Part 3: Safety and liveness

Safety vs. liveness

Safety: something "bad" will never happen

Liveness: something "good" will happen (but we don't know when)

Safety vs. liveness for sequential programs

Safety: the program will never produce a wrong result ("partial correctness")

Liveness: the program will produce a result ("termination")

Safety vs. liveness for sequential programs

Safety: the program will never produce a wrong result ("partial correctness")

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Safety vs. liveness for state-transition graphs

Safety: those properties whose violation always has a finite witness
("if something bad happens on an infinite run, then it happens already on some finite prefix")

Liveness: those properties whose violation never has a finite witness
("no matter what happens along a finite run, something good could still happen later")

This is much easier.

Safety: the properties that can be checked on finite executions

Liveness: the properties that cannot be checked on finite executions

(they need to be checked on infinite executions)

Example: Mutual exclusion

It cannot happen that both processes are in their critical sections simultaneously.

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Safety

Example: Bounded overtaking

Whenever process P1 wants to enter the critical section, then process P2 gets to enter at most once before process P1 gets to enter.

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Safety

Example: Starvation freedom

Whenever process P1 wants to enter the critical section, provided process P2 never stays in the critical section forever, P1 gets to enter eventually.

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Liveness

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Liveness

LTL (Linear Temporal Logic)

-safety & liveness

-linear time

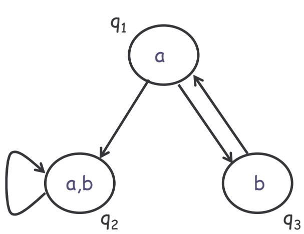
[Pnueli 1977; Lichtenstein & Pnueli 1982]

LTL Syntax

$\varphi \;::=\; a \mid \varphi \wedge \varphi \mid \neg \varphi \mid \bigcirc \varphi \mid \varphi \, U \, \varphi$

LTL Model

infinite trace $t = t_0 \, t_1 \, t_2 \ldots$
(sequence of observations)



Run: $q_1 \to q_3 \to q_1 \to q_3 \to q_1 \to q_2 \to q_2 \to$

Trace: $a \to b \to a \to b \to a \to a,b \to a,b \to$

Language of deadlock-free state-transition graph K at state q :

L(K,q) = set of infinite traces of K starting at q

$(K,q) \models^{\forall} \varphi$ iff for all $t \in L(K,q)$, $t \models \varphi$

$(K,q) \models^{\exists} \varphi$ iff exists $t \in L(K,q)$, $t \models \varphi$

LTL Semantics

$t \models a$	iff	$a \in t_0$
$t \models \varphi \wedge \psi$	iff	$t \models \varphi$ and $t \models \psi$
$t \models \neg\varphi$	iff	not $t \models \varphi$
$t \models \bigcirc \varphi$	iff	$t_1 t_2 \ldots \models \varphi$
$t \models \varphi \, U \, \psi$	iff	exists $n \geq 0$ s.t. 1. for all $0 \leq i < n$, $t_i t_{i+1} \ldots \models \varphi$ 2. $t_n t_{n+1} \ldots \models \psi$

$(K,q) \models^{\forall} \varphi$ iff $\neg (K,q) \models^{\exists} \neg\varphi$

Defined modalities

$\bigcirc$	X	next
U	U	until
$\Diamond \varphi = true \, U \, \varphi$	F	eventually
$\Box \varphi = \neg \Diamond \neg\varphi$	G	always
$\varphi \, W \psi = (\varphi \, U \, \psi) \vee \Box\varphi$	W	waiting-for (weak-until)

Important properties

Invariance	$\Box\, a$	safety
	$\Box \neg (pc1{=}in \wedge pc2{=}in)$	
Sequencing	$a \, W \, b \, W \, c \, W \, d$	safety
	$\Box (pc1{=}req \Rightarrow (pc2{\neq}in) \, W \, (pc2{=}in) \, W \, (pc2{\neq}in) \, W \, (pc1{=}in))$	
Response	$\Box (a \Rightarrow \Diamond\, b)$	liveness
	$\Box (pc1{=}req \Rightarrow \Diamond (pc1{=}in))$	

Composed modalities

$\Box\Diamond\, a$	infinitely often a
$\Diamond\Box\, a$	almost always a

Example: Starvation freedom

Whenever process P1 wants to enter the critical section, provided process P2 never stays in the critical section forever, P1 gets to enter eventually.

$\Box\Diamond (pc2{=}in \Rightarrow \bigcirc (pc2{=}out)) \Rightarrow$
$\Box (pc1{=}req \Rightarrow \Diamond (pc1{=}in))$

State-transition graph

Q	set of states	$\{q_1,q_2,q_3\}$
A	set of atomic observations	$\{a,b\}$
$\rightarrow \subseteq Q \times Q$	transition relation	$q_1 \rightarrow q_2$
$[\ ]: Q \rightarrow 2^A$	observation function	$[q_1] = \{a\}$

$(K,q) \models^{\forall} \varphi$

↓ Tableau construction (Vardi-Wolper)

(K', q', BA) where BA $\subseteq$ K'

Is there an infinite path starting from q' that hits BA infinitely often?

Is there a path from q' to p $\in$ BA such that p is a member of a strongly connnected component of K'?

```
dfs(s) {
   add s to dfsTable
   for each successor t of s
      if (t ∉ dfsTable) then dfs(t)
   if (s ∈ BA) then { seed := s; ndfs(s) }
}

ndfs(s) {
   add s to ndfsTable
   for each successor t of s
      if (t ∉ ndfsTable) then ndfs(t)
      else if (t = seed) then report error
}
```