A Multiscale Morphological Approach to Topology Correction of Cortical Surfaces

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Abstract. We present a topology correction method for automatic reconstruction of brain cortical surfaces. We take the volume-based approach by first correcting the topology of the white matter volumes followed by extracting the cortical surfaces. A multiscale method is taken so that topology errors are gradually corrected with respect to the correction cost. The special surface-likeness property of white matter and gray matter is considered in evaluating the cost of topology correction.

1 Introduction

Reconstruction of the brain cortical surface from magnetic resonance (MR) image is an important task for studying both the structure and the function of the human cortex[10]. Although human cortices are highly convoluted and fold in different ways, the cortical surface should be topologically equivalent to a sphere if the opening at the brain stem is artificially closed. Topology correctness is an important requirement in the cortical surface reconstruction process while topology defects or errors, mainly in the form of handles (see the left-top image in figure 4), may arise due to various artifacts in the MR images.

Deformable-model-based method integrates topology correction into segmentation[3]. It starts from a surface model with correct topology and preserves the topology while deforming the model for better segmentation configuration. Depending on the initial state, undesired topology correction may occur while the segmentation itself may also be trapped into local optima.

Most topology correction methods proceed as a post-processing step after image segmentation. A surface-based approach[11] first extracts the cortical surfaces and then corrects the topology of the surfaces while volume-based approach[2][5][9] first corrects the topology of a volume and then extract the surface preserving the correct topology. A particularly interesting volume-based approach corrects topology errors in a multiscale manner[2][9] such that handles are eliminated gradually in an order according to the correction costs.

Most automatic topology correction methods aim to eliminate handles while refraining themselves from modifying the object in a certain way. We think that a topology correction algorithm should consider the characteristic of the shape of the object as an attribute to preserve. Our method differs from existing

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methods mainly in that the special surface-likeness property of white matter and gray matter is considered in evaluating the cost of topology correction. We follow the volume-based approach by first correcting the topology of the white matter volumes followed by extracting the cortical surfaces. A multiscale method is taken so that topology errors are gradually corrected with respect to the correction cost.

The rest of the paper is organized as follows. In section 2, necessary background on digital topology and isosurface extraction is given. The core algorithm for topology correction at single scale is a filter described in section 3, based on which a multiscale procedure is presented in section 4 for topology correction of white matter. Experimental results are presented in section 5.

2 Background

A 3D binary image is defined as the quadruple $(Z^3, n, \overline{n}, \mathcal{F})$[4]. Z^3 is the 3D cubic lattice representing all voxels in the image. $\mathcal{F} \subset Z^3$ represents the set of foreground voxels and $\overline{\mathcal{F}}$ represents the complement of $\mathcal{F}$. n and $\overline{n}$ represent the adjacency in $\mathcal{F}$ and $\overline{\mathcal{F}}$ respectively. Three types of adjacency are commonly used in 3D: 6-, 18-, and 26-adjacency. Two voxels are 6-adjacent if they share a face, 18-adjacent if they share a face or an edge, and 26-adjacent if they share a face, an edge, or a corner. An *n-neighbor* of a point p is a point that is n-adjacent to p. The set of n-neighbors of a point p is denoted as $\mathcal{N}_n(p)$. Let $\mathcal{N}(p)$ denote $\mathcal{N}_{26}(p) \cup \{p\}$. A set of points S is *n-connected* if S cannot be partitioned into two subsets that are not n-adjacent to each other. Topologically compatible adjacencies/connectivities of $\mathcal{F}$ and $\overline{\mathcal{F}}$ are $(6, 26)$, $(6, 18)$, $(18, 6)$ and $(26, 6)$.

The central concept in digital topology is the definition of *simple point*[1]. A point in a binary image $(\mathcal{Z}^3, n, \overline{n}, \mathcal{F})$ is *simple* if it can be added to or removed from $\mathcal{F}$ without changing the topology of both $\mathcal{F}$ and $\overline{\mathcal{F}}$, i.e. without changing the number of connected components, cavities and handles of both $\mathcal{F}$ and $\overline{\mathcal{F}}$. A simple point p can be characterized by two *topological numbers* $T_n(p, \mathcal{F})$ and $T_{\overline{n}}(p, \overline{\mathcal{F}})$. When $m = 26$, the topological number $T_m(x, X)$ is the number of connected components in $\mathcal{N}_{26}(x) \cap X$; when $m = 6$, $T_m(x, X)$ is the number of connected components in $N_{18}(x) \cap X$ that are adjacent to x. Intuitively, $T_m(x, X)$ can be seen as the number of ports at which the point x is connected to a point set X in m-adjaceny. $T_{\overline{n}}(p, \overline{\mathcal{F}}) - 1$ can also be seen as the number of tunnels in the image $\hat{\mathcal{N}}(p) = (\mathcal{N}(p), n, \overline{n}, (\mathcal{N} \cap \mathcal{F}) - \{p\})$. A point p is a simple point iff $T_n(p, \mathcal{F}) = 1$ and $T_{\overline{n}}(p, \overline{\mathcal{F}}) = 1$.

Another concept critical to our method is the definition of *multisimple point*[8]. A point p is *multisimple* relative to $\mathcal{F}(\overline{\mathcal{F}})$ iff it can be added to or removed from $\mathcal{F}(\overline{\mathcal{F}})$ without changing the number of handles and cavities of $\mathcal{F}(\ \overline{\mathcal{F}})$. Splitting and merging connected components in $\mathcal{F}(\overline{\mathcal{F}})$ are allowed. Let $T_n^+(p, \mathcal{F})$ and $T_{\overline{n}}^+(x, \overline{\mathcal{F}})$ respectively denote the number of foreground and background components in the whole image excluding p that are adjacent to p. Then p is multisimple relative to $\mathcal{F}$ iff $T_{\overline{n}}(p, \overline{\mathcal{F}}) = 1$ and $T_n^+(p, \mathcal{F}) = T_n(p, \mathcal{F})$; p is multisimple relative to $\overline{\mathcal{F}}$ iff $T_n(p, \mathcal{F}) = 1$ and $T_{\overline{n}}^+(p, \overline{\mathcal{F}}) = T_{\overline{n}}(p, \overline{\mathcal{F}})$.

This paper mainly focuses on topology correction of a binary image($1 =$ foreground, $0 =$ background). We use a topologically consistent marching cubes isosurface algorithm[7] to generate the triangulated surface representation of the cortical surfaces. It is demonstrated that for $(26, 6)$ adjacency, an isovalue less than 0.25 should be used to avoid topological paradoxes.

3 A Morphological Topology Correction Filter

Our method for topology correction is mainly motivated by the observation about the surface-likeness of white matter(WM) and gray matter(GM). To preserve the shape of surface-like objects, the cost of handle cut B in figure 1 should be bigger than that of cut A because the object is "wider", in other words more like a surface, at B than at A. although the object is thinner at B. Similarly, we should fill the tunnel(i.e. cut the associate background handle) in the right object in figure 1 instead of cutting the foreground handle. In this section, we describe a morphological topology correction (MTC) filter that cut all handles at a specific scale of cost evaluated in terms of the wideness of the object.

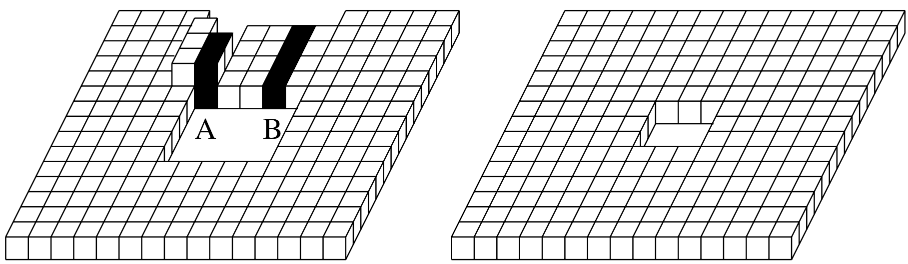

Fig. 1. Topology correction adapted to the surface-likeness of objects

Now it's time to give a precise definition of cut . A *cut* with respect to $\mathcal{F}$(or $\overline{\mathcal{F}}$) is a set of connected points in $\mathcal{F}$(or $\overline{\mathcal{F}}$) none of which is a multisimple point. Whenever a cut is removed from $\mathcal{F}$(or $\overline{\mathcal{F}}$), the number of handles of $\mathcal{F}$(or $\overline{\mathcal{F}}$) is decreased. A cut is minimal in that removal any proper subset of the cut from $\mathcal{F}$(or $\overline{\mathcal{F}}$) will lead to elimination of less or no handles.

We make another observation that any cut should pass the topology preserving curve skeleton of the object and take the approach of finding the cut via first finding the curve skeleton points. We designed an iterative shape and topology preserving erosion (ISTPE) operation for curve skeletonization. The cost of cutting handles at a skeleton point is determined by the iteration, or scale, at which the point emerges as a curve skeleton point in the procedure of performing ISTPE. At a specific scale, the emerged curve skeleton points are identified as candidate cut locations. Then several stages of carefully designed dilation operations are performed to find the final cuts.

3.1 Iterative Shape and Topology Preserving Erosion

To clearly describe ISTPE, we need to make some definitions on various point types (see figure 1) involved in the operation. First, we extend the definition of simple point to *thicksimple point*. In the rest of the paper, we use $\mathcal{X}$ to denote either $\mathcal{F}$ or $\overline{\mathcal{F}}$. $\mathcal{X}$ is in m-adacency, where $m = 26$ or 6. A point $p \in \mathcal{X}$ is a *thicksimple point* if p is a simple point and removal of p from $\mathcal{X}$ does not create new non-simple points in its 26-neighborhood $\mathcal{N}_{26}(p)$.

We define an *exterior point* $p \in \mathcal{X}$ as a point 6-adjacent to $\overline{\mathcal{X}}$. Any points in $\mathcal{X}$ that are not exterior points are referred to as *interior points* of $\mathcal{X}$. If an exterior point $p \in \mathcal{X}$ is not a simple point, it should be in a topology-preserving surface skeleton of $\mathcal{X}$. If $T_{\overline{m}}(p, \overline{\mathcal{X}})$ is greater than 1, then p is referred to as a *surface point*; otherwise, p is referred to as a *curve point*.

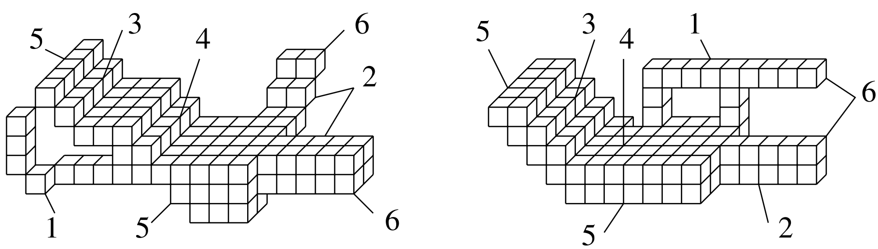

Fig. 2. Point types(left: 26-adjacenty; right: 6-adjacency). 1: curve, 2: thick curve, 3: surface, 4: thick surface, 5: thicksimple, 6: curve end.

If an exterior point p is a simple point but not a thicksimple point, then it should be in the object's topology-preserving thick surface skeleton. If removal of p from $\mathcal{X}$ creates a new surface point in $\mathcal{N}_{26}(p)$, then we refer to p as a *thick surface point*; otherwise we refer to p as a *thick curve point*. A thicksimple point p may be in one of the following situations:

- at the boundary of a volume portion of $\mathcal{X}$;
- at the edge of a thin or thick surface portion of $\mathcal{X}$'s surface skeleton;
- in a thick curve portion of $\mathcal{X}$'s surface skeleton(only when $m = 26$);
- at the end of a curve or a thick curve portion of $\mathcal{X}$. In 26-adjacency, a point p is a *curve end* if it is thicksimple and all points in $\mathcal{N}_{26}(p)$ are m-adjacent to each other. In 6-adjacency, a point p is a curve end if it is thicksimple and $\mathcal{N}_6(p)$ either contains exclusively curve or thick curve points, or contains one thicksimple point as well as curve or thick curve points.

When applied on a binary object $\mathcal{X}$, standard morphological erosion removes all exterior points without preserving topology and shape of $\mathcal{X}$. If we sequentially remove exterior points that are simple at the moment of removal, the topology of $\mathcal{X}$ is preserved but the shape of $\mathcal{X}$ may still be significantly changed. In addition to checking whether an exterior point is simple to impose topology preservation, ISTPE preserves object shape by thinning thick curve portions and eliminating only surface edges, and boundary points of volume portions. At a specific scale s, ISTPE is applied on $\mathcal{X}$ s iterations, each which consists of following steps.

1. Identify exterior points and classify them into thicksimple points, thick surface points, thick curve points, surface points, curve points, and curve ends.
2. Remove thick curve points and thicksimple points that are not curve ends if they are simple at the moment of removal until there exists no such point.

3.2 Geodesic Dilation with Topology Control

After ISTPE is applied on $\mathcal{X}$ at a specific scale, all curve points, denoted as set R, are identified in the erosion result $\mathcal{X}'$. Let $B = \mathcal{X}' - R$. We refer to R and B as residual set and body set respectively. Removal of R from $\mathcal{X}'$ may decrease the number of handles in $\mathcal{X}'$, which is exactly desirable. However it may also disconnects $\mathcal{X}'$ and R may be not minimal with respect to cutting handles in $\mathcal{X}'$. By several stages of geodesic dilation with topology control, we recover a subset of residual points and find the final cuts.

If removal of a residual point p from $\mathcal{X}'$ decreases the number of handles in $\mathcal{X}'$, p is referred to as a *handle residual point*; otherwise, p is referred to as a *finger residual point*. According to the mechanism of ISTPE, the object $\mathcal{X}$ is equally wide at each location of handle residual points. Every handle residual point is associated with the same cost of handle cutting and the dilation procedure will prefer those in the middle of the connected components of handle residual points.

The following three dilation stages iterate in the same manner. The dilation in every stage involves a seed set S and a condition set C. In each iteration of every stage, any points in C that are m-adjacent to S are marked first and then are recovered if they satisfy some additional conditions. Each stage of dilation terminates when no more points can be recovered. Note that whenever a point is recovered, B and S are augmented with the recovered point. C is also updated with the removal of the point.

In *stage 1*, $S = B$, $C = R$, and finger residual points are identified and recovered. In each iteration, a marked point p is identified as a finger residual point and recovered if $T_m(p, R) \leq 1$, and p is m-adjacent to a body component B_i with $d(B_i) = 1$ and is multisimple relative to B at the moment of recovery. Here $d(B_i)$ denotes the *degree* of a body component B_i and is defined as $d(B_i) = \sum_{j=1}^{N_R} T_m(r_j, B_i)$, where r_j represents a residual point and N_R represents the total number of residual points. $d(B_i)$ can be seen as the number of ports at which B_i is connected to residual points.

In *stage 2*, a minimal set of handle residual points $R^* \subseteq R$ is identified and any points in $R - R^*$ are recovered. The condition set $C = R$ and the seed set S is initialized as the set of points in B that are not finger residual points identified in stage 1. In each iteration of stage 2, a marked point is recovered if it is multisimple relative to B at the moment of recovery. Figure 3 gives an illustration on stage 1 and stage 2. In *stage 3*, $S = B$, $C = \mathcal{X} - B$, and a marked point is recovered if it is multisimple relative to B at the moment of recovery. Eventually, the original object $\mathcal{X}$ is recovered except for the points in the cuts.

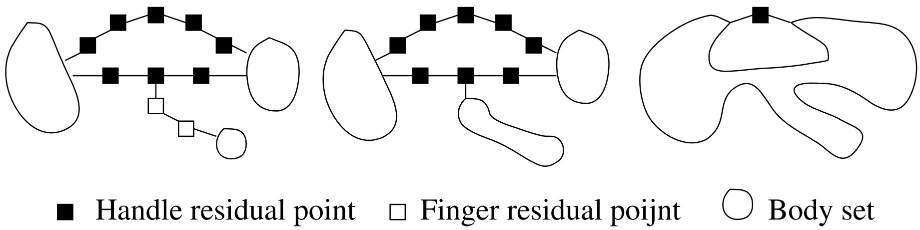

Fig. 3. Illustration of dilation. left: state before dilation; middle: state after first stage dilation; right: state after second stage dilation.

Since a recovered point has to be multisimple, the number of handles in $\mathcal{X}'$ is not changed. Merging of body components maybe happen and is desirable. In rare situations, however, there might be more than one connected components in the result body set and we only keep the largest component.

4 Multiscale Topology Correction of White Matter

Our algorithm for generating topologically correct cortical surfaces exploits the shape characteristic of surface-likeness of GM, WM as well as the layered structure in which CSF, GM and WM are organized. The input is the segmentation result in three sets $\mathcal{W}$, $\mathcal{G}$ and $\mathcal{B}$ representing WM, GM, and background respectively. We use 26-, 6- and 26-adjacency for them respectively. We require that $\mathcal{W}$, $\mathcal{G}$ and $\mathcal{B}$ each should form only one connected component. Furthermore, $\mathcal{W}$ should not be connected to $\mathcal{B}$. Necessary preprocessing operations are performed to enforce these constraints.

The following loop corrects the topology of $\mathcal{W}$ in a multiscale manner starting from scale s of 0. At the end, $\mathcal{W}$ should be homotopic to a ball. An object is homotopic to a ball if we can keep removing simple points from the object ending up with a single point.

1. Operate MTC at scale s on the foreground object in $(\mathcal{Z}^3, 26, 6, \mathcal{W})$ and move the voxels in all cuts from $\mathcal{W}$ into $\mathcal{G}$. The loop terminates if now $\mathcal{W}$ is homotopic to a ball.
2. Operate MTC at scale s on the background object in $(\mathcal{Z}^3, 6, 26, \mathcal{W} \cup \mathcal{G})$ and move the voxels in all cuts from $\mathcal{B}$ into $\mathcal{G}$.
3. Operate MTC at scale s on the foreground object in $(\mathcal{Z}^3, 6, 26, \mathcal{G})$. For each cut c, if it is connected to $\mathcal{W}$, move all voxels in c from $\mathcal{G}$ into $\mathcal{W}$; otherwise it is connected to $\mathcal{B}$ and all voxels in it are moved from $\mathcal{G}$ into $\mathcal{B}$. It is impossible for the cut to be connected to both $\mathcal{W}$ and $\mathcal{B}$. The loop terminates if now $\mathcal{W}$ is homotopic to a ball; otherwise, increase s by 1 and go to step 1.

5 Results

Figure 4 demonstrates the behavior of our algorithm on eliminating a handle in the white matter and an result cortical surface after topology correction. The handle in the white matter is removed by filling the associated tunnel, which is actually performed by cutting the corresponding handle in the gray matter. The algorithm can detect that the white matter handle is wider than the associated tunnel(i.e. the gray matter handle), despite that the former is thinner than the latter. Therefore, the algorithm chose to fill the tunnel instead of cutting the white matter so that the shape of the white matter is better maintained.

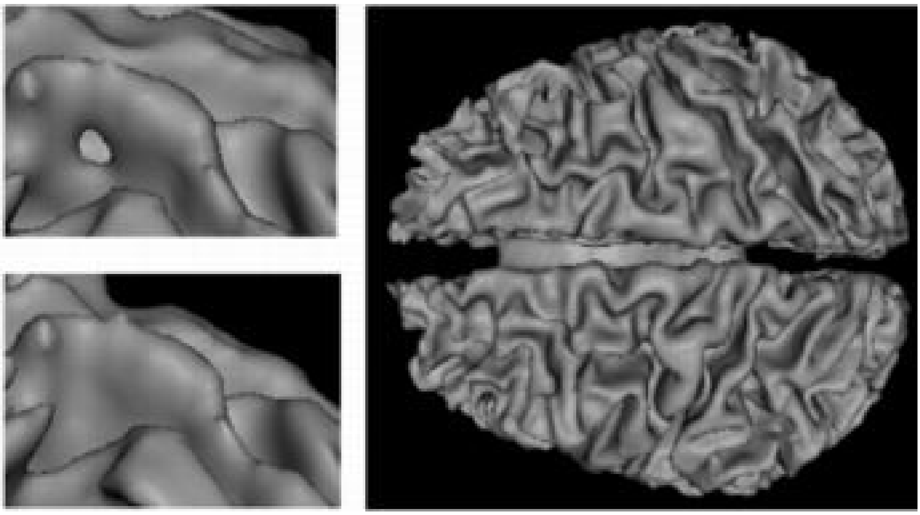

Fig. 4. Left top: before topology correction; left bottom: after topology correction; right: an inner cortical surface after topology correction

Table 1. Computation performance(time is measured in seconds)

Image	1	2	3	4	5	6	7	8	9	10
N	15	21	24	24	30	33	36	42	45	45
genus	98	104	79	163	65	106	79	63	58	44
Time	86	192	128	142	171	292	330	385	321	298

Table 1 shows the computation performance of our algorithm applied on the segmentation results of 10 MR images given by the "relative thresholding" method [6]. The resolutions of the MR images are between $1.4mm^3$ and $1mm^3$. The number of handles in the original white matter volume equals to the *genus number* given by $g = 1-(V-E+F)/2$, where V, E and F represent the number of vertices, edges and faces in the surface mesh. The computation time mainly depends on how many times (N) the morphological topology correction filter is performed. On a 1500MHz Itanium 2 processor, the average time of performing MTC in our experiments is about 7 seconds. We think that it can still be significantly improved by optimizing the code.

6 Conclusion

We proposed a multiscale morphological approach to correct topology of 3D white matter volumes, based on which cortical surface homotopic to a sphere can be generated. Attributed to the "shape and topology preserving erosion" operation, the algorithm evaluates the cost of cutting handles in terms of the modificaton to the "surface-likenss" of the white matter and tends to give rise to more appropriate topology correction solutions. We are investigating evaluation methods to determine the accuracy of our algorithm.

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