Modeling Denotational Semantics
and For

Axioms for Domain Theory

Part 1—

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unstructured domain.

The semantics underlying our

These axioms refer to the structure of

The Axiom List

\begin{align*}
\text{Axiom I:} & \text{ Mapping} \\
\text{Axiom II:} & \text{ Patrifying} \\
\text{Axiom II:} & \text{ Towering} \\
\text{Axiom I:} & \text{ Approximating} 
\end{align*}
Theorem: For \( x \in \text{Dom}(g) \), we have \( x \in \bigcup \{ \{ f \mid f \in \text{Dom}(g) \} \} \).

Definition: \( \text{Id} = \{ x \mid g \circ x = f \} \) and \( \text{Id} = \bigcup \{ \{ f \mid f \in \text{Dom}(g) \} \} \).

Least upper bound in \( \text{Dom}(g) \).

\( \text{Id} = \{ f \mid \exists x \in \text{Dom}(g) \text{ s.t. } f(x) = g(x) \} \).
$\mathbf{1} = \langle x', b \rangle \land x.d = 0 < \langle x', b \rangle \land 0 < x.d$

$\langle b \land b', d \rangle \land d = \langle b', d \rangle \land (b \land d)$

$\mathbf{1} > \langle b', d \rangle$

$\langle b \land b', d \land d \rangle = \langle b, d \rangle \land (b \land d)$

$\mathbf{1} = b \land (\mathbf{1} = d) \land \mathbf{1} = (b \land d)$

$0 = \langle 0, 0 \rangle$

$\mathbf{b} = \mathbf{d} \Leftrightarrow \mathbf{b} = \mathbf{d}$

$b \land \mathbf{d} = (b \land \mathbf{d})$

$0 = \mathbf{0} \land \mathbf{1} > \mathbf{d}$

**Lowering $\times$ Pairing**
Joint function will evaluate approximated by:

\[ f = (a - b) \land (b + d) = 0 < (a - b) \land 0 < (b + d) \]
\[ f = (x - b) \land x + d = 0 < (x - b) \land 0 < x + d \]
\[ \{ x \in \mathbb{R} \mid r \} \land s \iff (r - (b + d)x) + 1 \land s \iff (r - (b + d)x) + 1 \]
\[ f = (x \in \mathbb{R} \mid r \} \land s \iff \{ r = (b - d)x \iff 1 \land \]
\[ f = \{ x > d \iff \text{detrended } (b - d) \}

Mapping
A Universal Semilattice

Note: The three parts of $\mathcal{U}$ are each

$x = 1 \lor y \lor z$

where there are many ways of determining $\text{subsemilattice}$.

Definition: We take $\mathcal{U}$ as the minimal solution of:

$n = 0 \lor (n \times n) \lor (n \lor n)$.
Theorem: Some very basic properties.

Def: Homomorphism

Def: Domain Isomorphism
A Universal Domain
Continuous Functions
Function spaces

Theorem: In the theory of the structure of the universal

Definition: For continuous $f: \text{Dom}(g) \to \text{Dom}(f)$,
End of Part I
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Representing Integers

Question: How to interpret $\text{dom}(\{x\})$?

Theorem: $\text{dom}(\{x\}) = \{10\} \cup \{1\} | x \in \mathbb{N}.$

Definition: $\{x\} = \{9\} \cup \{1\} | x \in \mathbb{N}.$

Theorem: For all $n, m \in \mathbb{N}$.

Definition: For $n \in \mathbb{N}$ define $\text{reverse}(n).$
Theorem: Compositions of computable mappings are computable.

A computable mapping to a computable set is a recursive mapping. The composition of two recursive mappings is a recursive mapping.

Definition: The computable mappings f \( \text{dom}(f) \rightarrow \text{dom}(g) \) are those that are continuous and where the set of those that are recursive is recursive.

Definition: The computable elements of \( \text{dom}(g) \) are those that are recursively enumerable subsets of \( \text{dom}(g) \).

Definition: The computable elements of \( S \) are those that are recursively enumerable subsets of \( S \).

Computable Domains and Mappings
A Review of Semantical Constructs
Partial recursive functions.

The computable elements of \(\text{dom}(g)\).
Least Fixed Points

Theorem: Every continuous \( f : \text{dom}(g) \to \text{dom}(g) \) has a least fixed point.
Some Domain Equations
Moreover, if $\phi$ is continuous and $\phi(x) \not\in \text{dom}(\chi)$, then $\phi(x) \not\in \text{dom}(\chi)$. This follows from the definition of the application and the assumption that $\phi(x) \not\in \text{dom}(\chi)$. Hence, the values of the application and the counterapplication are disjoint.

Definition: A counterapplication for continuous $\phi(x)$ is

\[
\chi(x) = \{ y \mid \exists x \in \text{dom}(\phi) : \phi(x) = y \}.
\]
A Simple Recursion

\(\text{fact}(n) = \text{if } n = 0 \text{ then } 1 \text{ else } n \times \text{fact}(n-1)\)

\(\text{plus}(x, y) = \text{if } y = 0 \text{ then } x \text{ else } \text{plus}(\text{succ}(x), y-1)\)

\(\text{read}(\{ x | x \in \mathbb{N} \}) = \text{if } \exists y \in \mathbb{N} \text{ such that } y = x \text{ and } \text{read}(\{ y | y \in \mathbb{N} \}) \text{ then } \text{read}(\{ y | y \in \mathbb{N} \}) \text{ else } \bot\)

\(\text{write}(x, y) = \text{if } \exists y \in \mathbb{N} \text{ such that } y = x \text{ and } \text{write}(y, y) \text{ then } \text{write}(y, y) \text{ else } \bot\)
Partial Equivalence Relations

Definition. For $\rho \subseteq \text{dom}(\pi)$, let

$$\{ \langle p, q \rangle \mid \rho \} = \langle p \rho q \rangle$$

where, for all elements $p, q \in \text{dom}(\pi)$,

- Relations over $\text{dom}(\pi)$ are the subsets of $\pi \times \pi$.
- $\pi \rho$ is the collection of partial equivalence relations on $\pi$.
Question: Is it possible to place an ace among

\[(g - x) \times (g - y) = (g - (g + y))\]

and

\[(g - x) \times (g - y) = (g + x) - y\]

and

\[(g - x) + y = (g - (g + y))\]

and

\[x \times y + g + x \times y = y\]

Theorem: For all 

\[(a + b) \times (a - b) = a^2 - b^2\]

where

\[a = x + y\] and 
\[b = y\]

Theorem: For all 

\[(a + b) \times (a - b) = a^2 - b^2\]

where

\[a = g\] and 
\[b = y\]
Dependent Types
Systems of Dependent Types
Identity Types