1 Papers


2 Propositional Adjoint Logic

2.1 Syntax

Intuitionistic Types \( X ::= 1 \mid X \times Y \mid X \rightarrow Y \mid G A \)

Linear Types \( A ::= I \mid A \otimes B \mid A \rightarrow B \mid F X \)

Intuitionistic Contexts \( \Gamma ::= \cdot \mid \Gamma, x : X \)

Linear Contexts \( \Delta ::= \cdot \mid \Delta, a : A \)

Intuitionistic Terms \( e ::= () \mid (e, e') \mid \text{fst } e \mid \text{snd } e \mid \lambda x . e \mid e e' \mid G(t) \mid x \)

Linear Terms \( t ::= () \mid \text{let } () = t \text{ in } t' \mid (t, t') \mid \text{let } (x, y) = t \text{ in } t' \mid \lambda a . t \mid t t' \mid F(e) \mid \text{let } F(x) = t \text{ in } t' \mid \text{run}(e) \mid a \)
2.2 Rules

\[ \frac{\Gamma \vdash e : X}{\Gamma \vdash () : 1} \]  
(no 1E)

\[ \frac{\Gamma \vdash e : X \quad \Gamma \vdash e' : Y}{\Gamma \vdash \langle e, e' \rangle : X \times Y} \]  
\times I

\[ \frac{\Gamma \vdash e : X \times Y}{\Gamma \vdash \text{fst}(e) : X} \]  
\times E_1

\[ \frac{\Gamma \vdash e : X \times Y}{\Gamma \vdash \text{snd}(e) : X} \]  
\times E_2

\[ \frac{\Gamma, x : X \vdash e : Y}{\Gamma \vdash \lambda x. e : X \rightarrow Y} \]  
\rightarrow I

\[ \frac{\Gamma \vdash e : X \rightarrow Y \quad \Gamma \vdash e' : X}{\Gamma \vdash e' : Y} \]  
\rightarrow E

\[ \frac{\Gamma ; \vdash t : A}{\Gamma ; \vdash \text{run}(e) : A} \]  
GE

\[ \frac{\Gamma ; \vdash () : 1}{\Gamma ; \vdash () : 1} \]  
II

\[ \frac{\Gamma ; \vdash t : A \quad \Gamma ; \Delta \vdash t' : B}{\Gamma ; \Delta, \Delta' \vdash (t, t') : A \otimes B} \]  
\otimes I

\[ \frac{\Gamma ; \Delta, \Delta' \vdash (t, t') : A \otimes B}{\Gamma ; \Delta \vdash \lambda a. t : A \rightarrow B} \]  
\rightarrow I

\[ \frac{\Gamma \vdash e : X}{\Gamma ; \vdash \text{F}(e) : \text{F}(X)} \]  
FI

\[ \frac{\Gamma \vdash e : G(A)}{\Gamma ; \vdash \text{run}(e) : A} \]  
GE

\[ \frac{\Gamma ; \vdash t : A}{\Gamma ; \vdash \text{run}(e) : A} \]  
GE

2.3 Substitution Properties

Lemma 1. (Weakening) We have that:
- If \( \Gamma \vdash e : Y \) then \( \Gamma, x : X \vdash e : Y \).
- If \( \Gamma ; \Delta \vdash t : A \) then \( \Gamma, x : X; \Delta \vdash t : A \).

Theorem 1. (Substitution) We have that:
- If \( \Gamma \vdash e : X \) and \( \Gamma, x : X \vdash e' : Y \), then \( \Gamma \vdash [e/x]e' : Y \).
- If \( \Gamma \vdash e : X \) and \( \Gamma, x : X; \Delta \vdash t : A \), then \( \Gamma; \Delta \vdash [e/x]t : A \).
- If \( \Gamma; \Delta \vdash t : A \) and \( \Gamma; \Delta' \vdash t' : B \), then \( \Gamma \vdash [t/a]t' : B \).