Session-Typed Concurrent Programming
Lecture 2

Stephanie Balzer
Carnegie Mellon University

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Recap

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- Roadmap and learning objectives

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  - pi-calculus as formal model
  - nondeterminism

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we’ll resume here

Intuitionistic linear logic session types

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Types:

A, B	$\triangleq$	$A \otimes B$	multiplicative conjunction	"channel output"
		$A \multimap B$	multiplicative implication	"channel input"
		$A \,\&\, B$	additive conjunction	"external choice"
		$A \oplus B$	additive disjunction	"internal choice"
		$\mathbf{1}$	unit for $\otimes$	"termination"

Queue session type:

$$\begin{aligned} \mathsf{queue}\ A = \&\{&\mathsf{enq} : A \multimap \mathsf{queue}\ A, \\ &\mathsf{deq} : \oplus\{\mathsf{none} : \mathbf{1}, \mathsf{some} : A \otimes \mathsf{queue}\ A\}\} \end{aligned}$$

Typing judgment and rules

Intuitionistic linear sequent:

$$x_1 : A_1, \ldots, x_n : A_n \vdash P :: (x : A)$$

"Process P offers a session of type A along channel x using session A_1, …, A_n provided along channels x_1, …, x_n."

Inference rule:

premise / conclusion

$$\frac{\Delta' \vdash Q :: (x : A')}{\Delta \vdash P; Q :: (x : A)}$$

↑ bottom-up reading

Left and right rules:

$$\frac{\Delta', x : B \vdash Q :: (z : C)}{\Delta, x : A \diamond B \vdash P; Q :: (z : C)} \diamond_L \qquad \frac{\Delta' \vdash Q :: (x : B)}{\Delta \vdash P; Q :: (x : A \diamond B)} \diamond_R$$

Connectives so far

$$\frac{\Delta \vdash P :: (x : B)}{\Delta, y : A \vdash \textsf{send}\ x\ y; P :: (x : A \otimes B)} \otimes_R \qquad \frac{\Delta, x : B, y : A \vdash Q_y :: (z : C)}{\Delta, x : A \otimes B \vdash y \leftarrow \textsf{recv}\ x; Q_y :: (z : C)} \otimes_L$$

$$\frac{\Delta, y : A \vdash P_y :: (x : B)}{\Delta \vdash y \leftarrow \textsf{recv}\ x; P_y :: (x : A \multimap B)} \multimap_R \qquad \frac{\Delta, x : B \vdash Q :: (z : C)}{\Delta, x : A \multimap B, y : A \vdash \textsf{send}\ x\ y; Q :: (z : C)} \multimap_L$$

$$\frac{\Delta \vdash P :: (x : A)}{\Delta \vdash x.\textsf{inl}; P :: (x : A \oplus B)} \oplus_{R_1} \qquad \frac{\Delta \vdash P :: (x : B)}{\Delta \vdash x.\textsf{inr}; P :: (x : A \oplus B)} \oplus_{R_2}$$

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→ no unit for & and $\oplus$, since must consist of at least one label

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split context

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Let’s implement the queue!

We use the formal language SILL used in research papers

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Philip Wadler. Propositions as sessions. ICFP, 2012.

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- without contraction, every provider has at most one client
- thus, every provider has exactly one client

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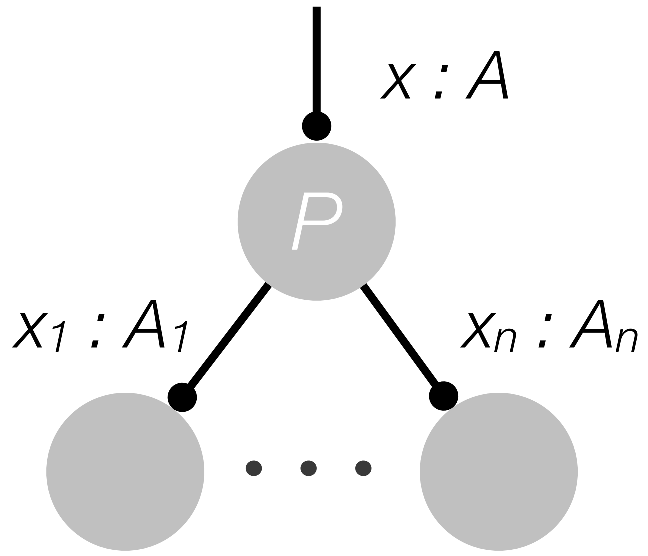

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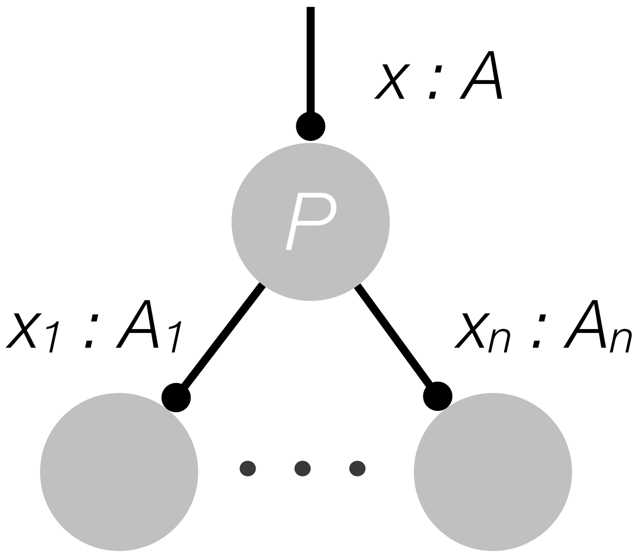

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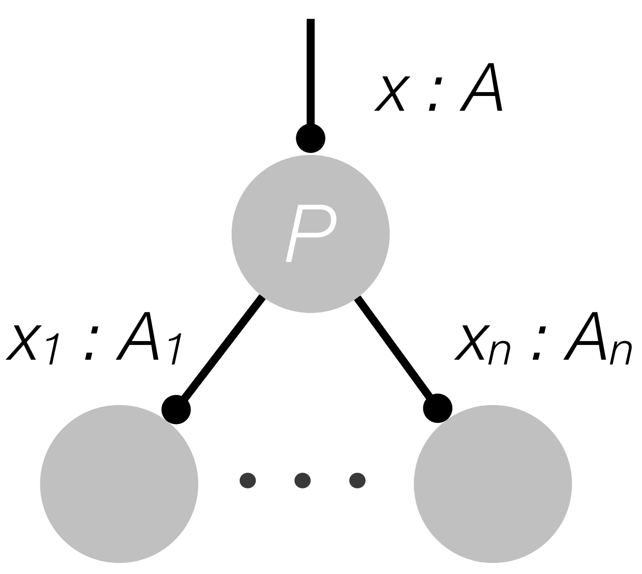

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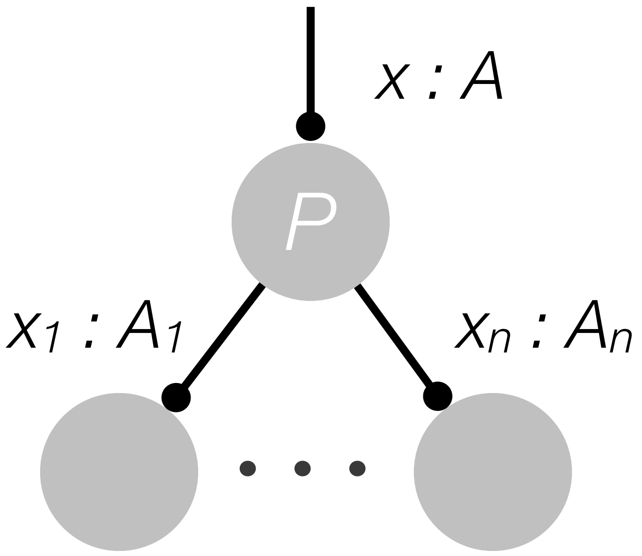

- we will use directedness for deadlock-freedom

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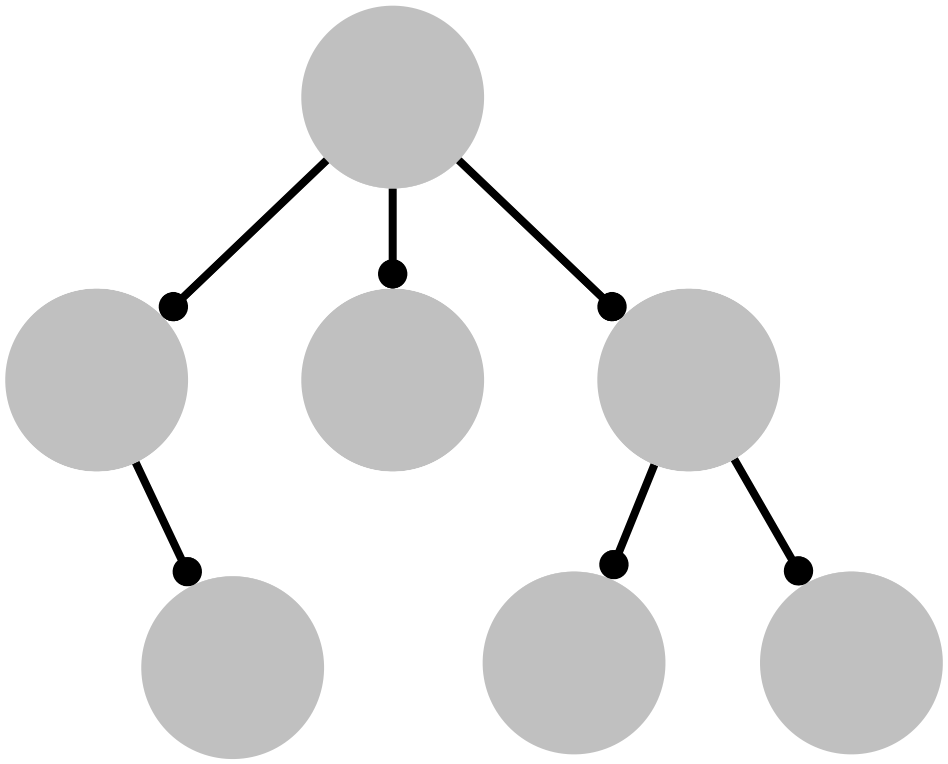

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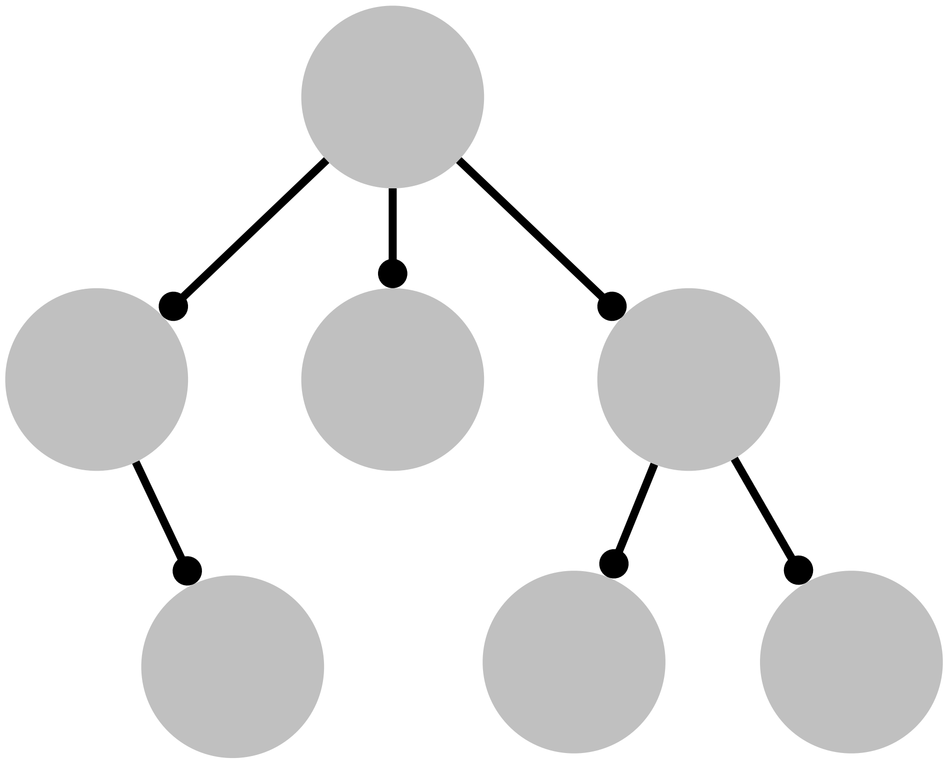

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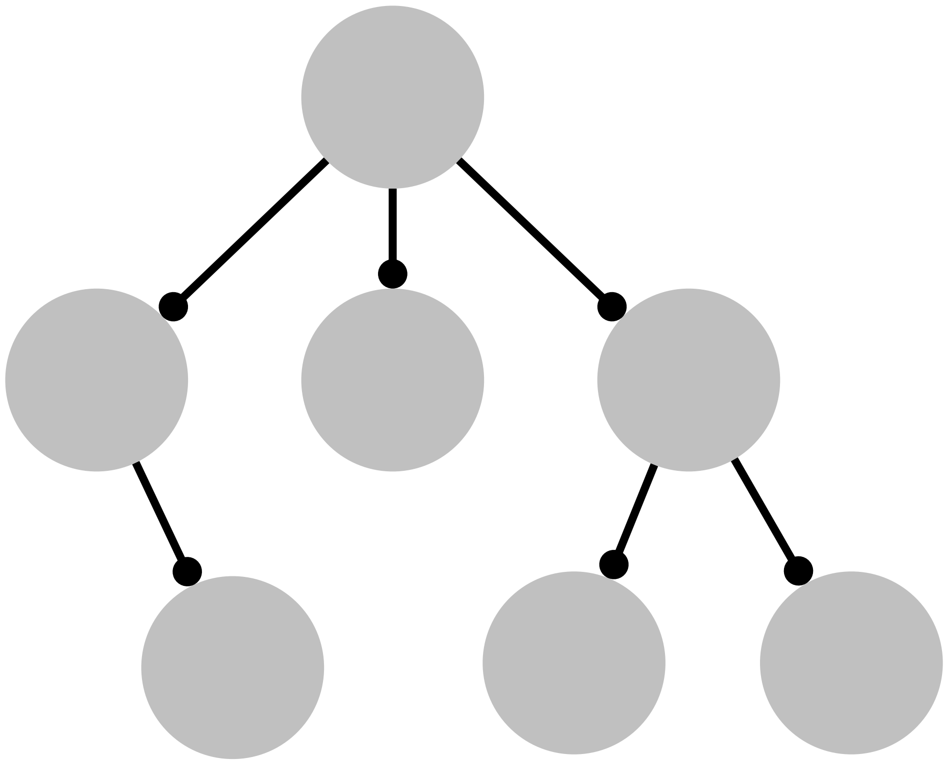

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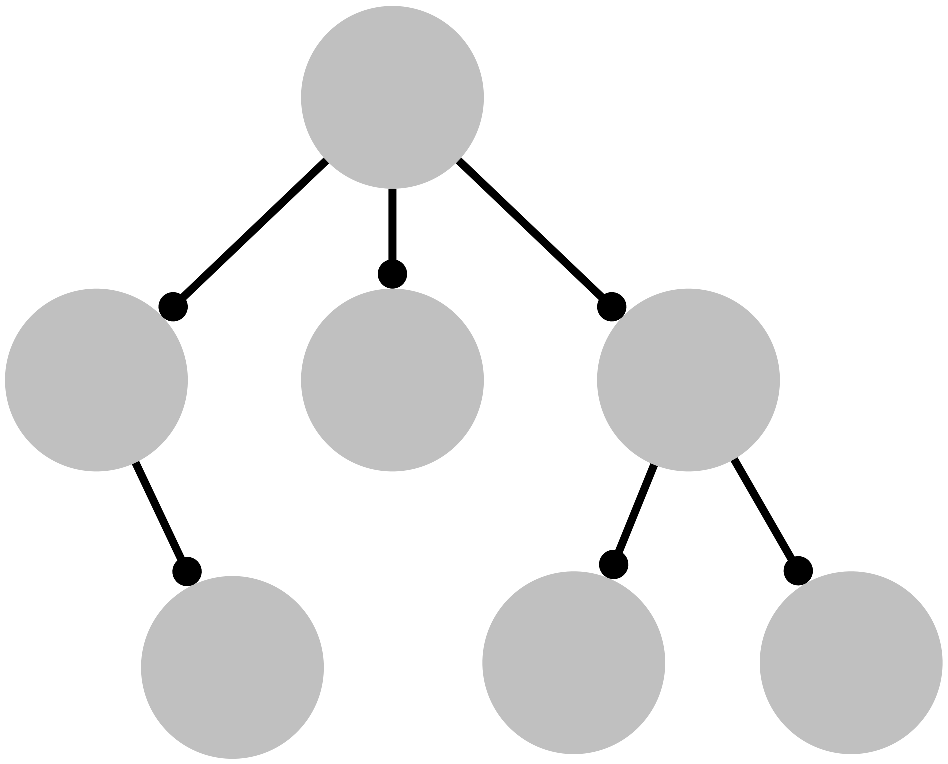

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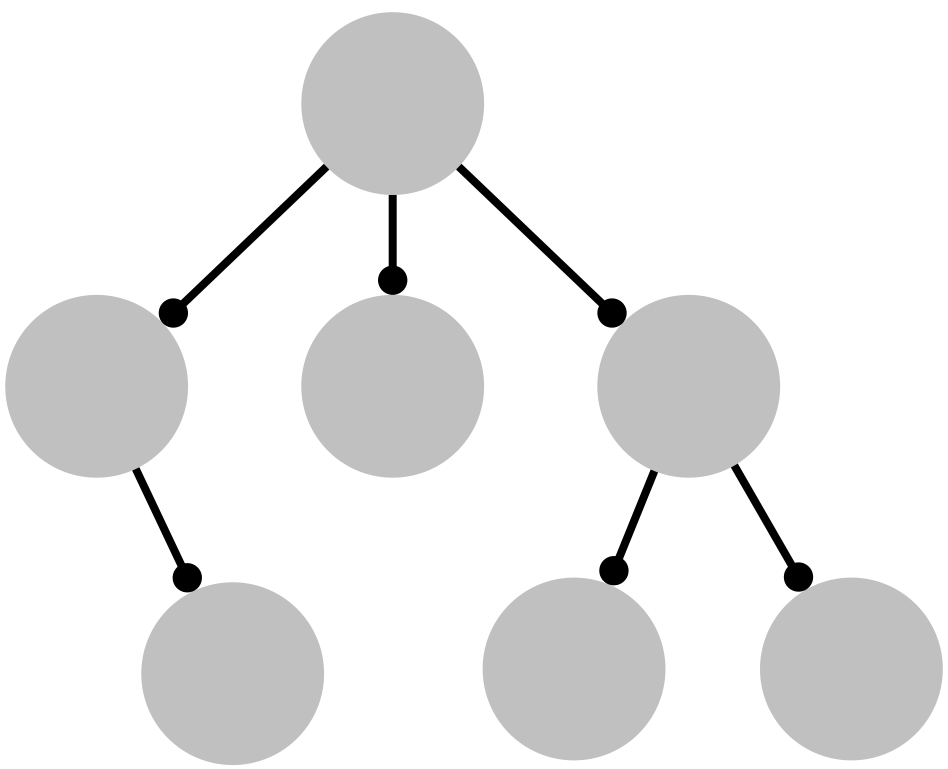

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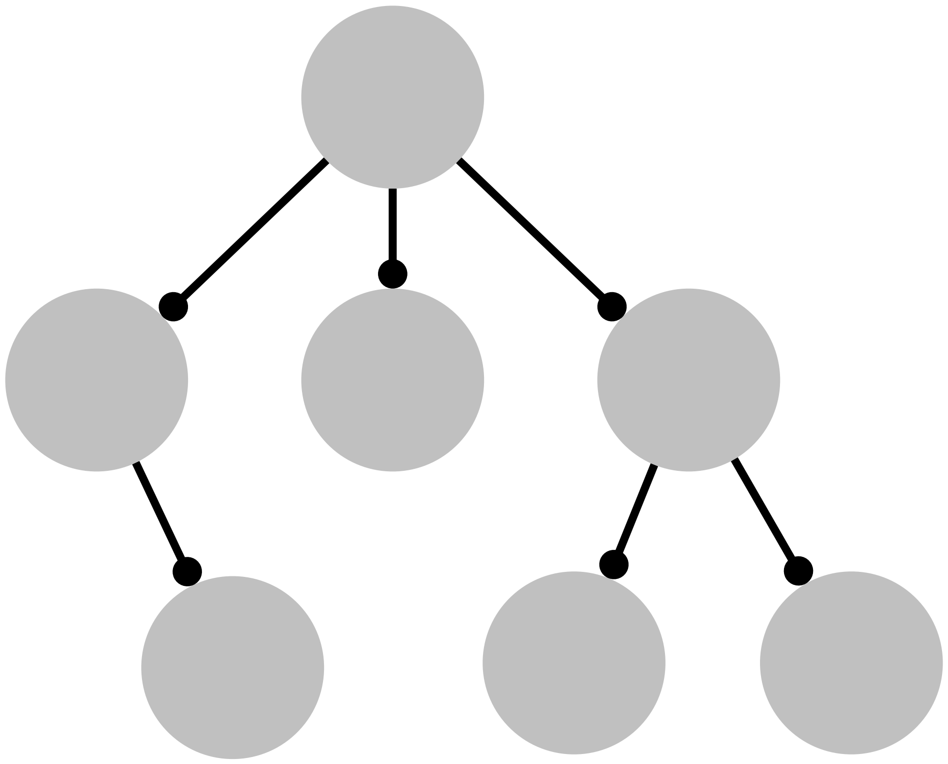

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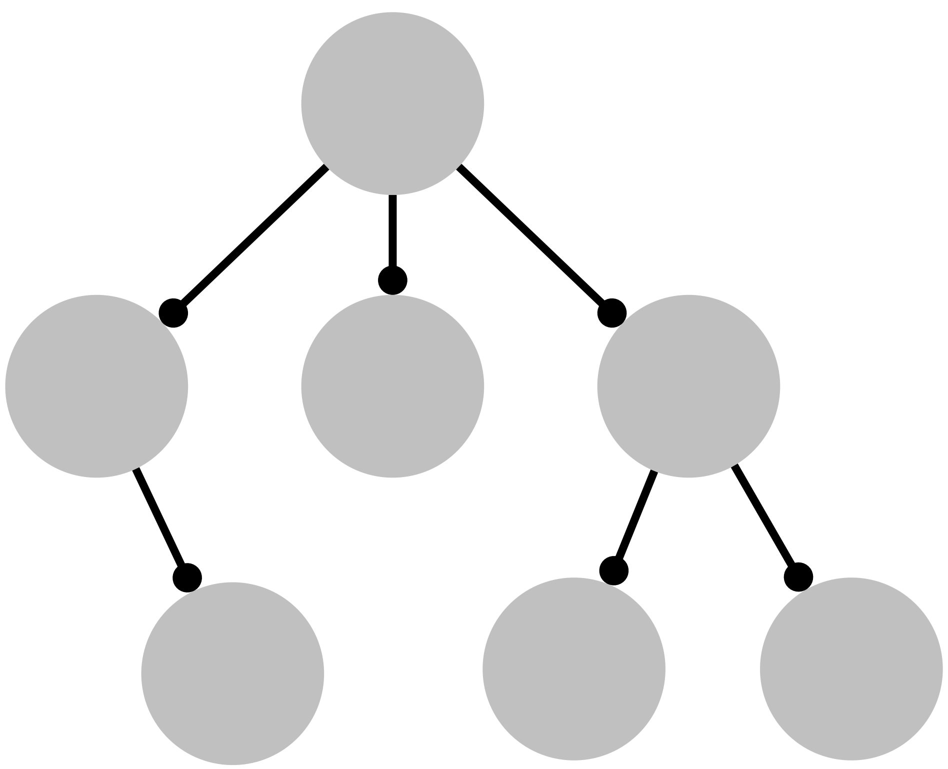

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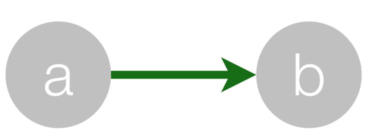

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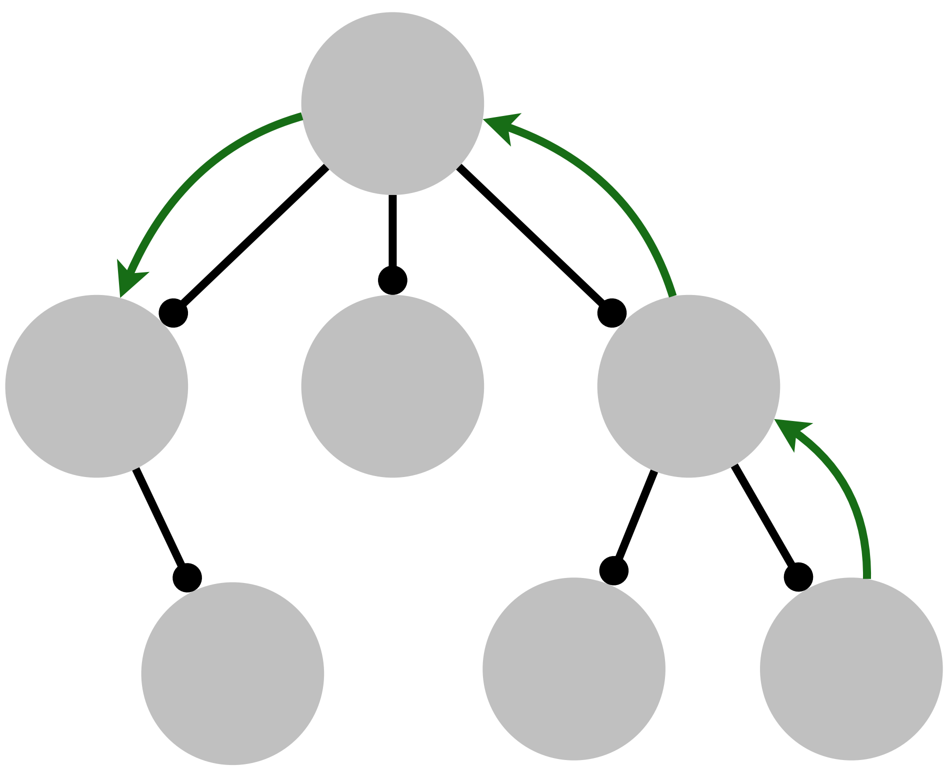

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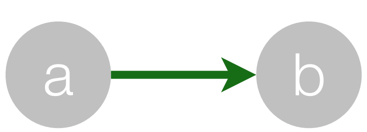

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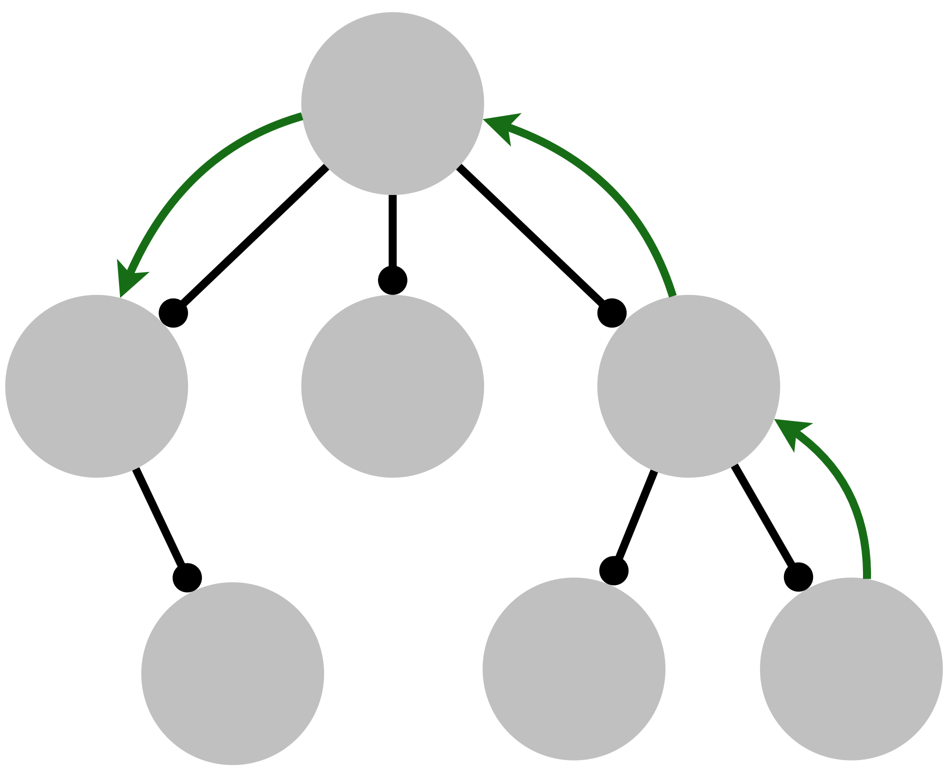

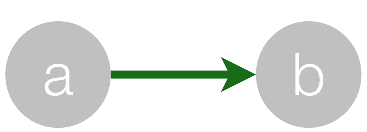

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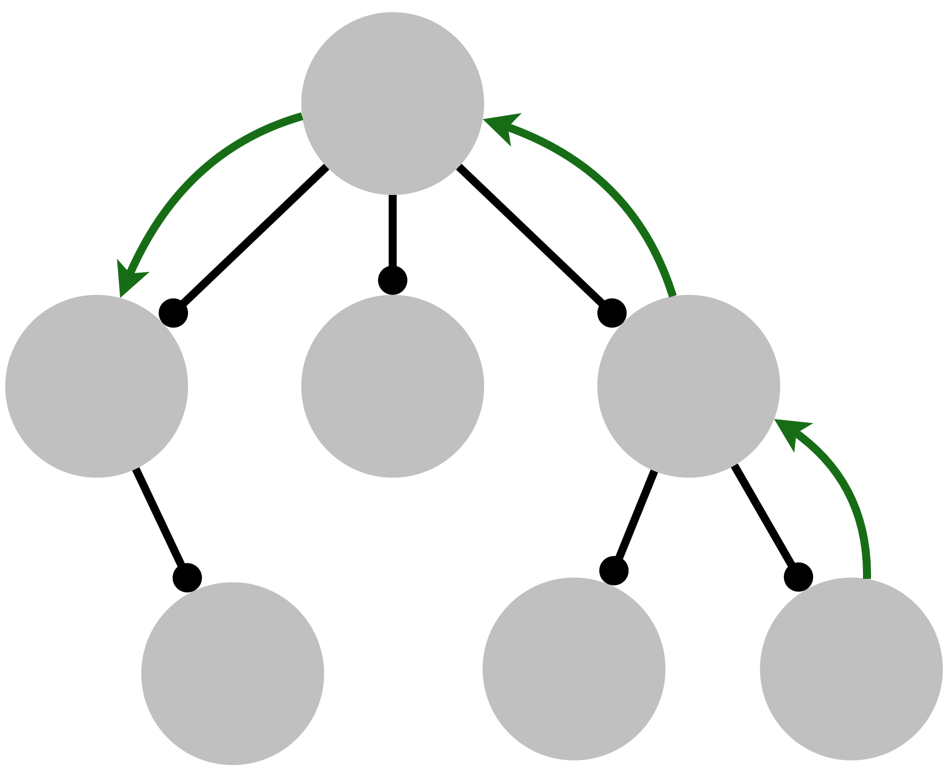

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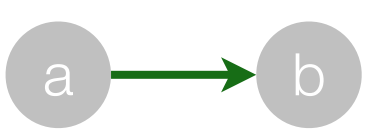

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asynchronous semantics: spawns off messages and links them with forward

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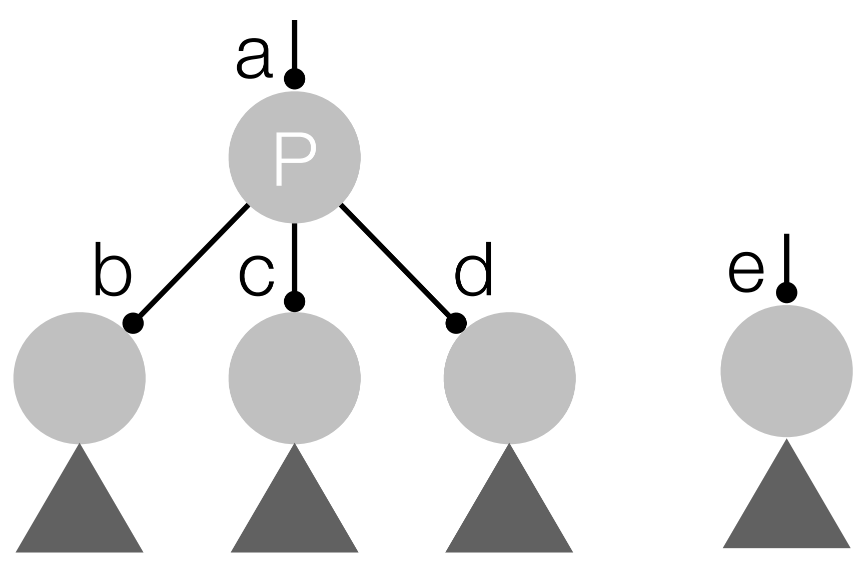

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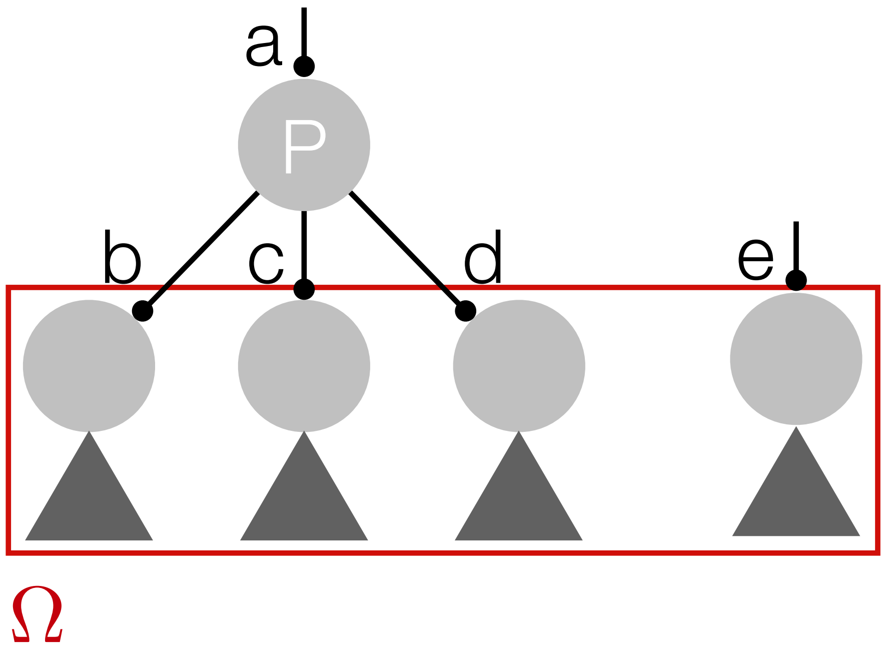

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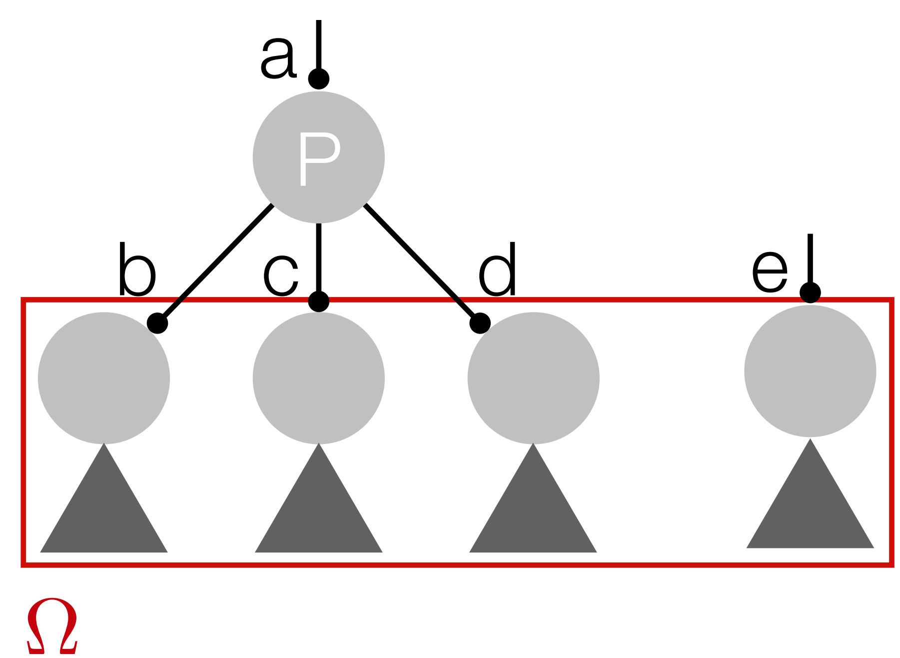

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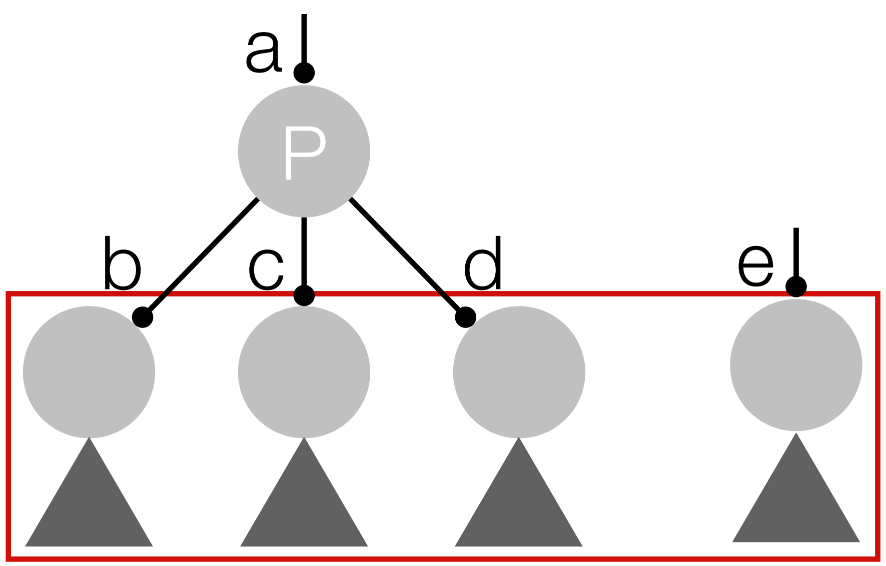

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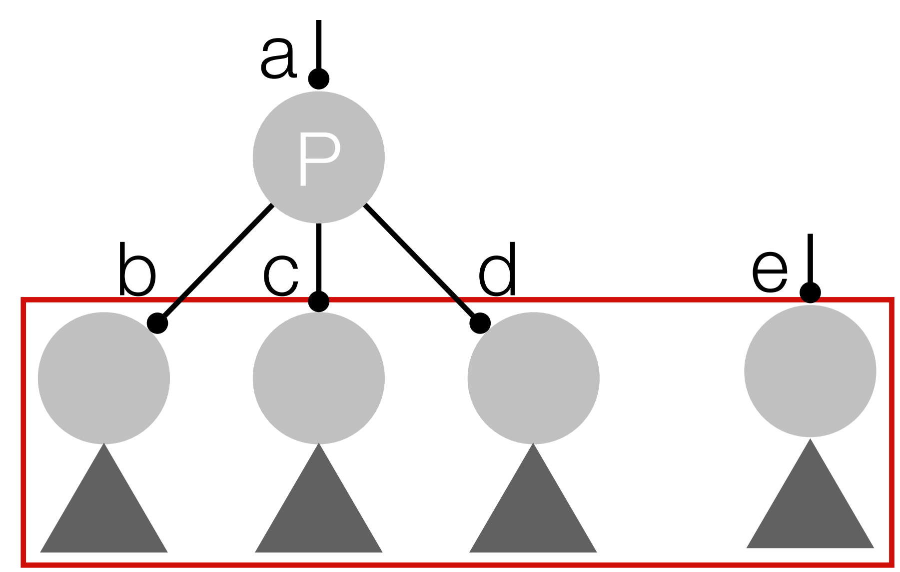

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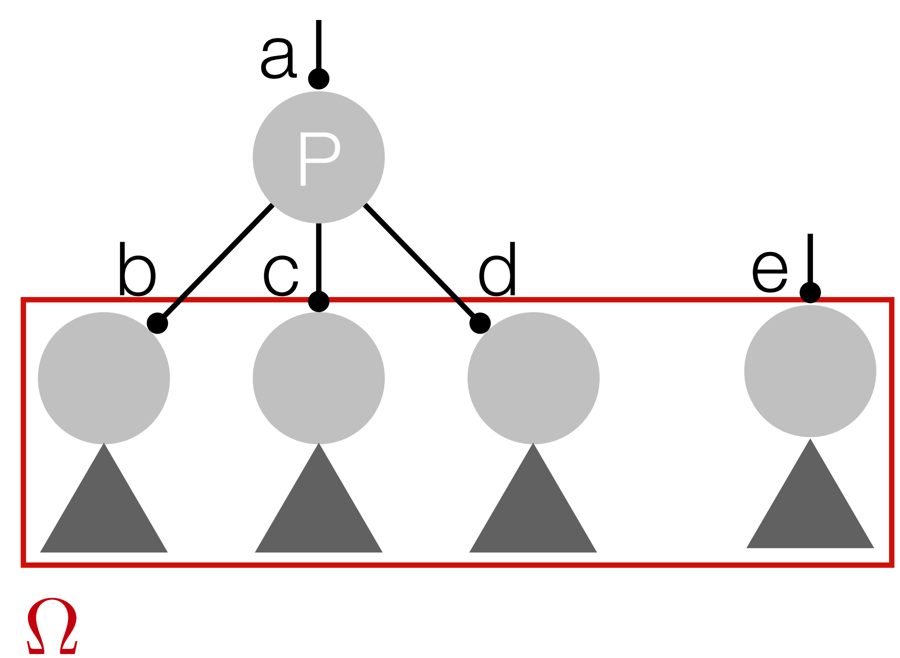

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- typing imposes forest structure and tree structure at top level
- a closed program offers a session of type $\mathbf{1}$, the top-level "main" process

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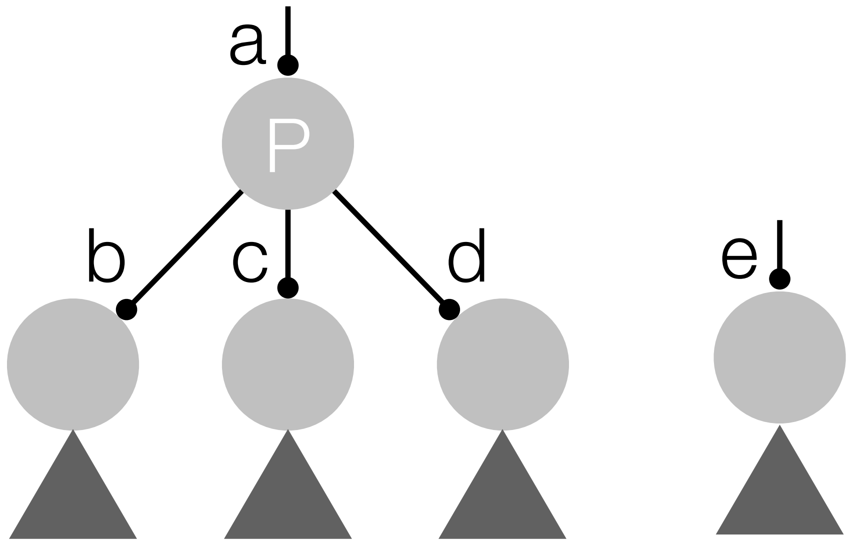

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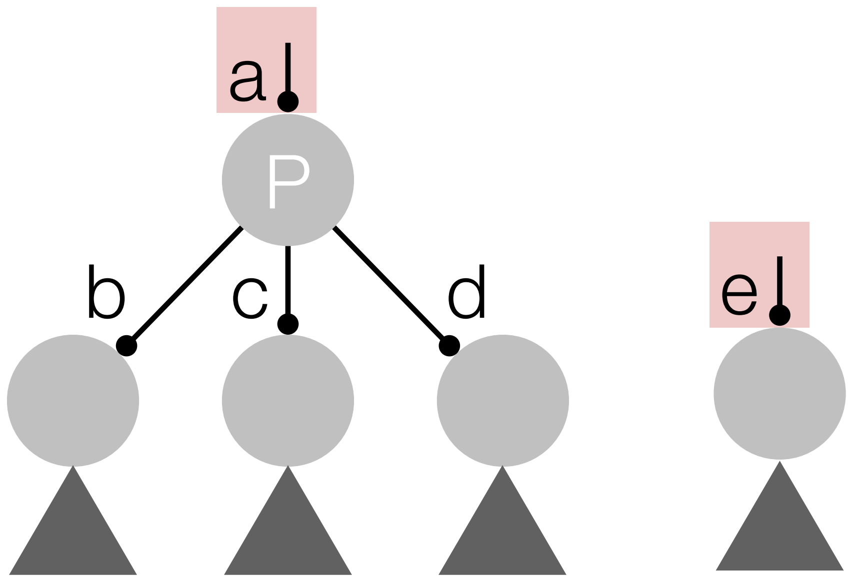

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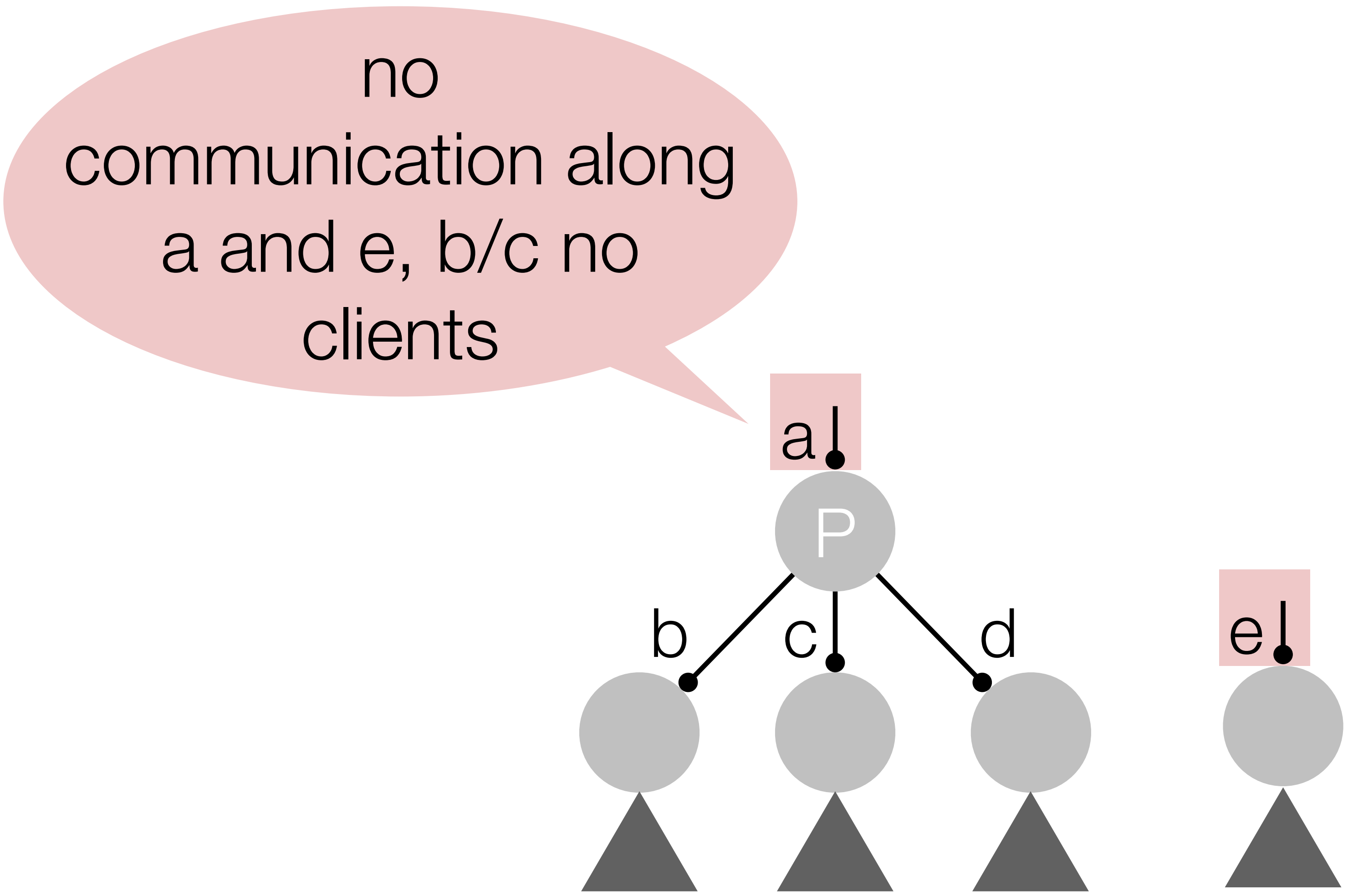

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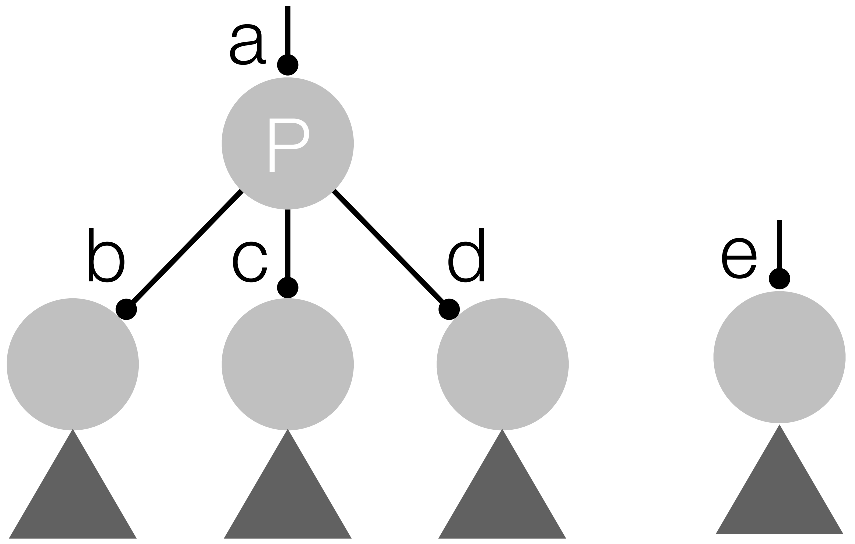

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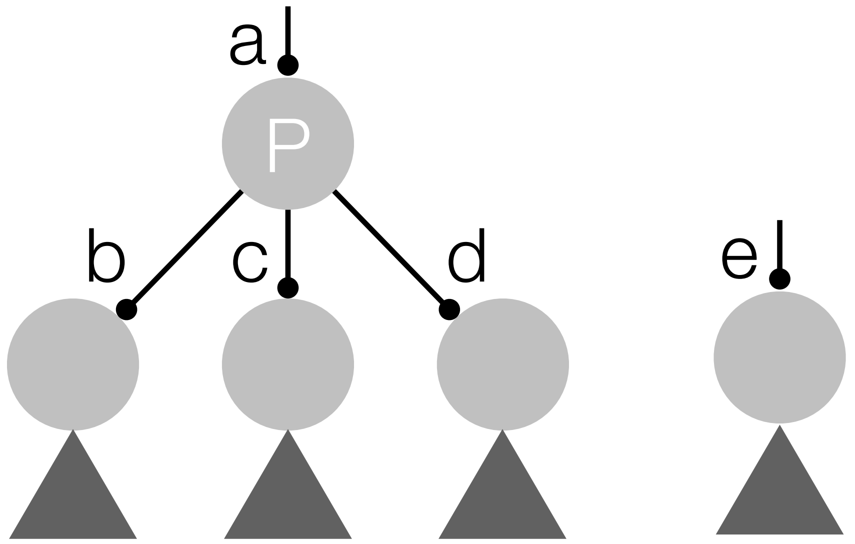

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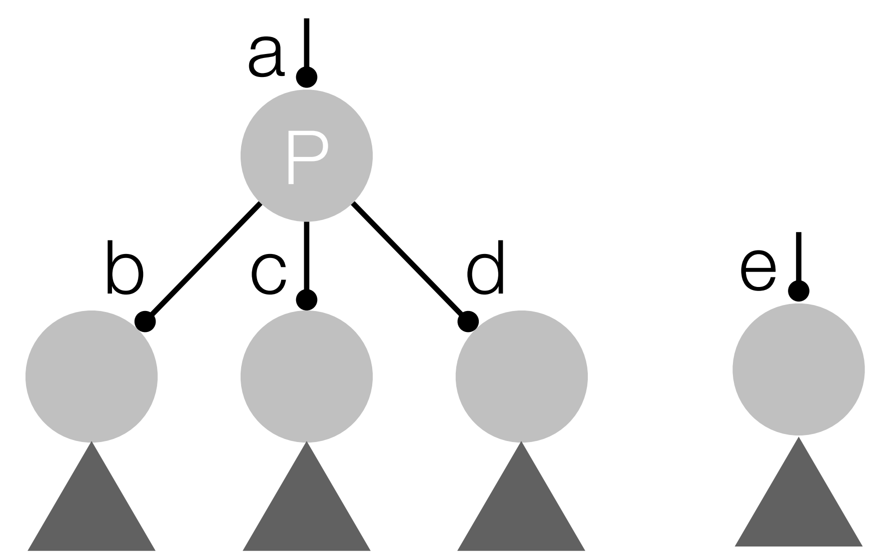

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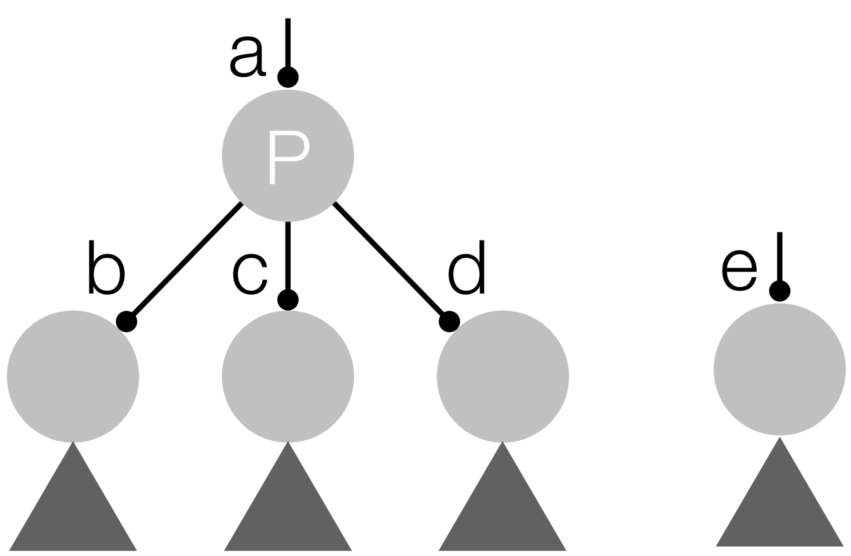
a poised process is ready to sync along offering channel



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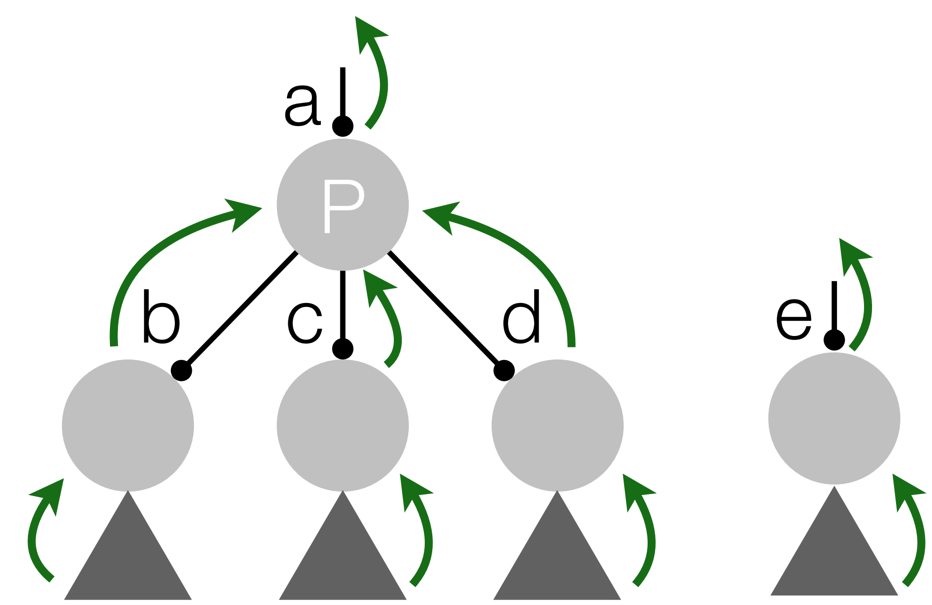
2. Ω *is poised.*

every process poised

a poised process is ready to sync along offering channel



Taking stock

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➡ linear session type language

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		$A \multimap B$	multiplicative implication	"channel input"
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		$!A$	"of course", persistent truth	"replication"