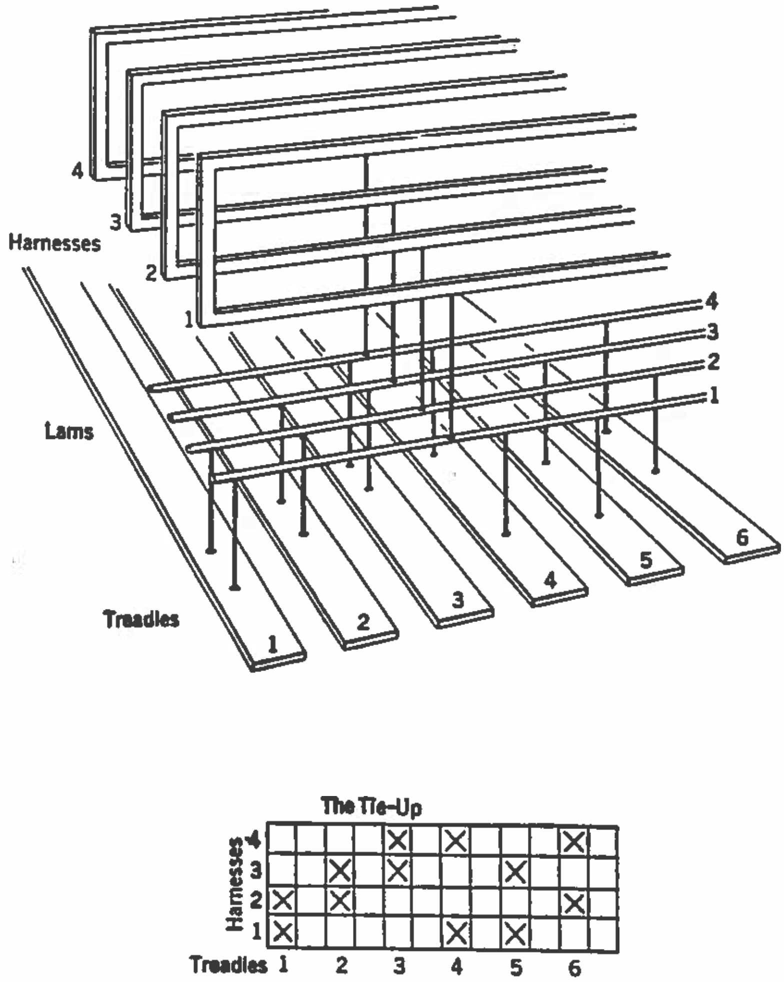


Figure 9. The Tie-up Diagram. Adapted from *The Joy of Hand Weaving* (Tod, 1964, p. 169).

The drawdown refers to the part of the draft that represents the finished weave; it usually appears opposite the tie-up and lines up with the threading and treadling. The drawdown reveals all the interactions of the warp and weft. A black square represents a warp stand passing over a weft strand; conversely, a white square represents a weft strand passing over a warp strand. Each column represents the interactions of its corresponding warp strand with all the weft strands. Each row represents the interactions of its corresponding weft strand with all the warp strands. The actual grid of black and white squares has the same appearance as a fabric in which the warp strands are black pieces of paper as wide as a grid square, the weft strands are white pieces of paper as wide as a grid square, and the strips are woven as close together as possible in the same structure as the one indicated by the drawdown. In most cases, however, the drawdown will not closely resemble the appearance of the finished fabric and will not take into account different warp and weft materials. The weaver can describe such qualitative information in words, pictures, or material samples that may accompany the draft.

The arrangement of the four parts of the draft can appear in any rotation, but this work will primarily use the arrangement that appears in Figure 7. Any clear binary representation of choice may appear in the draft. For instance, weavers often use tick marks or X marks in any of the parts of a draft consistently in place of black squares. To make the draft easier to read or use, weavers sometimes fill in the squares with harness or treadle numbers instead of blackening them (see Figure 10). The drawdown may also represent the weave structure with vertical and horizontal lines; a vertical line

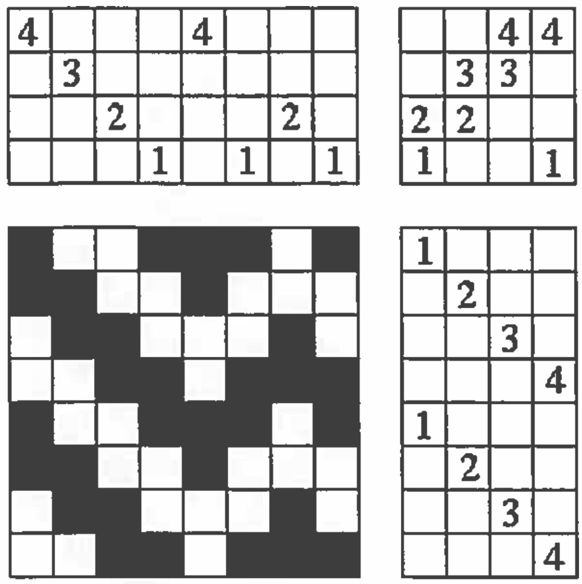


Figure 10. Notation with Numbers.

represents a warp strand showing on the face of the fabric and a horizontal line represents a weft strand showing on the face of the fabric in that given square (as in the top right corner of Figure 6).

When the weaver wants to indicate the coloring of the warp and weft strands, she might replace a black square in a column of the drawdown with a square of the color of the corresponding warp strand, and replace a white square in a row of the drawdown with the color of the corresponding weft strand. The weaver could also replace the black squares in the threading or treadling with colored squares or letters that represent the colors. As an alternative, the weaver might expand the draft to include a row above or below the threading and a column next to the treadling which records the colors (see Figure 11). The coloring method in this work will primarily be the latter method. Note that when the warp strand color is white or the weft strand color is black, the weaver will have to be more careful in being sure she replaces the correct squares with the new color. Here the notation may be ambiguous. The drawdown could represent the weave structure (with black warp and white weft) or the *visual design* of the fabric. The visual design can hide the real structure of the fabric, but indicates the *color-and-weave* effects that occur (see Figure 11). In this work, the visual design will appear in place of the drawdown in drafts and the weave structure will appear separately with a label.

Other weavers have invented different ways to simultaneously represent the weave structure and visual design. In her book, *The Structure of Weaving*, Sutton (1982) used a notation which uses vertical and horizontal marks to indicate the structure and thickens

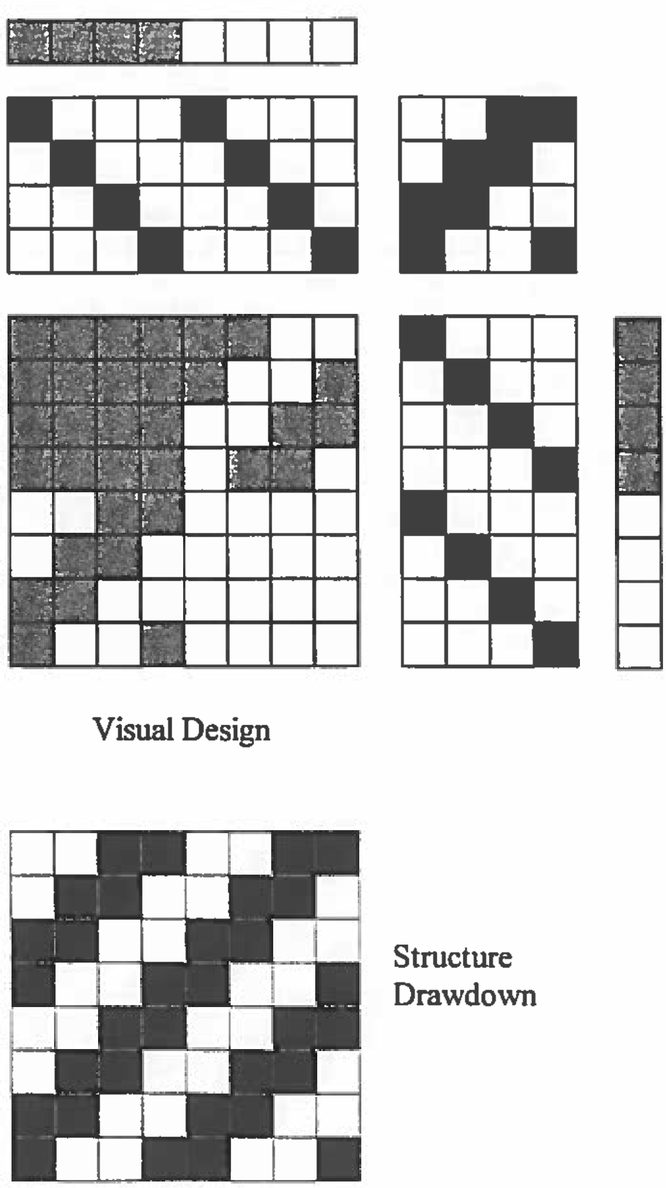


Figure 11. Notation for Color-and-Weave. The draft includes added information about the strand coloring and shows both the visual design and the structure. The coloring of the warp strands appears in a row above the threading (at the top left) and the coloring of the weft strands appears in a column beside the treadling (at the bottom right).

the lines that represent dark strands to indicate the color-and-weave effects (see Figure 12). Such a representation extends beyond the boundaries of binary notation, because it represent more than two possible choices for each square of the drawdown. The notation in this work will be consistent with the binary black and white grids, with color or shading appearing only in separate columns and rows that augment the basic draft.

Several other aspects of a fabric may appear in a draft, such as unbalanced sett in the warp and weft, multiple layers, different thicknesses of warp or weft materials, or repeated elements. Often weavers will show each layer in a separate drawdown. Multiple-layer weaves in which the two layers interact are difficult to represent except by cross-sections perpendicular to the fabric in the warp or weft (see Figure 13).

The draft is the weaver's forum for off-loom experimentation. On paper she can try different threadings, examine the effects of color-and-weave, combine weave structures, and plan the fabric. The experienced weaver can picture the finished fabric design based on information in the drawdown and a knowledge of the materials she will use. Then she can weave the fabric according to the desired draft, which conveniently stores the information necessary to weave the fabric. Chapters two and three will examine in detail the ways in which weavers construct the drafts and discuss the tools now available to weavers in weave design.

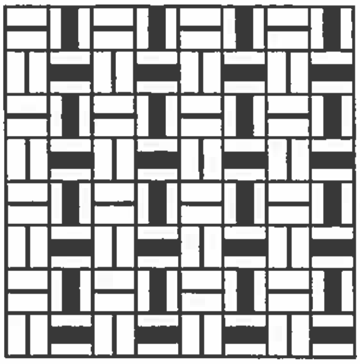


Figure 12. Alternate Color-and-Weave Notation. Thick lines denote a darker strand coloring and thin lines denote a lighter strand coloring.

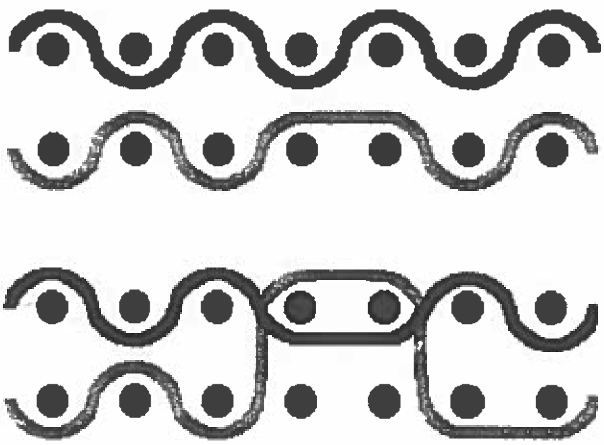


Figure 13. Cross Sections of Multi-layered Fabrics. The top drawing shows two distinct layers; the bottom drawing shows some interlacing between the two warps.

CHAPTER II

ADVANCEMENTS AND CONTRIBUTIONS

Many devices and machines arose to ease the weaver's task and to improve the speed of the process and efficiency of the loom. Other inventions allowed the weaver more freedom of design, making more complex designs possible. Later when computers became accessible to industry and the public, people could involve computers in the design process as well as the automation process. Soon weavers were making drafts on computers instead of graph paper, tweaking a weave and seeing the results of the change almost instantly.

Contributions to Computer Science

In 1804, Joseph Marie Jacquard created different kind of loom, called the Jacquard loom (see Figure 14). This loom did not have all the constraints that were present in earlier looms (such as those in Chapter 1). Instead of harnessing together groups of warp strands, this new loom allowed the weaver to control each warp strand independently of the others without having to control it by hand. In a sense, this loom had as many harnesses as there were warp strands--one harness for each warp strand. Each particular loom had a limit to the number of independent warp strands it could hold, but the idea was that the design of the Jacquard loom was expandable to as many warp-controlling

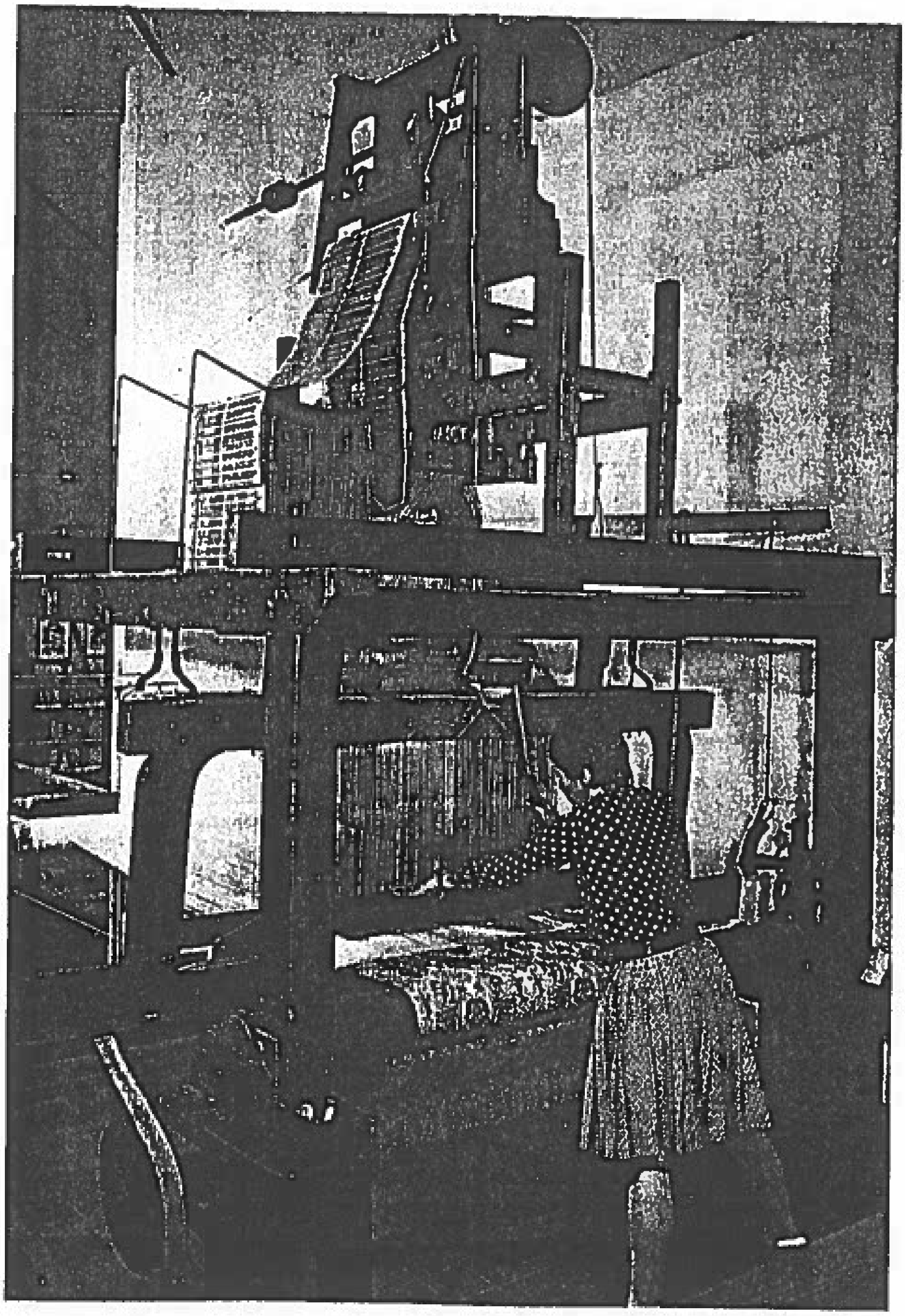


Figure 14. Loom Equipped with a Jacquard Mechanism. (Adrosko,1983, p. 11)

mechanisms as the design principles of mechanical engineering allowed (i.e., limits on the number of mechanisms that could fit into a reasonable space).

Jacquard built on ideas from past inventors who attempted to mechanize the drawloom. The drawloom was a tenth century A.D. Asian loom that employed a boy or girl to lift the harnesses in any combination the weaver required. It is the most complex non-mechanical loom in existence. Inventors later developed mechanisms to take the place of the person who lifted the harnesses, but Jacquard improved the mechanisms and put these ideas together into a loom that worked successfully.

The Jacquard loom operated from instructions in the form of punched cards. Each card had designated places (for holes) that correspond to each warp strand. Each warp strand connected via a lifting mechanism to a needle (see Figure 15). A spring pressed the needle against the corresponding position on the card. A punched hole in a given position let the needle pass through, which triggered the lifting mechanism to raise the corresponding warp strand. Each card represented a row of the weave, which was in fact the same as a row from the drawdown. The weaver laced the punch cards together in the desired order, often in a chain or loop. A square block held a single card against the needles, then rotated to bring the next card against the needles when the weaver pressed a treadle to advance the loom. The weaver could weave a whole fabric by pressing the treadle, inserting the weft strand, and repeating the two steps until she completed all the rows in the fabric. Figure 16 shows a complex fabric woven on a Jacquard loom.

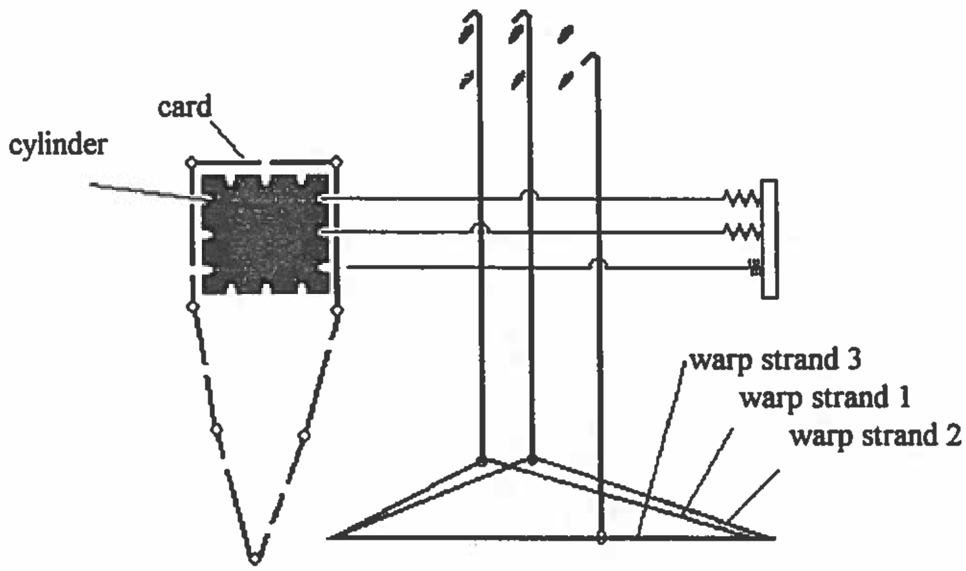


Figure 15 Jacquard Mechanism. Adapted from Adrosko (1983, p. 13).

Figure 16. Example of Jacquard Fabric. "Hansel and Gretel" based on a painting by Wunsch. (Adrosko, 1983, p. 51).

A weaver could potentially port any weave draft from a loom with any fixed number of harnesses to the Jacquard loom. Each square in a row mapped to a position on the card. If the row of the drawdown had a black square in a given position, the weaver punched a hole at the corresponding position of the card. The weaver could then string the cards together in the order in which the corresponding rows appear in the drawdown.

The loom input and executed a sequence of instructions that were stored in binary notation on punched cards, then output a desired product. Early computers performed the same operations, but this coincidence is not so amazing when we look at the origins of computers. Jacquard inspired one of the early pioneers in computing, Charles Babbage. Babbage was a mathematician who spent most of his life in the early 1800s trying to create machines to do automatic computation, including the Analytic Engine, part of which he based on Jacquard's ideas. His Analytic Engine design included an operator (which acted like a control unit in today's computers) to automatically process instructions on punched cards. The machine could solve a wide range of numerical problems. Countess Ada Lovelace, who worked closely with him in organizing the instructions, described the Analytic Engine thus: "We may say that the Analytic Engine weaves algebraic patterns just as the Jacquard Loom weaves flowers and leaves." (Schneider and Gersting, 1995, p. 189)

Babbage's ideas led the way for today's computers. Electronic computers emerged in the 1940s, and following in the late 1950s, computers began to use magnetic core memory for storage. Ideas from weaving influenced the organization of memory, which was like a plain weave fabric made of wire. In her article for the 1969 ACM

National Conference, Janice R. Lourie wrote: "Today many fabrics are woven of substances other than yarn, such as wire and other conducting media. These fabrics may have over 50 layers and the problem of merely expressing them is a significant one" (Lourie, 1969, p. 192). Some circuit wiring systems were woven. Chapter III covers weave algorithms that she describes in her article to help express and control the production of these new fabrics. In the 1960s, computers were advanced enough to make contributions to the field of weaving and return the favor.

Technological Advancements in Weaving

Many weavers do not have access to Jacquard looms, and instead usually use multi-shaft floor looms. On such a loom, the creation of fabric, from start to finish, is a long and labor-intensive process. The weaver plans a design, selects a weave, and drafts a sample of it on graph paper. Then the weaver threads the loom, ties up the treadles to the harnesses, and proceeds to weave the sample row by row. From the sample, the weaver can determine if any flaws exist in the design, such as extra long floats or uneven edges. The sample also provides an accurate model of the texture, balance, color, pattern, and structural integrity of the finished fabric. The weaver then changes the draft to accommodate necessary changes and expands the draft to represent the entire finished cloth. To weave the finished cloth, the weaver again threads the loom (and ties up the treadles again if any changes exist in the tie-up), and weaves the cloth row by row, following the draft. When finished weaving, the weaver releases the warp strands from the loom at the ends of the fabric and makes finishing touches to the fabric.

Errors in threading and treadling are common since weavers are human and the process of following the draft requires great concentration. If a weaver detects an error in treadling while part way through weaving, she must “unweave” the cloth row by row, unraveling the cloth until she eliminates the flaw, and then “reweave” the cloth. If a weaver detects an error in the threading once she has begun weaving, she might not be able to correct the error; if she detects it before weaving, she might be able to move, remove, or insert warp strands to correct the error, but that is also difficult. If she can’t correct the error, the flaw will be visible as a stripe that runs the entire length of the cloth. Designers who want to decrease the chance of weaver error can design cloths with small sections of threading and treadling that repeat so the weaver has less to handle and can memorize the small repeats.

The invention of the *dobby device* took over the task of remembering the complex lifting sequence and lifting the harnesses. A dobby loom needed no treadles, except perhaps one treadle that instructed the dobby device to change to the next row in the lifting sequence. The device read instructions from punched plastic tape or a wooden chain with holes in it; if a hole was present in a particular position, the device raised the harness that corresponded to that particular position. It resembled the Jacquard loom device, but offered less freedom of design since it could not control individual warp strands. This device controlled the loom based on preset instructions (as the Jacquard loom device did), freeing the worker's hands and requiring less concentration. The necessity for preset instructions limited the weaver's ability to experiment dynamically on the loom. When computers later entered the picture, the dobby device was adapted to take in a computer's electronic output, which replaced the plastic tape and wooden chain, so that the weaver could change the lifting order quickly and easily. The weaver could plan the lifting instructions at each step if she so desired, and in this way regained the ability to experiment with the fabric design while on the loom.

Some of the first computer programs that aided the weave designer, especially in the textile industry, arose in the 1960s. These programs took in a designer's sketch or drawing, calculated a weave structure and drawdown for the fabric, and output the design in some form that the weaver or the loom could use. If the picture contained shaded areas, the computer identified the shade and assigned a preset weave structure to the shaded area. The designer did not have much choice in the matter, nor freedom of design. The computer required many computations to satisfy requirements for the weave structure (such as a bounded float length). This type of limited interaction with the computer was called "off-line" design because the designer had to prepare the drawing and the weaves were pre-configured in the program before the computer executed the program and output the results.

At 21st National Conference for the ACM in 1966, Janice R. Lourie, John J. Lorenzo, and Abel Bomberault from IBM presented a new kind of software solution for textile designers in their article "On-line textile designing." The designer still created a drawing and scanned it into the computer before the program began, but because the solution offered "on-line" design, the designer could make changes to the drawdown and select from a variety of weaves to use as fillers for the shaded areas while the program was executing. This software let the designer use a "light pen" and cathode ray tube (CRT or rather a computer monitor) to make adjustments and design decisions, particularly at the boundaries between areas with different weaves. The designer could even create a new weave for the program library by generating a new weave or combining existing weaves. Figure 17 shows some example output from this software.

Such programs may have helped the textile industry, but they did not reach the home weavers. Weavers in general were resistant to the new computer aided technology. Some weavers used spreadsheet programs to store drafts (using a 0 to represent a weft running over a warp strand and a 1 to represent a warp strand running over a weft strand), and some even used computers to perform matrix multiplications in design drafting (Chapter III discusses these applications in detail), but weavers did not really make use of computers as a tool until user-friendly software arrived.

From most weaver's perspectives, the most important advantage that computers offer them is speed. Computers do not necessarily speed up the weaving process, rather they speed up the design process. Software programs now allow the weaver to work out a drawdown from a threading, tie-up, and treadling almost instantly, or even derive a threading, tie-up, and treadling for almost any drawdown. The weaver can use a mouse to input part of a draft or tweak existing drafts. The computer can generate many common

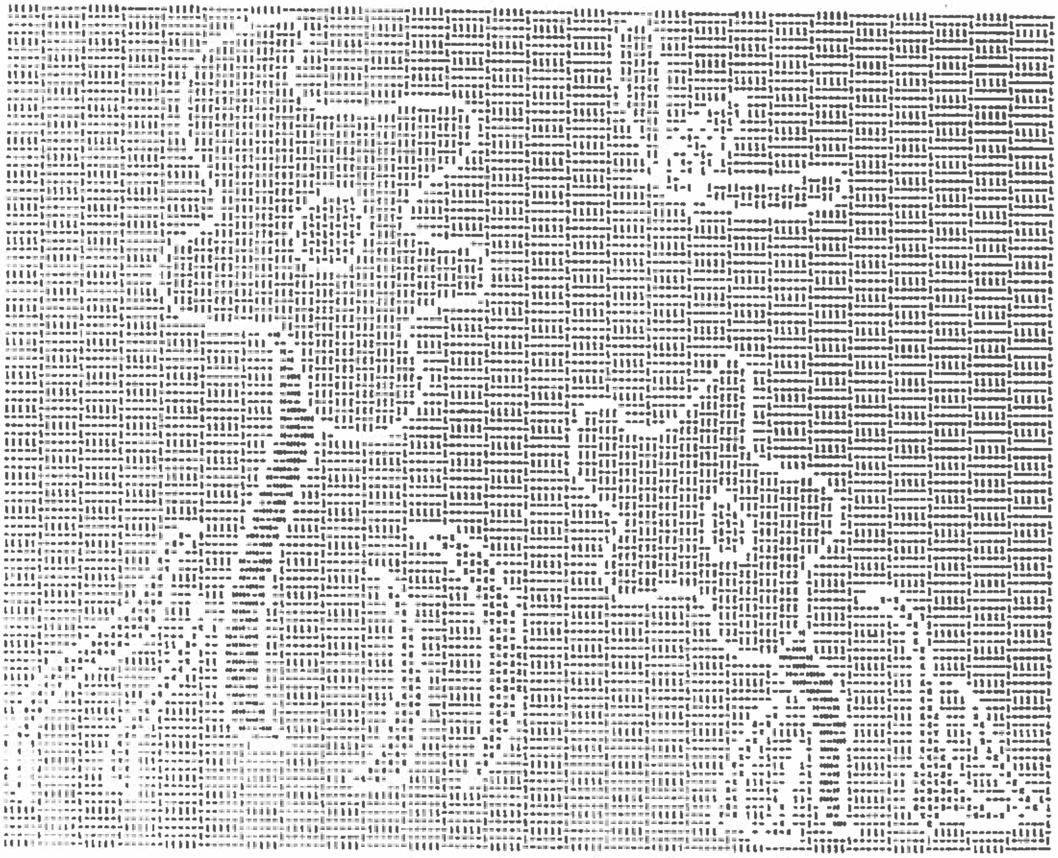


Figure 17. Output from an On-line Textile Design Program. (Lourie, Lorenzo, and Bomberault, 1966, p. 543).

weaves as a base for the design process; for instance, programs will generate a basic twill structure and then let the weaver experiment with the design. Most of these activities that the software performs are activities that weavers can do with pen and paper, but it would take the weavers much longer to do them than the computer does. The extra time allows the weaver to experiment with more designs and try more new ideas. In addition, the computer can reduce the number of errors, and saves the weaver from wasted time correcting problems while weaving on the loom.

Many programs attempt to represent the finished fabric, and render the visual image, in a more realistic way than just a drawdown. Programs now take into account color, sett, and thickness of materials. Advanced systems for computer aided design can render very realistic images of cloth including qualities such as texture and draping properties, but most weavers don't require such technology. These programs save time and costs because they allow the designer to sort of see the finished product before weaving the fabric. With the computer's help to render the image and also to check the weave for structural qualities, the weaver may not even need to weave a sample (or as many samples as usual), thereby wasting fewer of the weaver's resources. However, samples are still helpful in examining a fabric's tactile qualities and springiness, and weavers still weave them.

A handful of weavers use their computers and software programs to design in new and creative ways. The famous fashion designer Jhane Barnes uses her computer, along with a team of mathematicians, to create new patterns and designs for her fabrics. She has found ways to incorporate fractals and other geometric designs into her fabrics. She uses some experimental programs, and often *algorithms* that her mathematicians create, to create new and random designs. When the program generates a pattern or weave that interests her, Jhane Barnes might tweak the pattern or combine it with familiar weaves, or a great variety of creative ways to incorporate the new design into her next fabric. No computer can replace the designer, but it can augment the designer's creativity and ability to experiment.

The Jacquard loom was a contribution from weaving to computer science. In the 20^{th} Century, computer science was able to offer in return a contribution to weaving in the form of computer-loom interfaces and weave design software. Chapter IV introduces a new contribution to weaving from computer science. The next Chapter deals with the mathematics of weaving, some of which makes possible these weave design software programs. With such fundamental connections between weaving and computer science as this chapter presented, the future will likely hold still more connections between the two fields.

CHAPTER III

THE MATHEMATICS OF WEAVING

Any weaver who has ever tried to create a draft for a fabric must have a sense of the inherent mathematical nature of the technical part of the design process. She unknowingly performs operations from linear algebra. The binary nature of warp and weft interaction allows for mathematical description of cloth and the weaving process. Mathematically inclined weavers and interested mathematicians have studied many aspects of cloth in a variety of academic fields of mathematics. Janet A. Hoskins (1982) writes in her introduction:

> One of the major contributions of the twentieth century to hand-loom weaving can be the formalization and classification of the concepts and processes which weavers have developed and used intuitively for thousands of years.

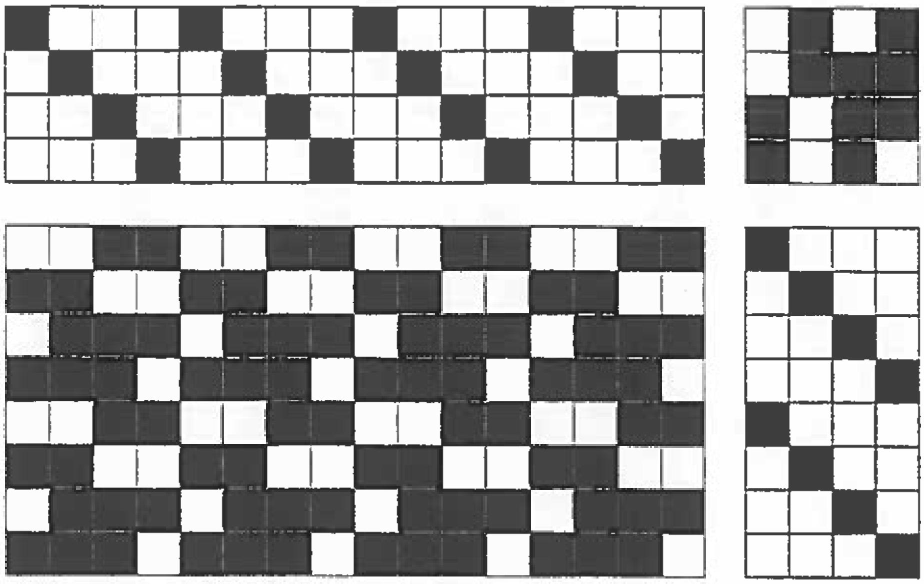
The focus of this chapter is the past efforts towards this formalization and classification of the concepts and processes in the field of weaving, particularly in an academic forum. The capabilities of current weave software show that some knowledge of weave algorithms must exist in some form, but some perhaps only in industry. Ideally, an academic forum allows for elegant or formal descriptions of weave structure and process. Several areas of mathematics become involved in weaving here, including linear algebra, Boolean algebra, combinatorics, number theory, and many others.

Mathematical Definition of Cloth

Each paper seems to describe fabric and the process of weaving in a slightly different way, but the basic idea is this: Fabric is the collection of interlacements of two orthogonal (or perpendicular) sets of parallel strands (the warp and weft) in the same plane; weaving is the process of interlacing the warp with the weft. Note that fabric could also have more than two sets of strands and they could cross at angles that were not right angles. For instance, a fabric could have warp strands in a spoke formation, and weft strands that wrap around like a spiral, running over and under the warp spokes; or it could have three sets of strands in a triangular formation. This work, however, will consider only fabrics with perpendicular warp and weft, and more specifically fabrics that can be woven on a loom with a fixed number of harnesses.

A fabric's structural representation can be a *matrix* of zeros and ones (see Figure 18). In Chapter II, a drawdown with black and white squares represented a fabric. By creating a matrix that has the same number of rows and columns as the drawdown, with zeros instead of white squares and ones instead of black squares, we map a drawdown to a matrix. Similarly, we can convert the threading, tie-up, and treadling instructions into matrix form by this process. For most looms, these instructions will yield binary matrices. As Janet Hoskins points out (1982), some special looms have harnesses that can hold in three positions: up, down, and neutral (whereas most looms only have up or down, or up and neutral). For such looms, the tie-up would need to be ternary, rather than binary, in

Drawdown



Matrix Representation

$$\begin{bmatrix} 1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0 \\ 0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0 \\ 0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1 \end{bmatrix} \quad \begin{bmatrix} 0&1&0&1 \\ 0&1&1&1 \\ 1&0&1&1 \\ 1&0&1&0 \end{bmatrix}$$

$$\begin{bmatrix} 0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1 \\ 1&1&0&0&1&1&0&0&1&1&0&0&1&1&0&0 \\ 0&1&1&1&0&1&1&1&0&1&1&1&0&1&1&1 \\ 1&1&1&0&1&1&1&0&1&1&1&0&1&1&1&0 \\ 0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1 \\ 1&1&0&0&1&1&0&0&1&1&0&0&1&1&0&0 \\ 0&1&1&1&0&1&1&1&0&1&1&1&0&1&1&1 \\ 1&1&1&0&1&1&1&0&1&1&1&0&1&1&1&0 \end{bmatrix} \quad \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix}$$

Figure 18. Matrix Representation of Fabric.

order to represent the instructions for which positions the harnesses should take when the weaver presses a given treadle. In this work, a matrix D will represent the drawdown, R will represent the treadling, T will represent the tie-up, and H will represent the threading.

These instruction matrices (R, T, and H) have certain properties, depending on the type of loom for which they work. The threading matrix H has one and only one 1 in each column (i.e., each column is a unit vector), for most looms configurations; but as Chapter I mentioned, some looms have long-eyed heddles so that each warp strand may go through more than one heddle, in which case the threading matrix might have more than one 1 in each column, but it must have at least one 0 in each column (otherwise the warp strand would be never interlace with the weft). The matrix H has dimensions h rows by n columns, where $h > 1$ is the number of harnesses and $n > 1$ is the number of warp strands.

The treadling matrix R usually has only one 1 in each row (unit row vectors); but weavers may use a *skeleton* tie-up and treadling and press two or more treadles simultaneously, so that each row of R may have more than one 1 in it, and again at least one 0. The skeleton tie-up and treadling allows the weaver to have a greater number of uniquely-patterned rows in the finished fabric, since it can accommodate more combinations than the number of treadles (on common looms, the number of treadles is often far less than the total number of possible combinations of harnesses). The matrix R has dimensions of m rows by t columns, where $m > 1$ is the number of rows in the treadling sequence and $t > 1$ is the number of treadles that the weaver uses.

The tie-up matrix T usually has more than one 1 in each row and column, since it allows for more patterned rows than there are harnesses. The matrix T has dimensions of h rows by t columns, where $h > 1$ is the number of harnesses (as above) and $t > 1$ is the number of treadles in use (as above). The algorithms and mathematical descriptions below apply to matrices H with only unit vector columns and matrices R with only unit vector rows, though they work for all tie-ups T, and may not work for other H and R.

Note also that the threading and tie-up diagrams number the harnesses from the bottom row to the top row, while the threading matrix H and tie-up matrix T number the rows 1 through h from top to bottom. The treadling and tie-up diagrams and matrices R and T all number the treadles and columns 1 through t from left to right. The drawdown D has dimensions of m rows by n columns for n and m as above, numbered from left to right and top to bottom. The columns of H and the rows of R are numbered in the same way as the drawdown D.

Weave Design Algorithms

When creating a fabric draft, weavers may begin with any of the four components of the draft. If a weaver knows the drawdown for a fabric, she can figure out the threading, treadling, and tie-up. If a weaver has a threading set on her loom, she can experiment with different tie-ups and treadling sequences to see how they affect the drawdown. A weaver may also want to use a basic or common threading and treadling (such as the ones in Figure 18) and vary the tie-up, then derive the drawdown. The process of deriving a drawdown from the other three parts is a bit easier than the inverse process (deriving the three parts from a drawdown). Because of the well-defined operations of the loom, the drawdown is predictable. A given drawdown, however, may have many possible threadings, tie-ups, and treadling sequences.

Let us say the weaver wants to derive the drawdown from the other three parts. She starts at the first row of the treadling, finds a column in that row that contains a black square and follows it up to a corresponding column above it in the tie-up (see Figure 19). For each row containing a black square in that tie-up column, she follows the row across to the threading. For each such row, if a black square appears in a column of that row, she fills in with black color that column entry square of the first row in the drawdown. She then repeats this process for each consecutive row in the treadling, corresponding to each consecutive row of the drawdown.

Another way of describing this process is to say that the weaver multiplies the first row of the treadling matrix R by the *transpose* of the tie-up matrix T to get a row vector v that has h entries, then multiplies v by the threading matrix H to get the first row of the drawdown matrix D. (Note: this multiplication performs the same function as a logical OR of the rows that correspond to the harnesses that are raised.) The weaver repeats this for each row in T. The whole process amounts to multiplying R by the transpose of T (denoted T^T), then multiplying that product by H to get the entire drawdown matrix D (see Figure 20). Since computers handle matrices and matrix operations well, this description of drafting lends itself well to computer algorithms.

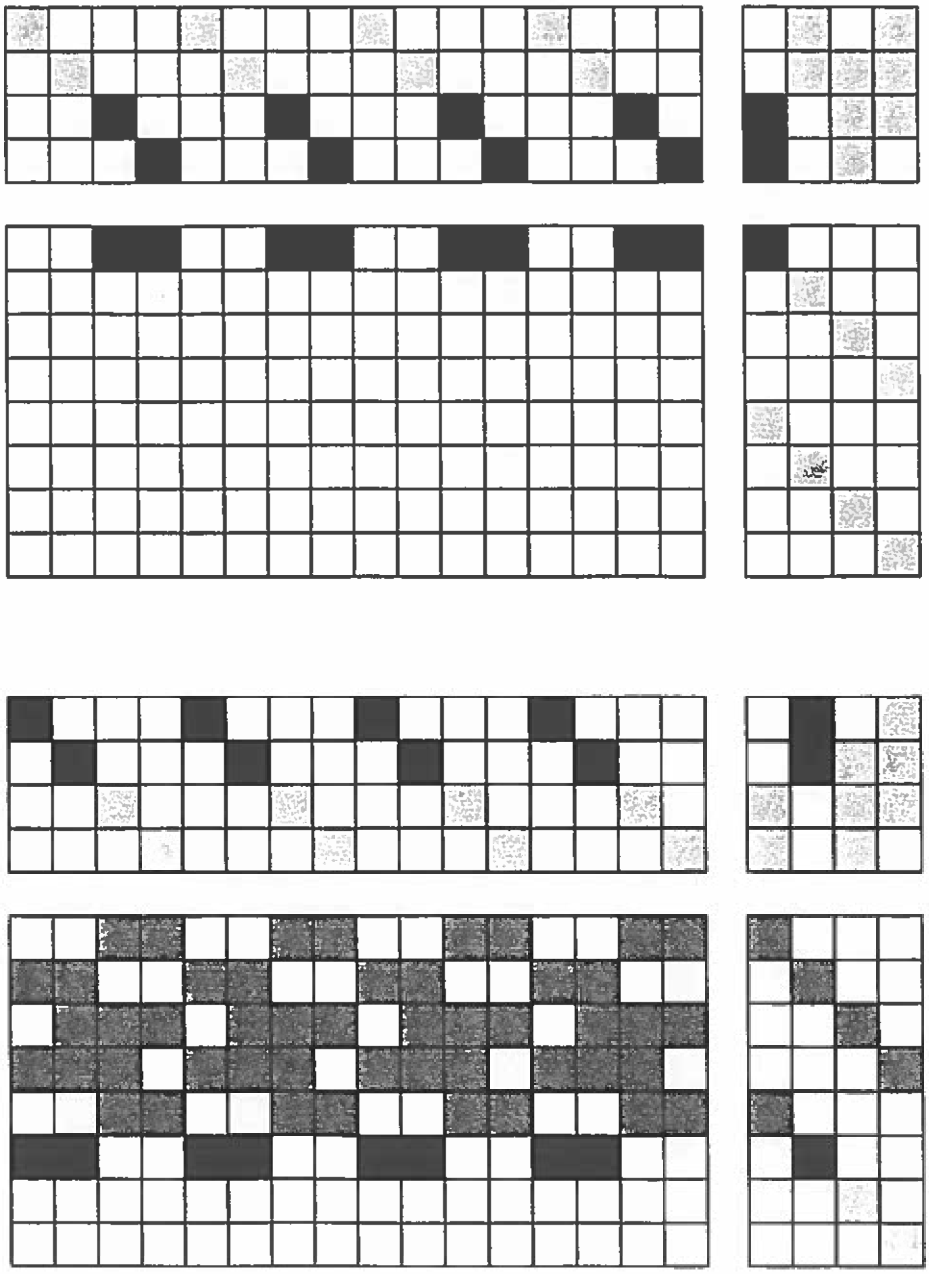


Figure 19. Constructing the Drawdown Row by Row.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Figure 20. Constructing the Drawdown by Matrix Multiplication. This diagram shows an intermediate step to produce a matrix L. $RT^{T} = L$ and $LH = D$ so $RT^{T}H = D$. Note that the matrix L is a horizontally mirrored image of the lifting matrix that would indicate dobby loom instructions. It is the mirror image because the harnesses in the threading are numbered from bottom to top and the entries in the lifting matrix are read from left to right.

Carrie Jane Brezine (1994) wrote her thesis on the linear algebra involved in weaving. She claimed and formally proved that the equation $RT^{T}H = D$ is true for matrices defined as above. Since the loom operates in predictable ways, she could show that for each interlacement (i.e., each entry in D), the loom created an equal corresponding element in the left-hand matrix via the matrix multiplication $RT^{T}H$ which occurs when the weaver performs the treadling sequence indicated by R and weaves in the weft.

Janice Lourie (1969) noted the relationship between the threading H, drawdown D, and a lifting matrix L which could serve as instructions for a loom with a dobby device. Dobby looms don't require a tie-up and treadling sequence, only a lifting sequence where each entry of a row corresponds to a harness (numbered from left to right in decreasing order if the threading is numbered from bottom to top, and vice versa). The lifting matrix has more than one 1 in each row since it represents which harnesses are raised to produce each shed (or row of the drawdown). The lifting matrix is simply the product of the treadling matrix (R) and transpose of the tie-up matrix (T^{T}).

Often a weaver will have some warp left over on a loom after weaving a project. That warp has the same threading as the previous project. Rather than wasting the warp, she might use it to weave a sample of some new design. In this situation, she already knows the threading matrix, and she can vary the tie-up. First the weaver could look at all the possible uniquely-patterned drawdown rows that are possible as combinations of the threading rows, noting which combination of the harnesses corresponds to the patterned-row by making a corresponding combination row (see Figure 21 for an example on a

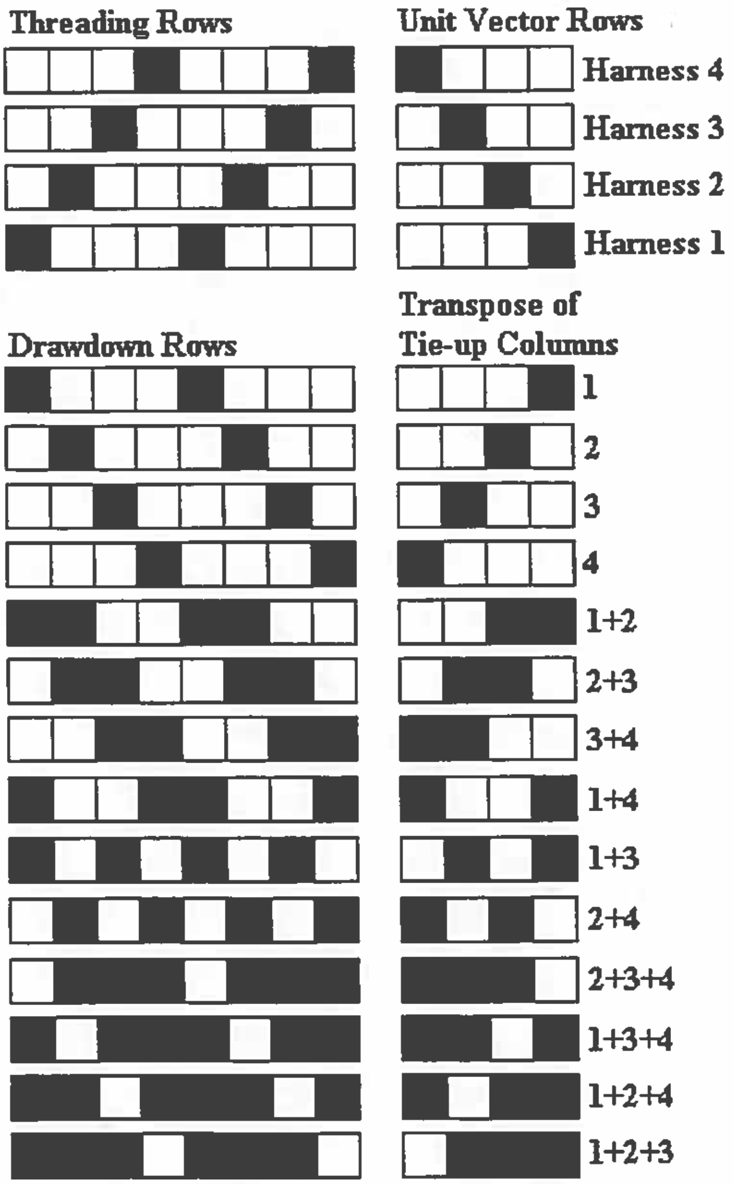


Figure 21. Designing from the Threading. All the possibilities for uniquely-patterned rows on a four-harness loom with the given threading above appear with their corresponding unit vectors that are transposes of tie-up columns.

four-harness loom) Each combination row is actually the transpose of a tie-up column for a four-harness loom and also a row of the lifting matrix for a dobby loom. Then she can experiment with those uniquely-patterned rows, make duplicates, and arrange them in different orders to make different patterns. The tie-up will become the transpose of the set of unique harness combination rows and the treadling will become a sequence of the unit row vectors that correspond to the appropriate columns in the new tie-up for each uniquely-patterned row. (For a dobby loom, the sequence of combination rows will become the lifting matrix.) As the weaver designs the cloth, she creates the tie-up and treadling sequences (or lifting matrix for a dobby loom).

Deriving the whole draft from the drawdown is more difficult, since it involves the reverse process of matrix multiplication. Several descriptions for deriving *R, T*, and *H* from *D* exist in the current literature (e.g., Sutton, 1982; Hoskins and Hoskins, 1981 and 1983; Brezine, 1994) , since weavers often want to replicate a fabric for which they can determine the drawdown but have no instructions for how to weave it on their looms. Lourie (1969) provides a description for deriving *R* and *L* for a dobby loom given *D*.

As Brezine (1994) points out, every drawdown *D* is weaveable, i.e., for every *D* we can find some *R,T,* and *H* that weave *D*. Let *H* be an *n* x *n identity matrix*, *R* be an *m* x *m* identity matrix, and *T* be equal to D^T. These instructions will lead to the drawdown *D* (see Figure 22). But in most cases, the drawdown *D* will not require *n* different harnesses nor *m* different treadles--this is fortunate since most looms to not have as many harnesses as the number of warp strands nor as many treadles as the number of weft strands.

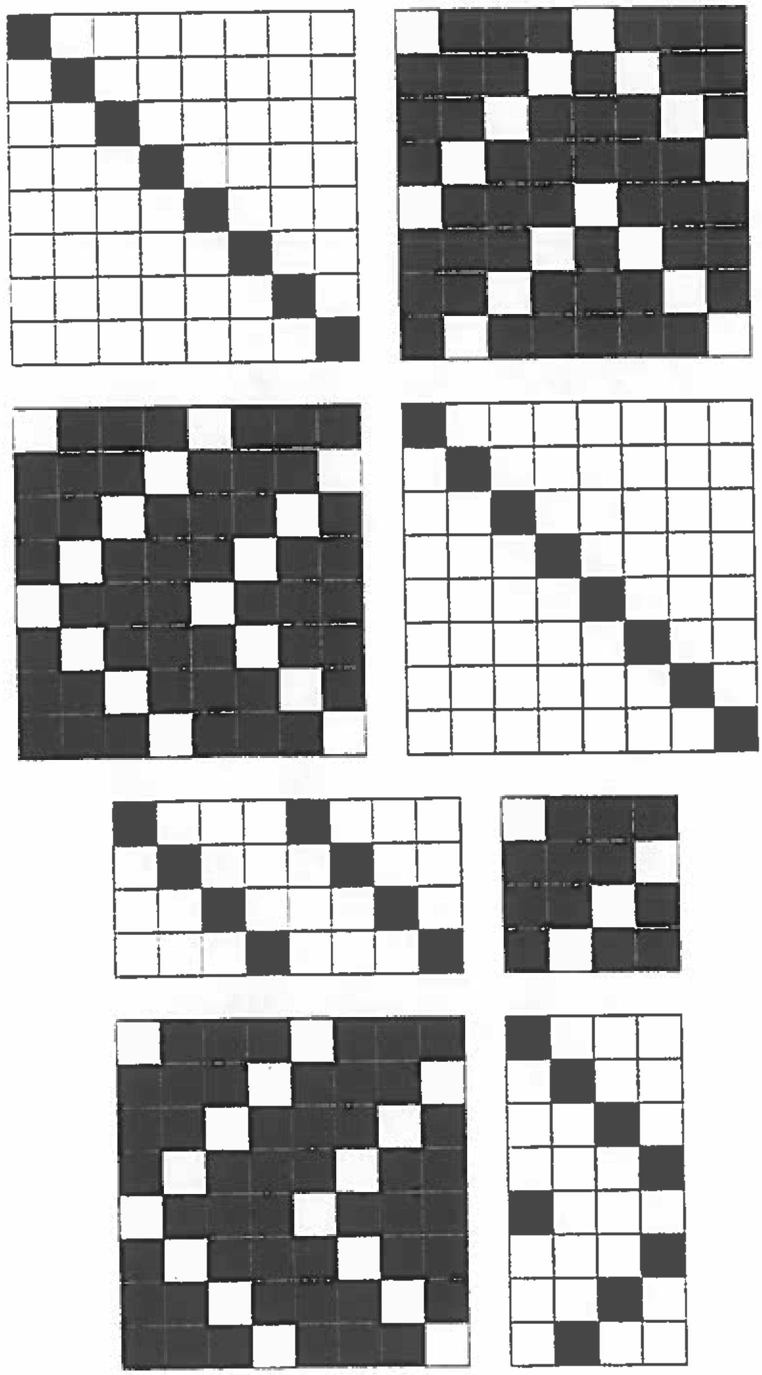


Figure 22. Minimal Draft. A possible draft and a minimal draft for the given draw-down. Note that the top draft allocates more harnesses and treadles than is necessary, while the bottom draft allocates the minimum number necessary.

We can minimize the threading and treadling so that they take up only as many harnesses and treadles as necessary. Notice that the tie-up matrix *T* has repeated columns and rows. The harnesses with repeated rows and the treadles with repeated columns are extraneous and we can "throw them out" or more precisely, we can do the following: For each row that appears more than once in *T*, combine the duplicate occurrences together with the first one by performing a logical OR of those entire rows of the draft (corresponding rows of *R* along with the rows of *T*) with the entire draft row of the first occurrence and then remove the duplicate rows from the draft. For each column that appears more than once in *T*, combine the duplicate occurrences together with the first one by performing a logical OR of those entire columns of the draft (columns of *H* along with the columns of *T*) with the entire draft column of the first occurrence and then remove the duplicate columns from the draft. At the end of this process, the new matrices *H,R,* and *T* will be minimal (Brezine, 1994) and *T* will have distinct rows and columns.

Another way of deriving *H,R,* and *T* is to follow the algorithm that Hoskins and Hoskins proposed (1981). It finds the minimal *H* and *R* and then applies linear algebra to determine *T* from the equation $RT^{T}H = D$. The process below determines *H*:

(1) Set j = 1, r = 1.

(2) Take the jth column of *D*, say, D_j, if it has at least one non-zero value in it, otherwise execute step 6, without incrementing r.

(3) If D_j is precisely equal to the kth column of *D* (k = 1, 2,..., t) then assign the kth position in the vector V the value 1, otherwise assign it a zero value.

(4) The vector V becomes the rth row of the threading matrix.

(5) Any column of D that corresponds to a 1 in V is assigned to a column of zeros.

(6) $J \leftarrow J + 1$, $r \leftarrow r + 1$ and if there are non-null columns left the process repeats from Step 2.

To determine the treadling, repeat the process with D^{T} and transpose the resulting matrix. The tie-up T is equal to the matrix product $((\sim R^{T})D(\sim H^{T}))^{T}$ where ~ denotes the *complement*. An example of this algorithm in process appears below.

Take the drawdown matrix D equal to the following:

$$\begin{bmatrix} 0&0&1&1&0&0&0&1&1&1&0&0&1&1 \\ 0&1&1&0&1&0&1&1&0&1&1&0&0&1 \\ 1&1&0&0&1&1&1&0&0&0&1&1&0&0 \\ 0&1&1&0&1&0&1&1&0&1&1&0&0&1 \\ 0&0&1&1&0&0&0&1&1&1&0&0&1&1 \\ 1&1&0&0&1&1&1&0&0&0&1&1&0&0 \end{bmatrix}$$

We start with the first column (j = 1 and r = 1). We find that it is precisely equal to columns 6 and 12 so the first row of the threading will be: [1 0 0 0 0 1 0 0 0 0 0 1 0 0]. We replace columns 1, 6 and 12 with zeros. The drawdown matrix now looks like this:

$$\begin{bmatrix} 0&0&1&1&0&0&0&1&1&1&0&0&1&1 \\ 0&1&1&0&1&0&1&1&0&1&1&0&0&1 \\ 0&1&0&0&1&0&1&0&0&0&1&0&0&0 \\ 0&1&1&0&1&0&1&1&0&1&1&0&0&1 \\ 0&0&1&1&0&0&0&1&1&1&0&0&1&1 \\ 0&1&0&0&1&0&1&0&0&0&1&0&0&0 \end{bmatrix}$$

Now j = 2 and r = 2. Since there are non-null columns left we look at column 2 and find that it is precisely equal to columns 5, 7 and 11 so the second row of the threading will be: [0 1 0 0 1 0 1 0 0 0 1 0 0 0]. We zero out columns 2, 5, 7, and 11. Now j = 3 and r = 3.

Since there are non-null columns left we look at column 3 and find that it is precisely equal to columns 8, 10, and 14 so the third row of the threading is equal to the following: [0 0 1 0 0 0 0 1 0 1 0 0 0 1]. Again we zero out the equal columns and now let j = 4 and r = 4. Since there are non-null columns left, we look at column 4, which is precisely equal to columns 9 and 13 so the fourth row of the threading is [0 0 0 1 0 0 0 0 1 0 0 0 1 0]. Now all the columns are null so we stop. The threading matrix is:

$$\begin{bmatrix} 1&0&0&0&0&1&0&0&0&0&0&1&0&0 \\ 0&1&0&0&1&0&1&0&0&0&1&0&0&0 \\ 0&0&1&0&0&0&0&1&0&1&0&0&0&1 \\ 0&0&0&1&0&0&0&0&1&0&0&0&1&0 \end{bmatrix}$$

We repeat the process for the treadling with D^T and get this result (transposed):

$$\begin{bmatrix} 1&0&0&0&1&0 \\ 0&1&0&1&0&0 \\ 0&0&1&0&0&1 \end{bmatrix}$$

The tie-up matrix $T = ((\sim R^T)D(\sim H^T))^T$ which is:

$$\begin{bmatrix} 0&0&1 \\ 0&1&1 \\ 1&1&0 \\ 1&0&0 \end{bmatrix}$$

In their 1983 article, Hoskins and Hoskins present a faster algorithm for this process that works well for computers but does not work well for hand computations. In fact, looking at the previous algorithm, we see how costly it is. A computer running that algorithm on a m x n drawdown D would have to first check each row to see if it contains all zeros (a total of m comparisons per row) and then compare all the entries in each non-zero column with the corresponding entries in the rest of the columns, which could even include some all-zero columns. The matrix multiplication to find the tie-up will cost $2mpq$ multiplications. The proposed faster algorithm makes some shortcuts that cut down the number of comparisons. It organizes the columns in buckets based on the first element where two columns differ, which saves some comparisons. When finding columns that are the same, instead of filling them with zeros, the algorithms "identifies" them, so we don't have to compare each entry of the columns with zero. In addition, we can reduce the drawdown to only its unique columns before transposing it and finding the treadling. This cuts down the number of comparisons in each row from n to q.

Note that the minimal number of harnesses is equal to the number of distinct columns in D and the minimal number of treadles is equal to the number of distinct rows in D (as long as the requirement that each row has only one 1 holds). Lourie (1969) gives a short proof by contradiction showing that the number of harnesses is greater than or equal to the number of distinct columns. If we assume that the number of harnesses is less than the number of distinct columns, then two distinct columns must correspond to strands on the same harness; but any two strands on the same harness must correspond to identical columns because of the way the loom behaves, which refutes our assumption. Therefore the number of harnesses cannot be less than the number of distinct columns.

Note also that *permutations* of the draft rows and columns preserve the relationship between all parts of the draft (see Figure 23), provided the entire row (including both threading and tie-up) or entire column (including both tie-up and treadling) are permuted. A weaver could take an existing draft and permute rows or columns of the

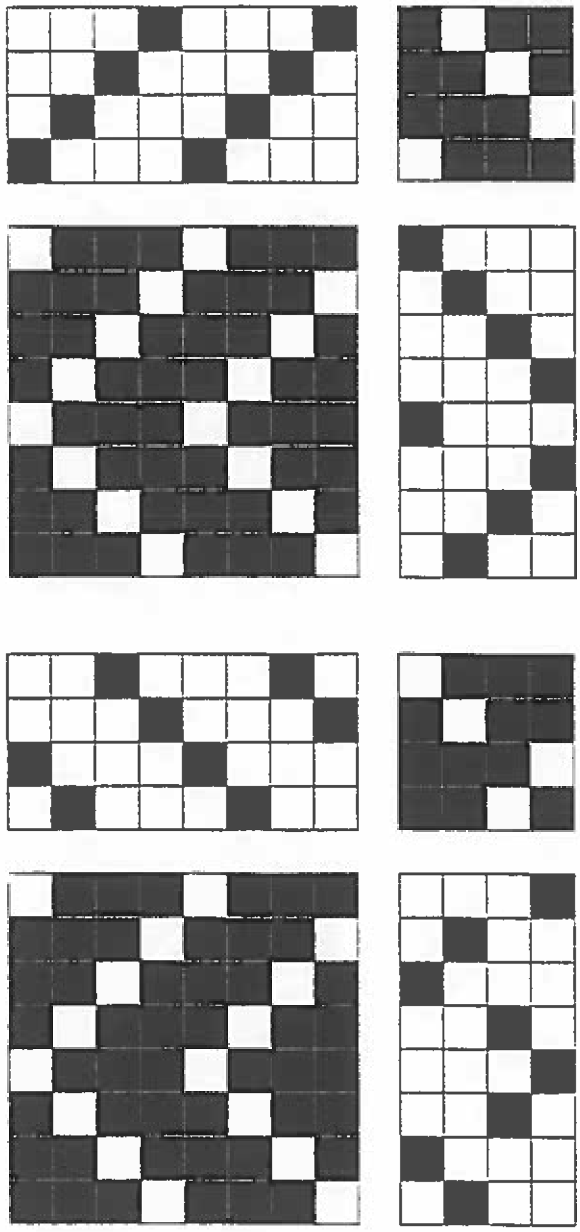


Figure 23. Permutations of *R*, *T*, and *H*. The top draft is a permutation of the draft in Figure 22, with the rows of *H* and *T* permuted. The bottom draft is a permutation of the same draft with the rows of *H* and *T* and the columns of *T* and *R* permuted.

draft that include the drawdown, thus generating a new fabric from an old one, and the draft would have all four parts. That would be an easy way to create new fabrics without having to derive any parts of the draft. Permuting the rows or columns of the draft also preserves the layering properties of the fabric (i.e., if the fabric is a single layer, then permutations will retain that fact; if the fabric has *n* layers, permutations will retain that fact as well). This property of the draft is important in the algorithm for determining whether or not a fabric hangs together, which is described below.

The drafts often show a *minimal block* or an even number of minimal blocks of a *periodic* fabric. For instance, a draft of plain weave may contain only a minimal 2 x 2 block, though the fabric has far more than 2 warp strands and 2 weft strands. Plain weave fabric has two distinct minimal blocks (1 0 / 0 1 and 0 1 / 1 0). For a given periodic fabric, more than one *m* x *n* minimal block exist. A fabric that has only a horizontal period of *n* has up to *n* different minimal blocks, which are all circular translations (i.e., translations that wrap around) of the columns. A fabric that has only a vertical period of *m* has up to *m* different minimal blocks, which are all circular translations of the rows. A fabric that is periodic in both directions has up to *mn* minimal blocks, which are all circular translations of both the rows and the columns. The following are some examples of 4 x 4 minimal blocks of the same fabric:

1 0 0 0	0 1 0 0	0 0 1 0	0 0 0 1	1 1 0 0
0 1 1 0	0 0 1 1	1 0 0 1	1 1 1 0	0 0 1 0
0 0 0 1	1 0 0 0	0 1 0 0	1 0 0 0	1 1 0 1
1 1 1 0	0 1 1 1	1 0 1 1	0 1 1 0	0 0 0 1

A fabric that *hangs together* is a fabric that is single-layered, i.e., the fabric cannot be divided into two sets of strands such that all the strands in one set lie on top of all the strands in the other set. Another word used to describe a fabric that hangs together is *irreducible*; a fabric that does not hang together is *reducible* (Hoskins, 1982). A randomly generated drawdown *D* may or may not hang together. As Clapham (1980) phrases it, "It may happen that a set of weft strands and a set of warp strands, themselves interwoven, can be lifted off the remaining sets of interwoven weft and warp strands." The fabric could even have several layers. Figure 24 shows some examples of fabrics that don't hang together. Some of the literature on weaving (e.g., Lourie, 1969; Kurtz, 1981) requires only that for a fabric to hang together, each row and column must have at least one 1 and one 0. While this guarantees that each strand will interlace with the set of strands that cross it, it does not guarantee that the fabric will hold together in a single layer. All the fabrics except for the top right fabric in Figure 24 have at least one 1 and one 0 in every row and column. Some of the fabrics even ensure at least two 1s and two 0s in each row, yet they also fall apart.

Grünbaum and Shephard (1980) introduced the concept of a fabric that hangs together and posed the problem in a mathematical context, noting a few designs that split into two sets of strands, one of which passed completely over the other. Clapham (1980) developed the idea and wrote an algorithm for determining whether or not a fabric hangs together. Clapham showed that whether or not a binary matrix represents a fabric that hangs together depends on the set of row-sums and column-sums, a dependency that is

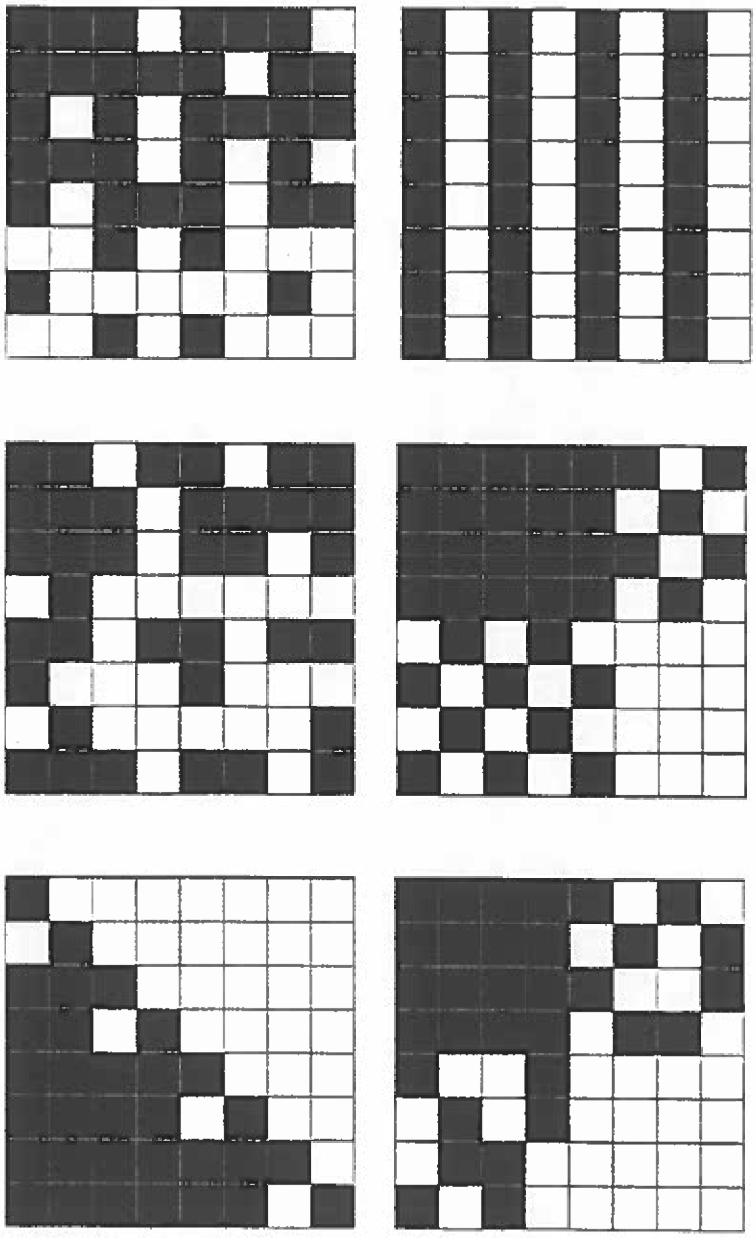


Figure 24. Some Fabrics that Don't Hang Together. The upper right fabric is a bunch of warp stands lying on top of the weft strands, with no interlacement at all. The other fabrics on the right and the bottom left are organized to show the separate white and black blocks. The top and middle left fabrics don't appear as if they would fall apart at first glance. The bottom left fabric has four different layers each of 2 x 2 plain weave.

not intuitive. Let $D = (d_{ij})$ be an m x n matrix of zeros and ones. The following are necessary and sufficient conditions for a set of weft strands S and a set of warp strands T to lift off of the remaining strands:

(1) The only warp strands that go over strands of S belong to T and

(2) The only weft strands that go over strands of T belong to S.

Those are intuitive, but these less intuitive conditions are equivalent to those above:

(1) $d_{\sigma j} = 0$ for all σ in S and j not in T and

(2) $d_{i\tau} = 1$ for all i not in S and τ not in T.

This results in the requirement that the matrix must be able, upon permutation of the rows and columns, to take this form (Brezine, 1994):

$$\begin{matrix} a & b \\ c & d \end{matrix}$$

where *b* is a matrix of zeroes, *c* is a matrix of all ones, *d* is a matrix representing a fabric that hangs together, and there is no restriction on *a*. For a black and white drawdown this would mean that rows and columns could be rearranged to show an all-black block and an all-white block in opposite corners such that the inside corners of the two blocks meet (see especially the middle right drawdown in Figure 24).

So how do the row- and column-sums fit in? Let r_i be the sum of the entries in the i-th row of *D* and c_j be the sum of the entries in the j-th column of *D*. Let also S have s elements and T have t elements (note that T and t are different from the tie-up matrix *T* and number of treadles *t* from earlier in this work). The necessary and sufficient conditions from above are also equivalent to the following equality:

$$\sum c_\tau - \sum r_\sigma = t(m\text{-}s) \quad \text{for all } \tau \in T \text{ and all } \sigma \in S$$

That is, the columns sums of the warp strands of T - the row sums of the weft strands is equal to the number of warp strands in T times the number of weft strands not in S if and only if the columns of T have all ones except for the shared elements of rows of S which may be 0 or 1 and the rows of S have all zeros except for the same shared elements of columns of T, which is precisely when the matrix can be arranged into blocks of zeros and ones as above (with the shared elements making up the matrix X). Note that when s = m and t = n, or when s = 0 and t = 0, the fabric has only one layer; hence we rule out those cases. Note also the equality holds in the following example of a reducible fabric (two permutations shown):

1	.	.	1	1	.	1	.
1	.	.	1	1	.	1	.
*	0	0	*	*	0	*	0
1	.	.	1	1	.	1	.
*	0	0	*	*	0	*	0
*	0	0	*	*	0	*	0
1	.	.	1	1	.	1	.
*	0	0	*	*	0	*	0

*	*	*	*	0	0	0	0
*	*	*	*	0	0	0	0
*	*	*	*	0	0	0	0
*	*	*	*	0	0	0	0
1	1	1	1	.	.	.	.
1	1	1	1	.	.	.	.
1	1	1	1	.	.	.	.
1	1	1	1	.	.	.	.

where "*" represents an element that is in both a row of S and a column of T and "." represents an element in a row not in S and a column not in T. Now assign the row and column sums such that $c_0 \geq c_1 \geq \cdots \geq c_n$ (decreasing order) and $r_0 \leq r_1 \leq \cdots \leq r_m$ (increasing order). Now we want to find s and t (if they exist)such that $c_0 + c_1 + \cdots + c_n - (r_0 + r_1 + \cdots + r_m) = t(m\text{-}s)$. We define a function E(s,t) = (the right hand side) - (the left hand side).

Then, by rearranging terms we get the equation (denoted *):

$$E(s,t) = r_1 + r_2 + \cdots + r_t + (m - c_1) + (m - c_2) + \cdots + (m - c_s) - st$$

When $E(s,t) = 0$, the fabric falls apart into a set of s weft strands and t warp strands and a set of m-s weft strands and n-t warp strands. Note that the row sums $r_1,\ldots, r_t$ are the t smallest row sums and the column sums $c_1,\ldots,c_s$ are the s largest column sums. This allows us to determine an algorithm that searches for the smallest s and t such that $E(s,t) = 0$, and if no such s and t exist (except $s = m$ and $t = n$) then the fabric hangs together. We can also note that $E(s,t) - E(s-1,t) = r_s - t$. Looking at the behavior of $E(s,t)$, we see that $E(s,t) \geq 0$ and when $r_s < t$, $E(s,t)$ decreases as s increases, and when $r_s \geq t$, $E(s,t)$ increases. Hence $E(s,t)$ will equal zero only at some minimum (if it does at all) and that will occur at the largest s such that $r_s < t$ for some t. The algorithm appears below (Clapham, 1980):

> If $r_1 = 0$, take $s = 1$ and $t = 0$ and the strand corresponding to the row with row-sum r_1 can be lifted off.
>
> If not, for each $t = 1,\ldots,n$, find the largest s such that $r_s < t$ (the r_s are increasing) and evaluate the corresponding $E(s,t)$ using formula (*). If any of these is equal to zero (excluding $E(m,n)$) then the strands corresponding to rows with row-sums $r_1,\ldots,r_s$ and columns with column-sums $c_1,\ldots,c_t$ can be lifted off. Otherwise the fabric hangs together.

Let's apply this algorithm to the top left drawdown in Figure 24. The drawdown is 8 x 8 so $m,n = 8$. The row sums are (in increasing order) 2,2,2,5,6,6,6,7 and the column-sums are (in decreasing order) 7,7,6,6,3,3,2,2. First, r_1 is not zero, so we go on for $t = 1$. Since no r_s is less than 1, we have $s = 0$ and $r_0 = 0$ by default so $E(0,1) = (8-7)$ which is not equal to zero. We go on to $t = 2$, and we have $s = 0$ again and $E(0,2) = (8-7) + (8-7)$ which is not equal to zero. For $t = 3$, the largest s such that $r_s < t$ is 3 so we check

E(3,3) = 2 + 2 + 2 + (8-7) + (8-7) + (8-6) - (3)(3) = 1. Since E(3,3) is not equal to zero, we move on to t = 4. Again, s = 3, so we evaluate E(3,4) = 2 + 2 + 2 + (8-7) + (8-7) + (8-6) + (8-6) - (3)(4) = 0. Hence the strands corresponding to the first three row-sums and the first four column-sums can be lifted off (those are rows 6,7, and 8 and columns 1,3,5, and 7).

Sometimes weavers purposefully design a two-layer or multi-layer fabric. Lourie (1969) introduced a method for representing three-dimensional fabrics on a two-dimensional drawdown. Since the general case for multi-layer fabrics includes the case for two-layer fabrics, I include here only the general case. Start with a separate drawdown for each layer of fabric (viewed from above) and arrange them in order of their layers, with D^i representing the drawdown for the i-th layer back. Create a matrix of the following form:

$$\begin{matrix} D^1 & 0 & 0 & \cdots & 0 \\ 1 & D^2 & 0 & \cdots & 0 \\ 1 & 1 & D^3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & D^n \end{matrix}$$

where all the entries below the D^i are matrices of ones and all the entries above the D^i are matrices of zeros. The drawdown in the lower left corner of Figure 24 is in this form, with a 2 x 2 block of plain weave for each layer. Then permute the columns and rows so that they are distributed evenly over the fabric. Please see (Lourie, 1969) for the mathematical description of how to distribute them using permutation matrices. Lourie provided this solution for how to represent multi-layer fabrics particularly so that industrial fabric designers would be able to represent them in an organized fashion.

Janet Hoskins has also researched the mathematical nature of multi-layered fabrics (Hoskins, 1983a, 1983b). She provides mathematical descriptions of how to represent several layers in a cross-sectional diagram (similar to Figure 13). Hoskins (1983b) introduces a "new classification system based on a mapping of the point diagram to a cross-sectional representation of the corresponding fabric." The point diagram she refers to is a typical drawdown. She refers to the cross-sectional diagram as a "sectional drawdown."

In addition to single-layered fabrics and reducible fabrics with distinct layers, Hoskins and others have worked to classify multi-layered structures that have some stitching or interlacement between them (e.g., some weaves have two different layers, but they alternate on the face and back of the weave, others only have stitches to hold the layers together in one piece). Though these additional structures hang together, they have distinct layers in which the warp and weft strands always remain in the same layers; sometimes these fabrics have large sections in which the layers do not interlace at all. Multi-layered fabrics have many practical uses (e.g., fabrics in which the middle layer is waterproof or has some important physical quality and the outer layers serve visual purposes), so a knowledge of their mathematical properties and classification can be quite relevant. Hoskins (1983b) particularly worked to classify each weft interaction with the multiple layers of warp. The interaction is no longer a binary choice, because the weft could go over or under a warp strand in any of the n layers. Hoskins developed a system to define the interactions mathematically.

Combinatorial Questions and Geometry

One of the most interesting and popular areas in the mathematics of weaving is the area of combinatorial questions about fabrics. The combinatorial questions often go hand in hand with a study of the geometric properties of certain kinds of weave structures. Chapter I introduced some of the simpler questions, such as "How many different sheds are possible on a loom with h harnesses and t treadles?" We might also ask, "How many different m x n drawdowns exist?" The answer to that question depends on whether or not we consider translations, reflections, or rotations of a pattern to be different drawdowns.

Combinatorics is about "counting without counting"--or rather finding a way of systematically counting or generating all the possible items that fit a requirement without actually counting or examining every single one exhaustively. For instance, we could try to answer the question "How many m x n drawdowns exist?" (assuming that permutations of a drawdown are different) by trying to draw all the drawdowns we can think of, or we could reasons thus: each drawdown contains mn squares, each of which could be either white or black, therefore we have two choices for each square, resulting in a total of 2^{mn} possible drawdowns.

A large number of works in circulation examine combinatorial questions about loom constraints, bounded float length, properties of twills and satins, and reducibility (e.g., Grünbaum and Shephard, 1980; Hoskins, Praeger, and Street, 1983a, 1983b;

Hoskins, Stanton, and Street, 1983; Hoskins, Stanton, and Street, 1984; Hoskins, Street, and Stanton, 1984; Delaney, 1986). Two particular concepts are central to discussion of the above topics. (1) Twills can be represented as a cyclic binary sequence or as a sequence of floats, and (2) Some fabrics can be classified as *mononemal* or *isonemal* (see below).

Because twills are periodic and because each row is a translation of the previous row, we need only know one row of a twill to represent it. We can represent that row in a cyclic binary sequence in this way: start at one element in the drawdown row and work through one repeat, let the first element of the sequence be a 0 if the weft crosses over the warp and a 1 otherwise, and let each of the following sequence elements correspond to elements in the drawdown row. Weavers often use another notation that represents the weft floats. For instance, the following are all representations of the same twill:

$$\frac{\quad 1 \quad 3}{1 \quad 2 \quad}$$

$$s = (0,1,0,0,1,1,1)$$

$$\begin{array}{ccccccc}
0 & 1 & 0 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 1 & 1
\end{array}$$

Grünbaum and Shephard (1980) determine the number of distinct twills of a given period n. Hoskins, Praeger, and Street (1983b) determine the number of n-harness twills with a given maximum float length. They also classify twills according to maximum float length and the number of breaks (when the sequence switches from 0 to 1 or from 1 to 0) in the twill. They also determine (1983a) the number of balanced n-harness twills (a balanced twill has an equal number of zeros and ones).

Hoskins (1982) gives an overview of some geometric properties of fabrics that often lead to appealing designs, and have mathematical interest:

> Rows are considered to be *equivalent* if one can be changed into the other by complementation, reversal, cyclic rotation, or any combination of these. *Mononemal fabrics* have rows and columns which all have equivalent interlacement sequences and *simple isonemal fabrics* are ones in which, in addition to having rows and columns with equivalent interlacement sequences, use a constant rule to obtain each row from the previous one. The relationship between each column and its neighbours must also be constant and the same as for the rows.

As she points out, the above characteristics are desirable in fabrics because they often give fabrics a balance that leads to even tension in the fabrics. Research into these topics has produced new classes of fabrics and given weavers some formal guidelines for constructing desirable fabrics.

Grünbaum and Shephard (1980) prove that a twill corresponding to a binary sequence with at least two pairs of distinct neighbors is an isonemal fabric. They also examine the geometric properties of satins. Define an (n,s)-satin as a fabric with period n and a step of s units (i.e., a row is the same as the previous row moved s units to the right). Then they prove that an (n,s)-satin is isonemal iff $(s^2 \pm 1) \bmod n = 0$. They also determine the number of distinct isonemal (n,s)-satins for a given n.

The issues above are only a few of the combinatorial questions and answers that have been researched. Such questions are of interest to both mathematicians and weavers because the answers lead to new and interesting fabric structures (and also visual designs that have the same appearance as the structures) that have desirable qualities and because they lead to new classifications of fabrics.

The 20th Century brought a variety of contributions from the field of mathematics to weaving, each of which helped weavers design fabrics. When mathematicians and weavers joined together to formally define the mathematics implicit in functions of the loom and the relationship between the four parts of the draft, the results increased the weave designer's productivity. New algorithms helped the designer derive missing parts of a draft more accurately and efficiently than the previous hand methods. The algorithm that determines whether or not a drawdown represents a fabric that hangs together allows weavers to experiment with new drawdown designs and check before weaving whether or not the fabric will be a single layer. Research on the combinatorial questions and geometric properties of certain weaves has produced new classifications of weaves, so that the designer can make a more informed decision in selecting a weave that is appropriate for a desired finished product. These connections create more connections as people seek further classifications and try to improve the algorithms.

CHAPTER IV

NEW IDEAS

This chapter contains my ideas (unless otherwise identified), which are original, to the best of my knowledge. The work here expands on the current knowledge (from Chapter III) and poses new problems in a mathematical form.

Graphical Representation of Fabric

The previous chapters showed common ways of representing fabrics. In this chapter I will explore a new way of representing fabric, a *fabric graph*. Graphs in computer science and mathematics provide a theoretical representation of a data in the form of a set of *vertices* and a set of *edges* that connect the vertices. Edges are usually represented by a pair of vertices (u,v) such that the edge goes from *vertex u* to vertex v. Many computer science problems are defined in terms of graphs, and because of this, a large number of algorithms exist that work with graphs. To visualize a graph, consider the vertices as points, circles, or spheres and consider the edges as line segments or curves (or in some cases arrows) that run from vertex to vertex. The graph definitions here are based on those of the text *Introduction to Algorithms* (Cormen, Leiserson, and Rivest, 1990).

A fabric graph F is a *directed graph,* i.e., the edges are ordered pairs, they have direction, they leave one vertex and enter another, and can be represented by an arrow from one vertex to another. The fabric graph F has vertices that represent warp or weft strands and edges that represent the interaction between warp and weft (i.e., warp over weft or weft over warp). The edge (u,v) represents the interaction of the strand u passing over strand v. (See Figure 25 for some fabrics and their fabric graphs.) Note that if an edge (u,v) exists, then the edge (v,u) cannot exist because a strand simultaneously passing under and over the same strand is not well defined in a fabric. Note also that we cannot have any edges (u,u) since a strand passing over itself is not well defined; and we cannot have any edges $(w1,w2)$ such that $w1$ and $w2$ are both in the set of warp strands or both in the set of weft strands since the warp and weft are at right angles in the fabric.

A fabric graph F is then by definition necessarily *bipartite,* i.e., we can partition the vertices into two sets V_1 and V_2 such that all edges (u,v) imply that either u is in V_1 and v is in V_2 or u is in V_2 and v is in V_1. For a fabric graph, those two sets are the set of warp strands and the set of weft strands. (Note that the term bipartite usually describes undirected graphs, but the same concept applies here for this directed fabric graph.) The graph is also *simple*, i.e., it is a directed graph with no self-loops, because no edges (u,u) can exist in the graph.

For a fabric graph F of an m x n drawdown, each warp vertex will interact with all m weft vertices, so the *degree* of any warp vertex in F is equal to m; similarly, the degree of any weft vertex in F is equal to n. The *in-degree* of any vertex is the number of strands that pass over it and the *out-degree* of any vertex is the number of strands that pass under it. Naturally, the degree of a vertex is equal to its in-degree plus its out-degree.

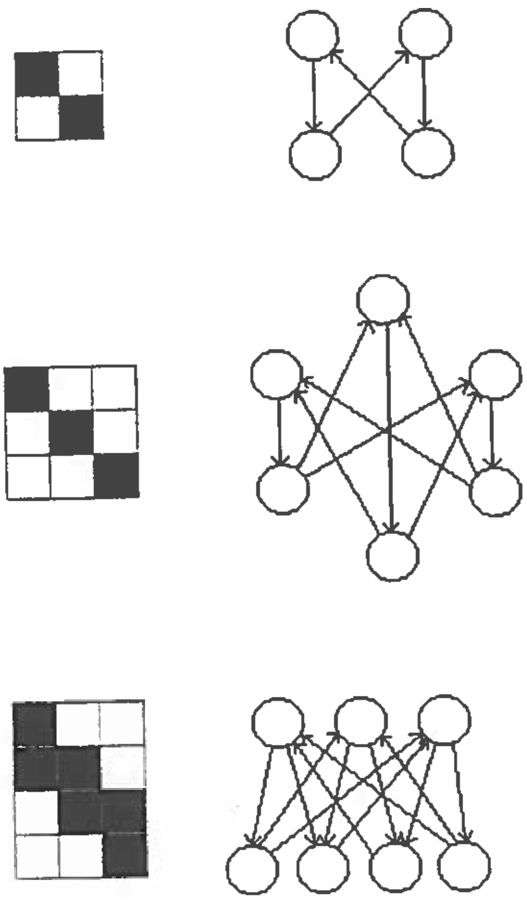


Figure 25. Fabrics and their Fabric Graphs. The warp strand vertices appear at the top of the graph, ordered from left to right as the strands are ordered in the drawdown. The weft strand vertices appear at the bottom of the graph, ordered from left to right as the strands are ordered from top to bottom in the drawdown.

Furthermore we can say that a vertex is *balanced* if its in-degree is equal to its out-degree. Since a vertex represents a fabric strand, a balanced vertex represents a strand that is equally distributed on the face and back of the cloth. Because balance in a fabric is desirable (often to avoid tension problems), a fabric graph with balanced vertices is desirable. Note that a row-sum is equal to the in-degree of that row's corresponding vertex, and a column-sum is equal to the out-degree of that column's corresponding vertex.

Graphs can hold data with their vertices and edges. Edges have *edge-weights* and vertices can have associated variables. We could store the degree, in-degree, and out-degree in each vertex, as well as any other data we want to store, such as the color, thickness, texture, identity, warp/weft membership, and other properties of the corresponding strand. If the fabric is multi-layered, each vertex could contain the identity of the layer that contains it. Edge-weights could possibly hold the color of the strand showing in the face of the fabric at that interlacement, but that is not necessary, since we could obtain this information at any time by noting the color of the vertex from whom the edge is incident. Each vertex must at least have an identity, which in the fabric graph for an m x n drawdown will be a number. A warp strand vertex will have an identity j, such that $1 \leq j \leq n$; a weft strand vertex will have an identity i, such that $n+1 \leq i \leq n+m$.

Graphs generally store their data structure in an adjacency-list or an adjacency-matrix. An adjacency-list is a list of vertices, and each vertex u keeps a list of all the edges (u,v). An adjacency-matrix is an n x n matrix, for n equal to the number of vertices. The (i,j) entry of the adjacency-matrix holds a value (usually one or zero) to indicate the presence or lack of an edge (i,j), or in some cases the weight of the edge (i,j). This particular adjacency-matrix would be very sparse for a fabric graph (since there are no warp-to-warp or weft-to-weft edges and half of the warp-to-weft and weft-to-warp edges cannot exist). However, the drawdown itself is an adjacency matrix that represents the edges, and it is more compact. For normal graphs, the adjacency matrix has a value for each edge, including both (i,j) and (j,i), but in our drawdown matrix, the (i,j) entry is equal to 1 if the edge (j,i) exists and is equal to 0 if the edge (i,j) exists. So we can use our more compact drawdown matrix to store the edge information for the fabric graph.

A very convenient property of fabric graphs is that we can easily see whether or not a fabric graph represents a fabric that hangs together and how many layers it has. To determine whether or not a fabric graph represents fabric that hangs together, we can run Clapham's (1980) algorithm, with slight modifications to make it fit the graph. We noted above that a warp vertex out-degree equals the warp's column-sum and the weft vertex in-degree equals the weft's row-sum. The in-degree of a weft vertex equals the (total) degree - out-degree, which equals n - out-degree for an m x n fabric. Hence we only need to calculate and store the out-degree for each vertex. In Clapham's algorithm, the column-sums were assigned in decreasing order and the row-sums were assigned in increasing order. Here we can arrange them both in decreasing order of out-degrees, and when we need the row-sum, we calculate n - out-degree. Define a function $d(u)$ = the out-degree of vertex u to make the notation easier. Now we can run the following modified version of Clapham's algorithm on our fabric graph:

(1) Find d(u) for vertices $u = 1$ through $u = n$ and assign the column-sums = d(u) in decreasing order to $c_1, c_2, \cdots c_n$. Find d(u) for vertices $u = n + 1$ through $u = n + m$ and assign the row-sums = m - d(u) in decreasing order of d(u) to $r_1, r_2, \cdots r_m$.

(2) Then if $r_1 = 0$, the strand with the row-sum r_1 can be lifted off.

(3) If not, for each $t = 1,\ldots,n$, find the largest s such that $r_s < t$ and evaluate E(s,t) = $r_1 + \cdots + r_s + (m - c_1) + \cdots + (m - c_t) - st$. If any of these equals zero (except E(m,n)) then the strands corresponding to the row-sums $r_1, \cdots r_s$ and the column-sums $c_1, \cdots c_t$ can be lifted off. Otherwise the fabric hangs together.

That algorithm is not the only algorithm we can use to determine whether or not a fabric graph represents a fabric that hangs together. An alternative algorithm takes advantage of *strongly connected components* in a graph. A strongly connected component is a subset of the graph in which all vertices in the graph are "reachable from" the other vertices via some path (the path consists of edges and vertices in the graph). A *strongly connected graph* is a graph that is itself one strongly connected component. For instance, the entire top graph in Figure 25 is strongly connected and itself a strongly connected component because the four vertices are in a cycle and each vertex can reach the others by traveling the path of the cycle. If we can show that iff (if and only if) the entire graph is a strongly connected component, the fabric represented by the graph hangs together, then we can apply an algorithm that detects strongly-connected components to determine whether or not the fabric hangs together. Note that one layer passes completely over the others if all edges from top layer to bottom layers have the same direction.

THEOREM 1. *A fabric graph represents a fabric that hangs together if and only if the graph is strongly connected. Otherwise it falls apart into two or more pieces.*

The following *lemma* (Delaney, 1986, p. 74) will be useful in our proof:

LEMMA. *Two rows (or columns) of a drawdown are in the same layer iff there is a circuit consisting of alternate 0 and 1 corners, to which both rows (or columns) contribute at least one corner.*

PROOF of THEOREM 1: We begin by applying the lemma to a fabric graph. Weft vertices represent rows of the drawdown and warp vertices represent columns of the drawdown.

A circuit consisting of alternate 0 and 1 corners in the drawdown corresponds to a circuit in the fabric graph since edges go from a warp vertex to a weft vertex at a "1 corner" and go from that weft vertex to a warp vertex at the next corner (which is a "0 corner") and so on around the circuit until the last edge connects the weft vertex with the starting warp vertex.

Furthermore, any circuit in a fabric graph corresponds to a circuit of alternate 0 and 1 corners in the drawdown because the graph is bipartite with all edges going from a weft to a warp vertex or vice versa and so the circuit must alternate 0 and 1 corners. Hence we can say that two weft vertices (or warp vertices) are in the same layer iff there is a circuit connecting those vertices in the fabric graph (denote this statement as **).

Now let us assume that the graph is strongly connected. [We want to show that the fabric hangs together.] Hence every vertex is reachable from every other vertex. Hence any two row vertices or column vertices are reachable from each other. Since row or column vertices cannot connect to another vertex of the same type directly, each must go through at least a small circuit of one vertex of the other type. Hence any two row or column vertices are connected by a circuit and by (**) the row or column vertices are in the same layer. Hence all row vertices are in the same layer and all column vertices are in the same layer. Because the graph is strongly connected, each vertex in the row layer is reachable from all vertices in the column layer and vice versa, so the two layers interlace. Hence the vertices all form one layer and the fabric hangs together.

Now let us assume that the fabric hangs together. [We want to show that the fabric graph is strongly connected.] Hence the rows are all in the same layer, the columns are all in the same layer, and the two layers interlace at one or more points. Hence by (**), any two row or column vertices are connected by a circuit, and hence all the vertices of the same type are reachable from each other (strongly connected). Hence, if any row vertex is reachable from any column vertex or vice versa, then all the vertices are reachable from each other. And we know that this must happen for at least one row vertex or column vertex because the two layers interlace at one or more points. Hence all vertices are reachable from each other and hence the graph is strongly connected. ■

Now we need an algorithm that determines whether or not a graph is strongly connected. Cormen, Leiserson, and Rivest (1990) provide an algorithm to accomplish this (using depth-first searches) that runs in $\Theta(mn)$ time (see Appendix A for *Θ-notation*). Compare this with the time it takes for the modified version of the Clapham algorithm (which is the same for Clapham's original algorithm). In setting up the row- and column-sums alone, we perform *mn* operations, and then sort the sums in decreasing or increasing order (which we will say takes $\Theta(n + m)$ time); we could calculate the sums of the row-sums $r_1 + r_2 + \cdots + r_s$ for $s = 1$ to m and the values of $(m - c_1) + \cdots + (m - c_t)$ for $t = 1$ to n ahead of time, each sum adding one term to the previous sum for a total of $\Theta(m + n)$ operations; then during step 3, in a loop $\Theta(n)$ times, we find the largest s such that $r_s < t$ (the search can be done in $\Theta(\log m)$ comparisons with a binary search) and calculate $E(s,t)$ (which takes constant time). Hence an informed implementation of Clapham's algorithm runs in $\Theta(mn)$ time, which is the same as our new graph algorithm. The new graph algorithm, however, performs different operations and avoids calculating E(s,t), so for some applications it may be easier to use the fabric graph and check for strongly-connectedness. Note that any algorithm to determine whether or not a fabric hangs together necessarily takes at least $\Theta(mn)$ time since the size of the input is *mn.*

The graph algorithm (using depth-first searches) not only determines if the graph is strongly connected; it determines the number of strongly connected components. When two strongly connected components appear in a fabric graph, all the edges between the two components must have the same direction (or else there would be a cycle connecting the two components, making them one strongly connected component). Since the edges between the two components are all the same direction, all the vertices in one component are over all the vertices of the other component or vice versa, and hence the two strongly connected components represent two different layers of the fabric. A similar argument applies for more than two components, and so by determining the number of strongly connected components in the fabric graph, we determine the number of layers in the fabric it represents.

For instance, the graph for the top right drawdown in Figure 24 has four warp vertices with all edges going out and four warp vertices with all edges coming in, and each weft vertex has in-degree equal to out-degree. The fabric falls apart into 16 pieces (one for each strand), as does the graph. A smaller example with four warp and four weft vertices appears in Figure 26. Note that having at least one vertex with all incoming or all outgoing edges is a sufficient condition but not necessary condition for the reducibility of the fabric (i.e., a graph having at least one vertex with all incoming or all outgoing edges represents a fabric that cannot hang together, but it is not true that a graph with no such vertices will represent a fabric that hangs together).

Another interesting fact in graph theory is that two graphs are *isomorphic* if we can map every vertex in one graph to a vertex in the other graph (for instance, by reassigning vertex identities) while preserving the edge relations. For fabric graphs, this means that when two fabrics have the same minimal block or are translations or permutations of the same drawdown, then their fabric graphs are isomorphic.

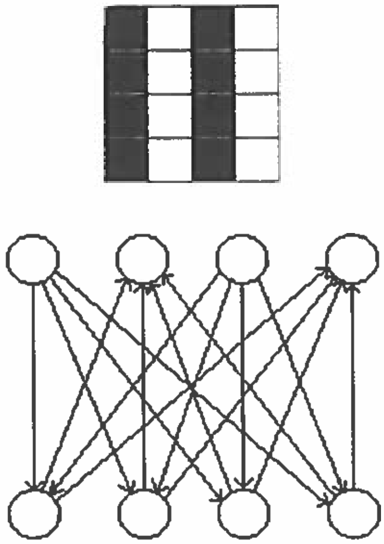


Figure 26. A Fabric Graph that Falls Apart. Notice that each vertex is its own strongly connected component because no other vertices both reach and are reachable from each vertex. Even with only one vertex with all incoming edges or all outgoing edges, that vertex would cause a dead end in any cycles so the graph would necessarily represent a fabric that does not hang together.

Before leaving the topic of fabric graphs, an outline of the advantages and disadvantages of representing fabrics with graphs will be useful. The disadvantages may already be obvious. The graphs are hard to represent on paper, and in that case the drawdowns are more convenient and illustrative at first glance. Moreover, large graphs are exceedingly difficult to represent on paper because each vertex must interact with all vertices that are not of the same kind (warp vs. weft). Fabric graphs also make it more difficult to determine the floats and maximum float lengths of warp and weft, in comparison to a drawdown. Since the graph vertices have a collection of incoming and outgoing edges, not necessarily ordered spatially by their identities, and so the graph does not reveal at a glance which strands it passes over for a contiguous stretch.

Nevertheless, this lack of spatial ordering can help the weaver better understand the fabric's structure as it relates to other structures of its kind and in its class. The weaver can see at a glance whether or not the weave is balanced. Though the vertices each have identities that preserve the initial order of rows and columns, the weave designer could at any time ignore the identities and look at the fabric in a less restricted sense. A rendering of the graph could represent visually all the interactions of one row or column as a ring of spokes around the corresponding vertex, showing how the minimal block repeats and wraps around the borders of the conventional drawdown. The graph has many possibilities for use in modeling circular or spherical cloths where the edges of the drawdown connect (the same effect occurs when the weaver cuts out a drawdown and shapes the paper into a cylinder, or attempts to shape it into a circle).

Other advantages of the fabric graph include the ease of determining the fabric's reducibility (e.g., with the algorithm above), and the graph's capacity for storing other information about vertices and edges. People interested in the structural integrity of the fabric could write algorithms that make use of the stored information about each vertex (including its in-degree and out-degree; strand size, elasticity, strength, fiber content). In addition, these graphs could be very useful in storing information about color-and-weave effects. Because so much work in the field of graph algorithms exists, other algorithms that relate to a process in weaving or determine properties of fabrics may arise out of an analogy with graph algorithms. Weaving could bring new graph problems to the study of graphs in the field of computer science.

Color and Weave Research

When the visual design or motif of a fabric is different from its structural drawdown, color-and-weave effects occur. These effects happen when the warp and weft are not a unified color, or when "the colour and intersection representations [do] not correspond directly" (Hoskins, 1982, p. 308). Sometimes when the structural drawdown matches the visual design of a fabric, the fabric falls apart. We say that the motif is reducible. Janet Hoskins proposes (1982, p. 308):

> An area for possible future interest could well be to devise a systematic algorithm for turning reducible motifs into irreducible ones by altering the correspondence between the interlacement and colour arrays (for example, by using a striped warp and weft).

Delaney (1986) proved, however, that some reducible motifs are *essentially reducible* (i.e., they cannot be woven, even with color-and-weave, in a single-layered fabric). In particular, "If E is an essentially reducible design then it (or its transpose) has at least one white row, one black row, and the non-monochromatic rows form an irreducible design" (p. 76). So an algorithm such as Hoskins proposes would have to first check to see if the motif is essentially reducible. Another difficulty with such an algorithm is that there could be more than one way to weave that motif with an irreducible structure and a striped warp. Even if a systematic way of making a reducible motif into an irreducible one existed, the algorithm would be somewhat irrelevant unless it could produce all the irreducible structures for the given motif or the "best" one (i.e., one that conformed to certain guidelines for a structure such as maximum float length). Ideally an algorithm should produce all the irreducible structures for the motif that conform to certain guidelines. Then the weaver could choose the "best" structure for the desired finished product.

The visual design or motif only enforces the following requirements:

If the (i,j) square of the motif is white, then either a white warp passes over some color weft so the (i,j) square of the structural drawdown is black or a white weft passes over some color warp so the (i,j) square of the structure drawdown is white.

If the (i,j) square of the motif is black, then either a black warp passes over some color weft so the (i,j) square of the structural drawdown is black or a black weft passes over some color warp so the (i,j) square of the structure drawdown is white.

But the motif does not directly tell us the color of the strand under the top strand, nor which of the two possibilities (warp or weft on top) applies for each square. Since this mapping is not one-to-one, we cannot find a function that takes in only the motif and outputs the structural drawdown or vice versa. The warp and weft coloring are variables that enter into play.

We define a warp and weft coloring by a 1 x n matrix C_1 and a m x 1 matrix C_2 such that each entry of C_1 is a number or letter that corresponds to the color of that warp strand and each entry of C_2 is a number or letter that corresponds to the color of that weft strand. In addition, we say that a fabric with C_1 and C_2 has a strand coloring C.

When we know two pieces of the puzzle, we may be able to find the third. For instance, if we know the m x n structural drawdown W and the warp and weft colorings C_1 and C_2, we can derive the motif M by the following algorithm:

(1) Initialize M to the structural drawdown W.

(2) Let a matrix A be the complement of W (i.e., take the opposite of each element in W, 0 or 1).

(3) For j = 1,...,n, multiply the j-th column of M by the j-th entry of C_1.

(4) For i = 1,...,m, multiply the i-th row of A by the i-th entry of C_2.

(5) Perform the logical OR of the corresponding elements in A and M and store it in M.

At the end of the algorithm, M will contain the motif, in full color representation, provided that C_1 and C_2 were in full color representation. Whether or not this algorithm has previously been formally outlined, the basic idea involved follows the procedures that weavers perform when they work out a color-and-weave drawdown on paper.

Note for the algorithm above that when C_1 and C_2 contain values other than zero and one (for instance, letters g and y to indicate colors green and yellow) the logical OR returns the color accordingly, since we never have a situation in the algorithm in which we try to perform the logical OR of two non-zero color values (for instance, g OR y). Here is an example, with more than two colors represented:

Step 1,2:

C_1

g b g b

C_2 (transposed)

g g y b

W

1	0	0	1
1	1	0	0
0	1	1	0
0	0	1	1

A

0	1	1	0
0	0	1	1
1	0	0	1
1	1	0	0

Step 3,4:

g	0	0	b
g	b	0	0
0	b	g	0
0	0	g	b

0	g	g	0
0	0	g	g
y	0	0	y
b	b	0	0

Step 5:

finished motif M

g	g	g	b
g	b	g	g
y	b	g	y
b	b	g	b

Note also that this method works well for hand computation, but a more efficient algorithm for computers could merely assign to each (i,j) entry of M the j-th entry of C_1 (if the entry of W was a 1) or the i-th entry of C_2 (if the entry of W was a 0).

Unless we know *C* for a given fabric, we will have trouble trying to derive *W* from *M*. Even if we do know that some given *C* works for a given fabric, we may have more than one solution for *W*, since a warp and weft of the same color cannot enforce an interlacement based on color alone. If we try some random *C* for a fabric, we may not find any solutions, since a square in the motif may have the opposite color of the two strands that make the corresponding intersection in the fabric.

We could, however, easily determine all the possible *W*, given *M*, and *C*, since we could create a different *W* for each combination of decisions of unenforced warp and weft interactions. Specifically, for each interaction of a warp and weft of the same color, we could assign some rank (say, warp over weft) to get one *W* and assign the opposite rank (in this case, weft over warp) to get another *W*. Furthermore, we can apply Clapham's algorithm (or the graph algorithm if we represent each possible *W* as a graph), to the possible *W* to find all the *W* that hang together.

One approach to finding all the irreducible structures *W* and their colorings *C* for a reducible motif *M* could be to examine the possible warp and weft colorings and interactions at each square. The following approach works only for binary colorings (i.e., only two colors possible for any warp or weft strand). For instance, a black square at (i,j) in a black and white motif implies that the j-th warp strand is black or the i-th weft strand is black (or both). Furthermore, this means that if the warp strand is white, then the weft strand must be black and vice versa. We can represent this as an expression:

$$\sim C_1[j] \rightarrow C_2[i]$$

where ~ denotes the opposite color value, $C_1[j]$ denotes the color of the j-th warp strand and $C_2[i]$ denotes the color of the i-th weft strand. We may assign a similar expression for a white square at (i,j) in the motif ($C_1[j] \rightarrow \sim C_2[i]$). To find all the possible colorings C for the motif M, we find the corresponding expression for each square in M. We then find any and all solutions to the set of expressions (if it is possible to solve such a set of expressions simultaneously). Since we know these C (if any exist) must work, we are guaranteed to find at least one structure W for each C. We could then input these C and the motif M into another algorithm that could find all the possible structures W that hang together.

So far I have attempted to outline a solution to the problem of how to determine all the irreducible weave structures and warp and weft colorings for an irreducible motif. In creating this outline, I suggest approaches to solving the problem and hint at some algorithms. I hope that my research into fabric graphs and color-and-weave will open up new connections between mathematics, computer science, and weaving and inspire further research in these areas.

CHAPTER V

CONCLUSION

In January 1996, I enrolled in an introductory weaving course, an introductory computer science course, and a discrete mathematics course. At the time, I did not know how much these three fields would captivate my interest, nor how deeply interconnected they are. Over the course of the term, some of the fundamental connections leaped out at me, and in my enthusiasm to know more, I began my search for all the connections--real or imagined--between mathematics, computer science, and weaving. I have presented in the previous chapters the history and possible future of some important and interesting connections between the fields of mathematics, computer science, and weaving. In this last chapter I leave the reader with some open questions and ideas for future research.

All the areas in Chapter III have open questions that call for future work. From a computer science perspective, we could analyze the efficiency of current algorithms in weaving and try to write more efficient algorithms. From a pure mathematical perspective, we could try to answer questions about fabric graphs, and probe further into the combinatorial theory of weave structure. Most importantly, from a weaver's perspective, we can take into account the mathematical nature of weave structures to create new aesthetically pleasing or functional fabric designs; we can use the algorithms to design "good cloths" in more efficient ways.

Chapter IV provided some new approaches to solving problems from weaving by introducing a new data structure (the fabric graph) as a representation of woven cloth. An interesting topic for further research would be to look at how symmetries in fabrics relate to the corresponding fabric graphs. I suspect that a study of some kinds of isomorphic graphs might fit in nicely with the classification of isonemal structures. In computer science, researchers could create new algorithms based on the fabric graph or other algorithms in this work. Any algorithm for a fabric graph that is designed to solve a particular problem in the field of weaving might be applicable to other graph problems in computer science, and vice versa.

The color-and-weave algorithms could also be applicable to other problems in computer science. One problem I discuss in Chapter IV is the problem of finding all the possible weave structures and warp and weft colorings that produce a given motif. Future contributions to weaving from mathematics and computer science could include solving this problem and formally defining the algorithms that lead to its solution. In addition, some useful and relevant algorithms could examine the set of possible weaves and warp and weft colorings to determine which ones satisfied certain requirements that designers could set (e.g., maximum float length, balance, structure must be a twill or some other fabric type). A program that could solve these problems would allow the weaver to use color-and-weave designs that the weaver might otherwise have discovered only by a lucky guess, trial and error, or time-consuming experimentation.

The contributions in Chapter II were the results of advancements in each field. The Jacquard loom was an amazing technical feat that advanced the freedom of design in weaving. The ideas at work in the loom inspired the mathematicians who contributed to the development of computer science. Advancements in computer science led to computer software and hardware that were later applied to the field of weaving. The major future connections between these fields may arise out of research in either field.

Existing research on connections between mathematics, computer science, and weaving is often hard to find, so I direct the interested reader to Appendix B for a listing of sources that can hopefully serve as good starting points for future research. This thesis possibly only scratches the surface of the many ways to interlace the three fields. Future research in the combined fields could lead to major breakthroughs in any or all of the fields. Mathematics, computer science, and weaving are so strongly interlaced and the connections between them are so fundamental that the future promises many more connections for years to come.

APPENDIX A

GLOSSARY

algorithm - a well-defined computational procedure that takes a set of values as input and produces some set of values as output (Cormen, Leiserson, and Rivest, 1990)

balanced - when pertaining to a vertex, balanced means that the in-degree of the vertex equals the out-degree

bipartite - a graph is bipartite if the vertices can be partitioned into two sets such that all edges connect a vertex from one set with a vertex from the other set

color-and-weave - an effect that occurs when the warp or weft are not a uniform color, causing the visual design to differ from the structure of the fabric

complement - when pertaining to binary matrices, the complement is the "negative" of a matrix, i.e., the (i,j) entries of the complement are equal to the opposite of the original (i,j) entry (if the original entry is a 0, then the complement entry is a 1, and vice versa)

compound weaves - structures composed of two (or more) complete and correlative weave structures (warp and weft) that are either separate or interwoven (Emery, 1980)

degree - when pertaining to a graph, the degree of a vertex is the number of edges that include that vertex

directed graph - a graph in which all the edges have a direction; the edges are represented as arrows going from one vertex to another vertex

dobby device - a device attached to a loom that mechanically raises and lowers appropriate harnesses according to input instructions

draft - a graphical representation of instructions for how to set up the loom to weave a cloth, includes four parts: the threading, tie-up, treadling sequence, and drawdown

drawdown - a representation of all the interactions of warp and weft in a cloth, can be in a grid form or matrix edge - when pertaining to a directed graph, an edge is an ordered pair of vertices that conceptually connect the vertices

edge-weight - a data value associated with an edge

essentially reducible - a visual design or motif is essentially reducible if it cannot be woven as a single-layered fabric regardless of the coloring of the warp and weft

fabric graph - a representation of fabric in the form of a graph data structure; each vertex corresponds to a strand, and each edge corresponds to an interaction between a warp strand and a weft strand

float - a weft or warp strand that passes over a group of two or more strands; the length of a float is the number of strands passed over

floor loom - a complex loom with two, four, eight, sixteen, or thirty-two harnesses that uses treadles (pedals) to control which harnesses are lifted

hangs together - holds together in a single layer; a fabric hangs together if the strands cannot be partitioned into two sets such that all the strands of one set pass completely over all the strands in the other set

harness - a frame with one or more heddles that each hold a single warp strand so that the warp strands raise up together when the frame is raised

heddle - a string or wire with a loop in the middle through which a warp strand may be threaded; the string or wire is attached to a harness frame and cannot move independently of the frame's movements in a vertical direction

identity matrix - a binary matrix with ones in the diagonal entries from the top left to bottom right corner (entries with equal row and column indices) and zeroes in the others

in-degree - when pertaining to a vertex in a graph, the in-degree is the number of in-coming edges to that vertex

irreducible - holds together in a single layer; an irreducible fabric cannot be partitioned into two sets of strands such that all the strands in one set pass over the strands in the other set

isomorphic - two graphs are isomorphic if we can relabel the vertices of the first graph to be vertices of the second graph, maintaining the corresponding edges

isonemal - (pertaining to a fabric design) having mononemal properties and also having a constant rule that determines each row from its previous row (Hoskins, 1982)
lifting matrix -a matrix that denotes which harnesses are raised for each row of the weave

lemma - a proposition used to demonstrate another proposition (such as a proof)

lifting matrix - a matrix that denotes which harnesses are raised for each row of the weave

long-eyed heddle - a heddle with a loop or eye that is long enough to allow a warp strand held in it to be raised (by another heddle) even when the heddle is not raised

loom - a device that holds and tensions a warp

matrix - a two-dimensional structure to store data in rows and columns of entries

minimal block - the *m* x *n* block (of a design or fabric) that when translated horizontally and vertically creates the whole fabric represented by the block

mononemal - having rows or columns whose interlacements can be changed into the other by complementation, reversal, cyclic rotation, or any combination of those (Hoskins, 1982, p. 304)

out-degree - when pertaining to a vertex in a graph, the out-degree is the number of out-going edges from that vertex

overshot - a weave structure that has a tabby (or ground) weft and a pattern weft

periodic - having repeated translations of a block placed one after another in one or more given directions

permutation - rearrangement of the order

plain weave - the simplest most highly interlaced weave, with a checkerboard structure

reducible - cannot hang together; a fabric that is reducible falls apart into two sets such that all the strands in one set pass over all the strands in the other set

relatively prime - sharing no common divisors other than 1; two numbers are relatively prime if their greatest common divisor is 1 satin - a weave structure in which each row is a translation of the previous row by some number of units (greater than 1) to the right

sett - the number of strands per inch in the warp and weft

shaft - another name for a harness

shed - the V-shaped opening between two groups of warp strands created when a subset of the warp is raised away from the plane of the fabric

simple - when pertaining to a graph, simple means that the directed graph has no self-loops, no edges that go from and to the same vertex

skeleton tie-up - each treadle is tied up to only one harness, so that the weaver creates a shed by pressing one or more treadles simultaneously

strand - a string or thread, strip of paper, or length or any material including metal wire or groups of fibers

strongly connected graph - a graph that has only one strongly connected component, i.e., each vertex is reachable from all the other vertices via some path along the edges

strongly connected component - a subset of vertices in a graph that are all reachable from each other (via some path along the edges)

structure - the set of interactions between the warp and the weft of a fabric

tabby - another name for plain weave

threading - the assignment of warp strands to the harnesses, or the matrix that denotes this assignment

tie-up - the assignment of sheds (combinations of harnesses) to the treadles; a matrix that denotes this assignment

transpose - when pertaining to a matrix, the transpose is an m x n matrix is an n x m matrix with each entry (i,j) equal to the (j,i) entry of the original matrix; a matrix in which the original i-th row (for all i between 1 and n) is the i-th row of the original matrix

treadle - a foot pedal that indirectly connects (via a lam) to one or more harnesses treadling - the order of depressing the treadles; a matrix that denotes this order

twill - a weave structure in which each row is a replica of the previous row shifted one unit to the left or right

vertex - an element of a graph that may or may not be connected to other vertices by edges; conceptually represented as a point, circle, or sphere

vertices - plural of vertex

visual design - the pattern or two-dimensional arrangement of colored strands on the surface of a fabric

warp - "the warp" refers to a set of warp strands

warp strand - a strand that runs length-wise or vertically in a fabric

weave - (verb) to construct cloth by interlacing weft with warp; (noun) an interlacement of the weft and the warp

weaves with compound sets of elements - weave structures with more than one distinct warp or more than one distinct weft, such as overshot

weaving - the process of constructing cloth by interlacing a weft with a warp, usually at right angles

weft - "the weft" refers to a set of weft strands in a fabric

weft strand - a strand that runs cross-wise or vertically in a fabric

Θ-notation - a notation for describing the asymptotic behavior of a function; when a program executes $\Theta(f(n))$ operations, it executes a number of operations that is dependent on the size of the input n and that lies between two constant multiples of $f(n)$ for large enough n (i.e., the number of operations is asymptotically bound by $f(n)$)

APPENDIX B

BACKGROUND SOURCES

General Weaving Resources

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