Session-Typed Concurrent Programming
Lecture 3

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Carnegie Mellon University

OPLSS 2021
June 25, 2021

Today’s lecture

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Recap

- Type system and dynamics for the intuitionistic linear session types language SILL
- Curry-Howard correspondence
- SILL readily guarantees session fidelity and deadlock-freedom

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- Type system and dynamics for the intuitionistic linear session types language SILL
- Curry-Howard correspondence
- SILL readily guarantees session fidelity and deadlock-freedom

Next

- Extend SILL with persistent truth (of course!)
- Then, switch gears and introduce shared session types

Follow-up on Slack

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$$\frac{\Delta_1 \vdash P :: (x : A) \qquad \Delta_2, x : A \vdash Q :: (z : C)}{\Delta_1, \Delta_2 \vdash x \leftarrow P; Q :: (z : C)}\ Cut$$

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\begin{array}{lll}
(\text{D-}Cut) & \mathsf{proc}(c, x \leftarrow P_x ; Q_x) & \\
& \longrightarrow\ \mathsf{proc}(a, [a/x]\, P_x), \mathsf{proc}(c, [a/x]\, Q_x) & \quad (\text{a fresh})
\end{array}
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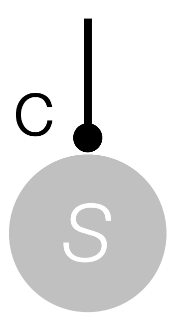

$$S = x \leftarrow P_x; Q_x$$

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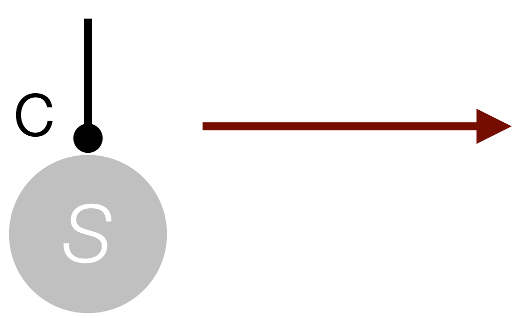

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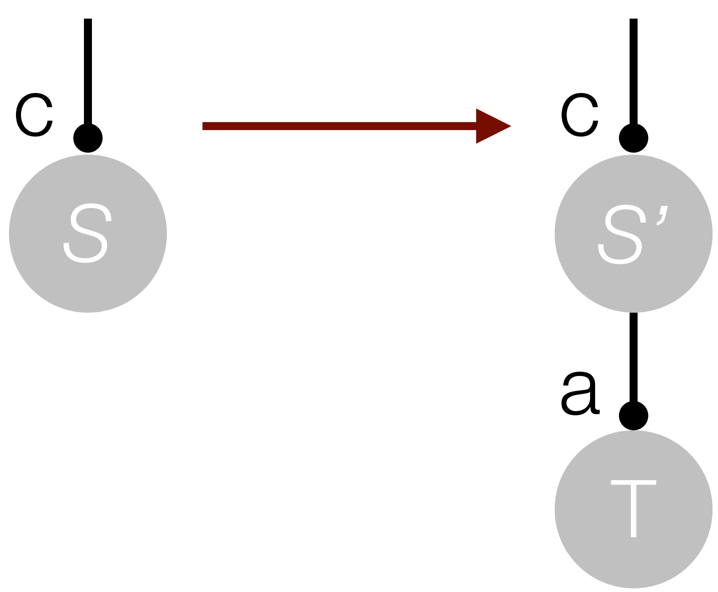

$S = x \leftarrow P_x ; Q_x$

$S' = [a/x]\, Q_x$

$T = [a/x]\, P_x$

Follow-up on Slack

homework:
make drawings for other
tensor and lolli!

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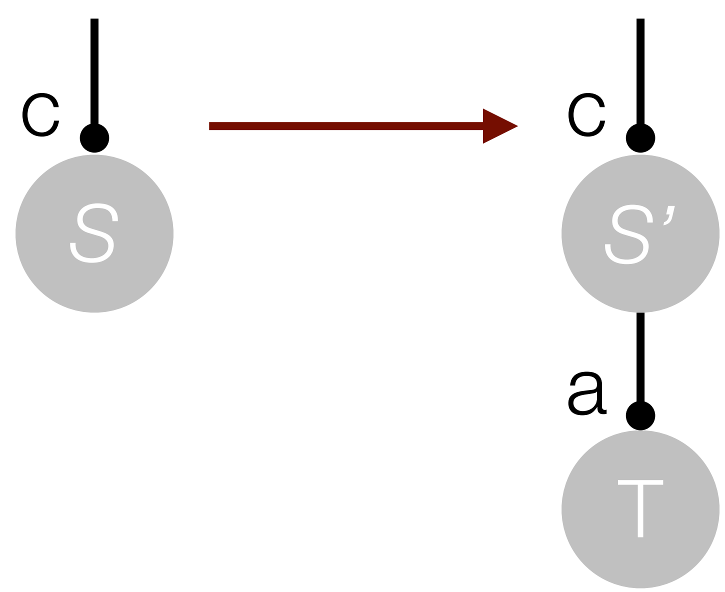

$$S = x \leftarrow P_x ; Q_x$$
$$S' = [a/x]\, Q_x$$
$$T = [a/x]\, P_x$$

Intuitionistic linear logic session types with !

Of course!

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one connective from linear logic still missing: persistent truth

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Types:

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a process of type !A can be used arbitrarily often, i.e., can have any number of clients

Of course!

What is the computational meaning of “of course”?

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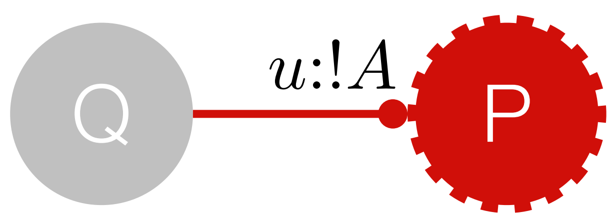

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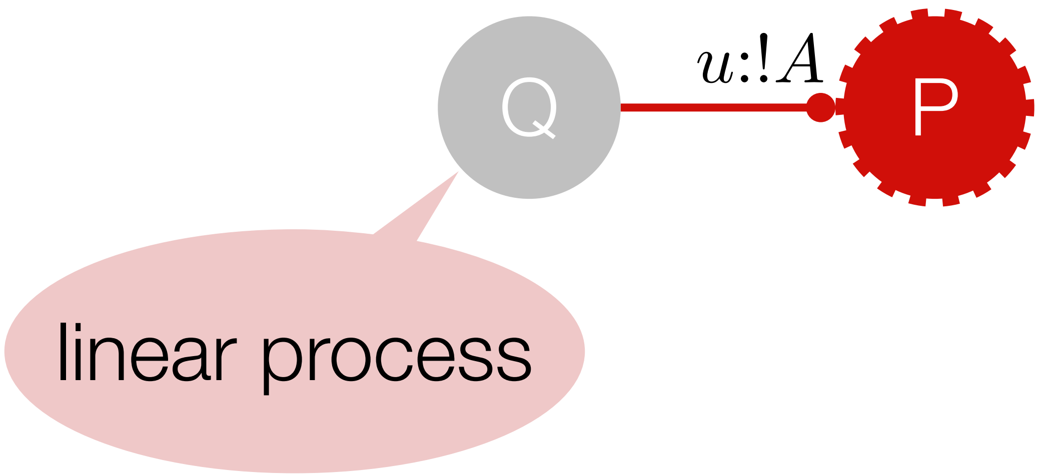

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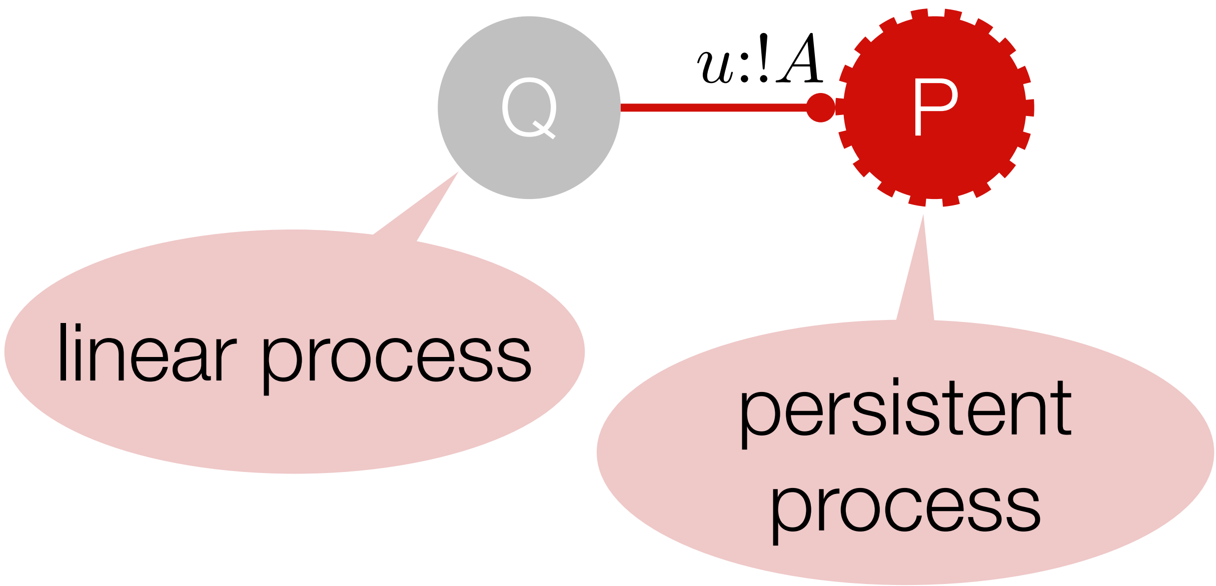

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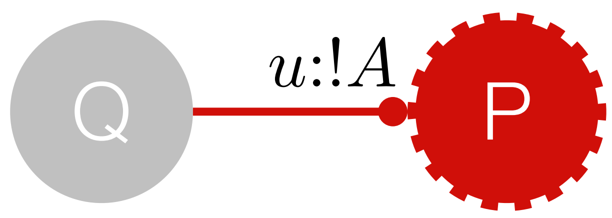

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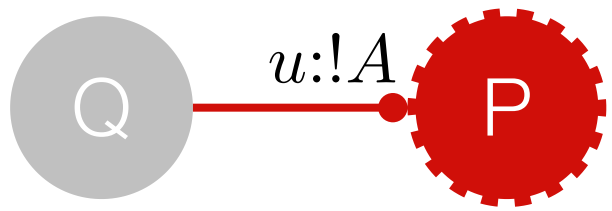

obtain a linear copy P’ of unrestricted process P

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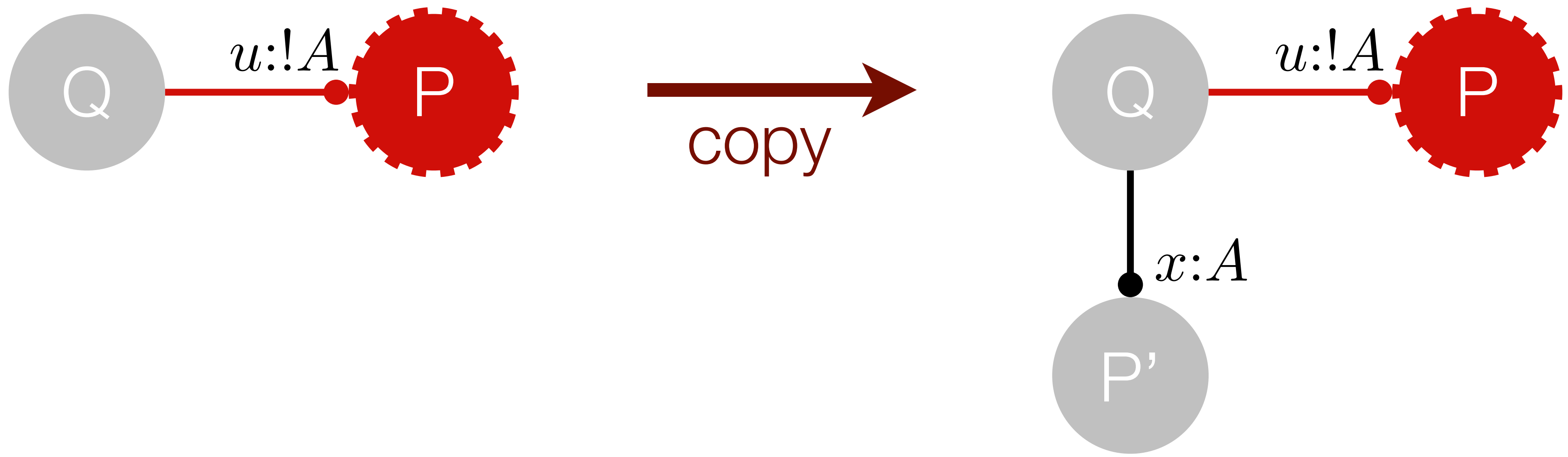

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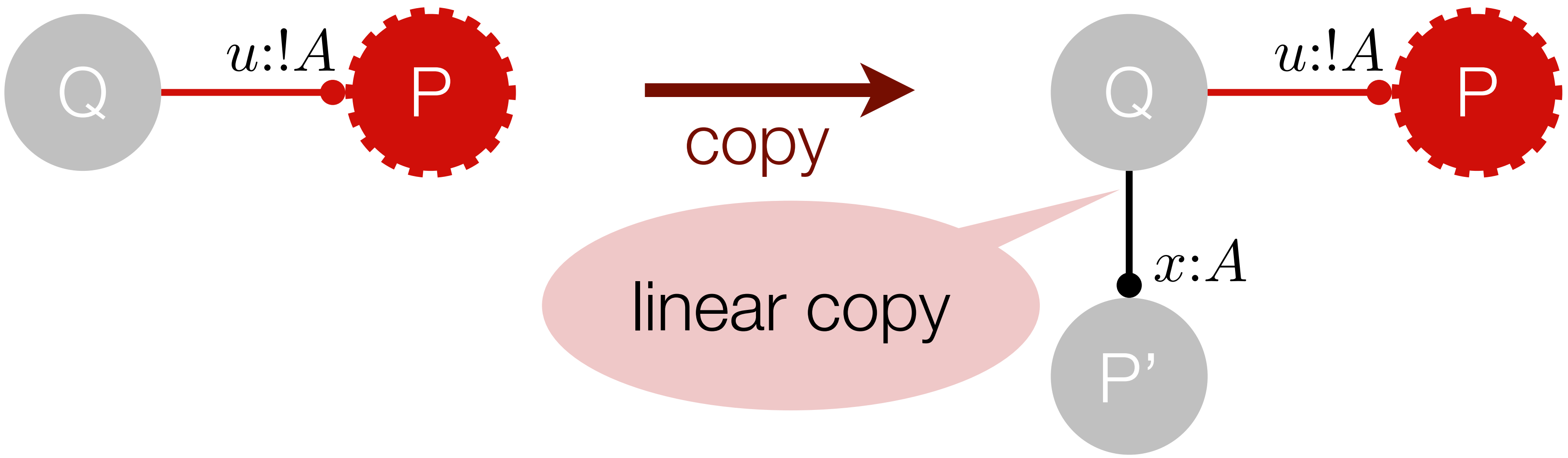

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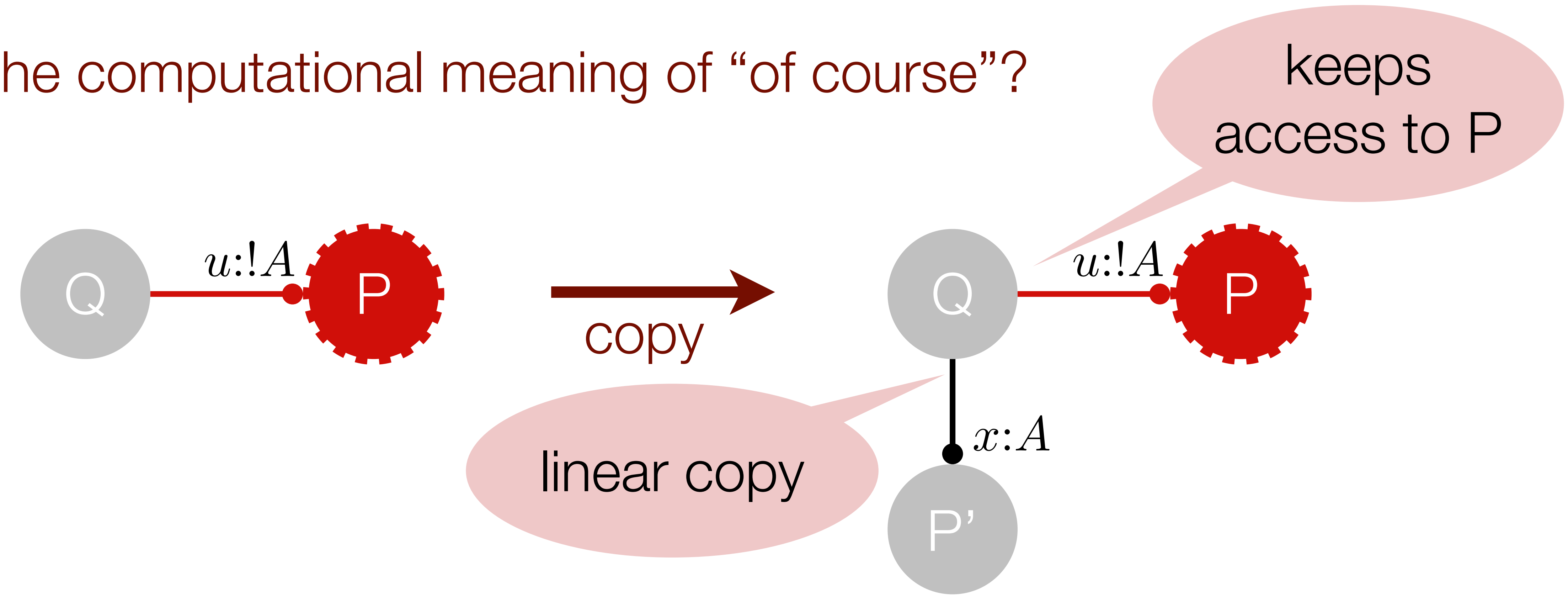

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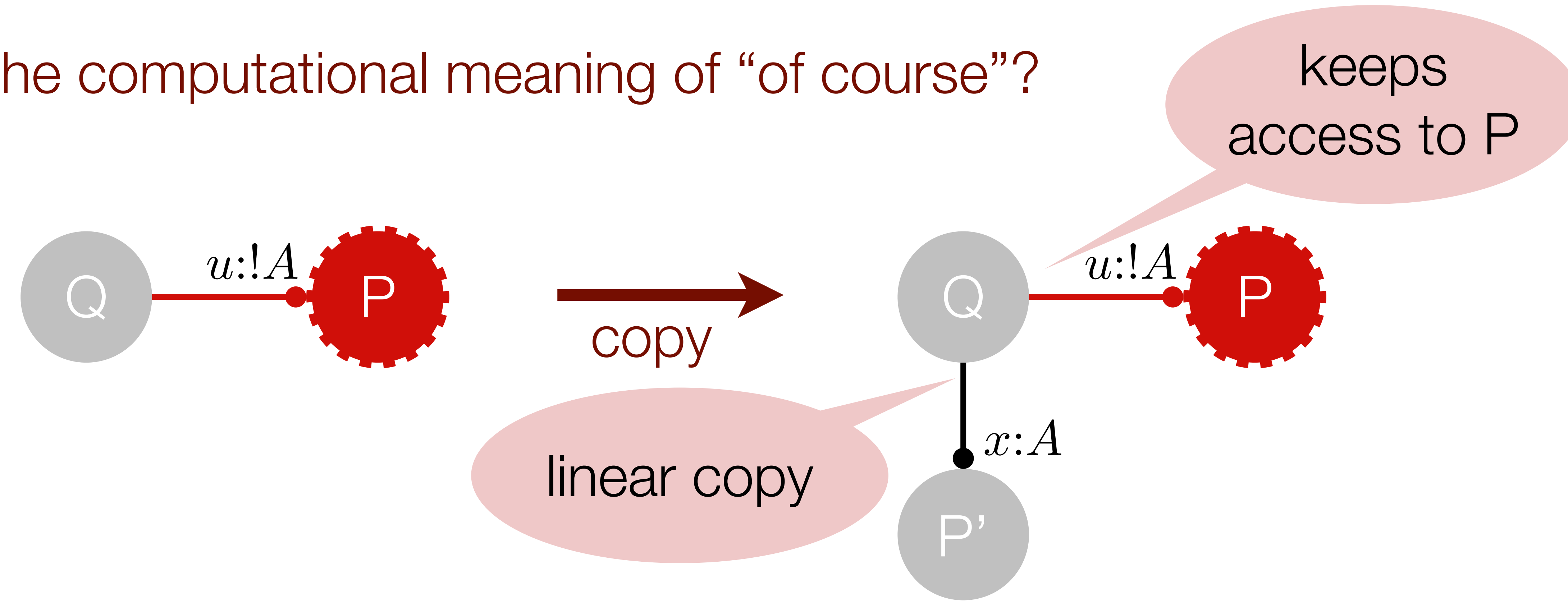

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- corresponds to replication in the pi-calculus

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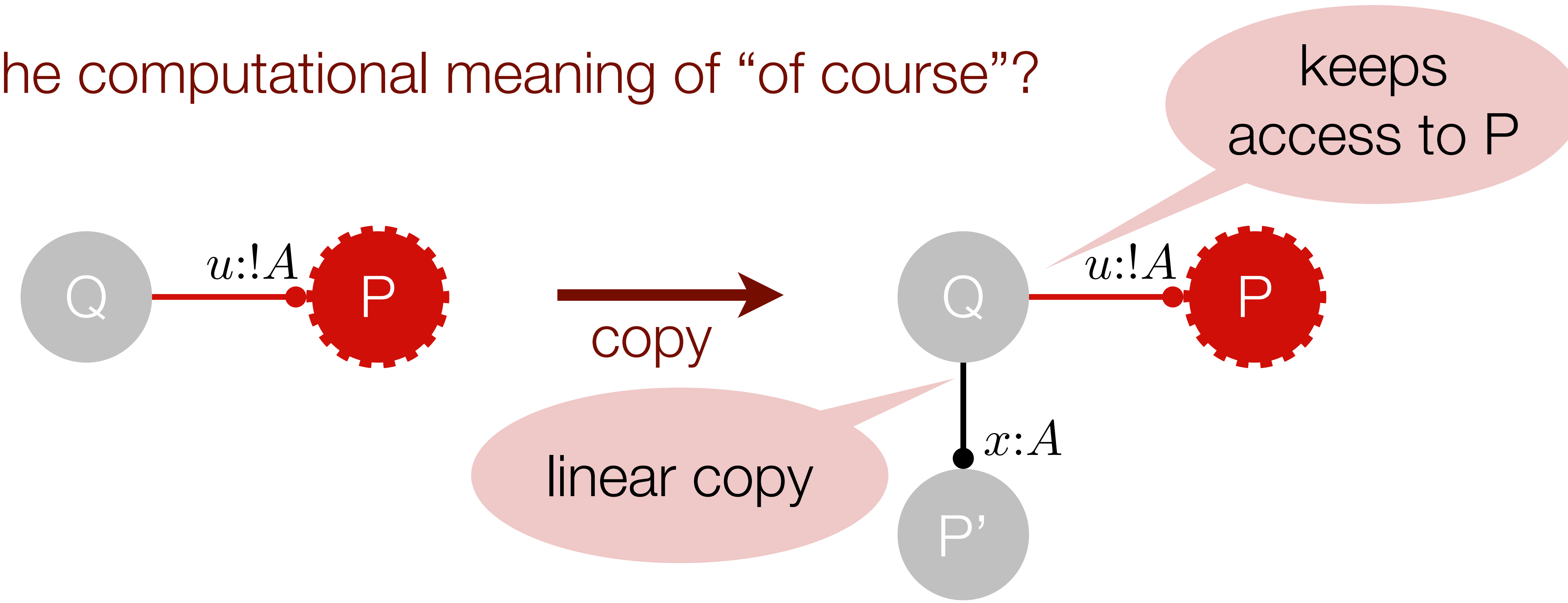

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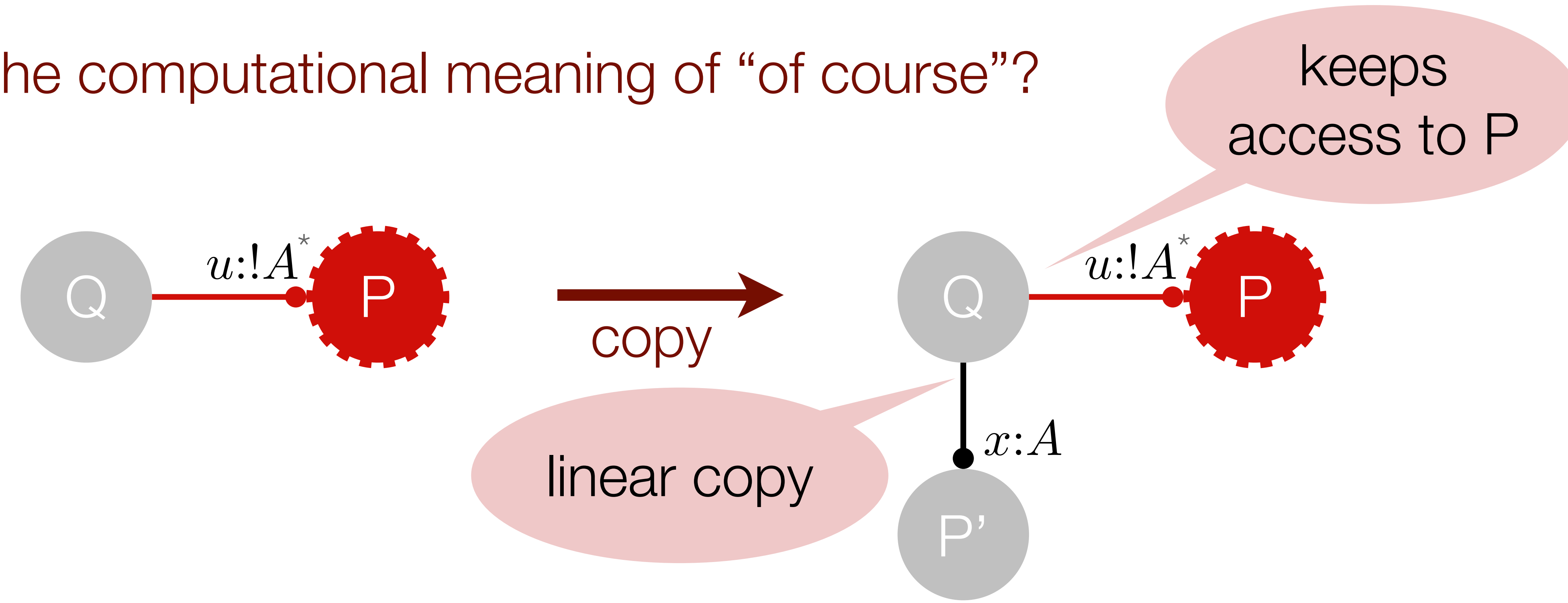

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- let’s look at typing rules and dynamics

(*) copy rule operates on structural context, so it should be u: A because A is judgmentally persistent

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Typing judgment:

$$\Psi; \Delta \vdash P :: (x : A)$$

Of course!

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structural context,
i.e., permits weakening and
contraction

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dyadic formulation

$$\Psi = u_1 : B_1, \ldots, u_n : B_n$$

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persistent channels in Δ are of type !A

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implicitly !-typed

Judgmental rule copy

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Typing rule:

$$\frac{\Psi, u : A; \Delta, x : A \vdash Q_x :: (z : C)}{\Psi, u : A; \Delta \vdash \mathsf{send}\, u\, (\mathsf{new}\, x); Q_x :: (z : C)}\ copy$$

Judgmental rule copy

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→ obtain a linear copy of a persistent server

Judgmental rule copy

Typing rule:

contraction!

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Dynamics:

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obtain a linear copy of a persistent server

Dynamics:

persistent!

remains available in post-state

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Typing rule:

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→ spawning a persistent server

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spawning a persistent server

Dynamics:

$$(\text{D-}cut!) \quad \textsf{proc}(c, u \leftarrow !(x \leftarrow \textsf{recv}\ u; P_x); Q_u)$$
$$\longrightarrow !\textsf{proc}(a, x \leftarrow \textsf{recv}\ a; P_x), \textsf{proc}(c, [a/u]\, Q_u) \qquad (\text{a fresh})$$

Rules for of course

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Typing rule:

$$\frac{\Psi; \cdot \vdash P_y :: (y : A)}{\Psi; \cdot \vdash \mathsf{send}\, x\, (\mathsf{new}\, u); !(y \leftarrow \mathsf{recv}\ u; P_y) :: (x :!A)}\ !_R$$

$$\frac{\Psi, u : A; \Delta \vdash Q_u :: (z : C)}{\Psi; \Delta, x :!A \vdash u \leftarrow \mathsf{recv}\ x; Q_u :: (z : C)}\ !_L$$

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→ spawning a persistent server

Dynamics:

$$\text{(D-!)} \quad \mathsf{proc}(a, \mathsf{send}\, a\, (\mathsf{new}\, u); !(y \leftarrow \mathsf{recv}\ u; P_y)), \mathsf{proc}(c, u \leftarrow \mathsf{recv}\ a; Q_u)$$
$$\longrightarrow !\mathsf{proc}(b, y \leftarrow \mathsf{recv}\ b; P_y), \mathsf{proc}(c, [b/u]\, Q_u) \qquad (\text{b fresh})$$

Taking stock

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Replication — clients are shielded from each others effects

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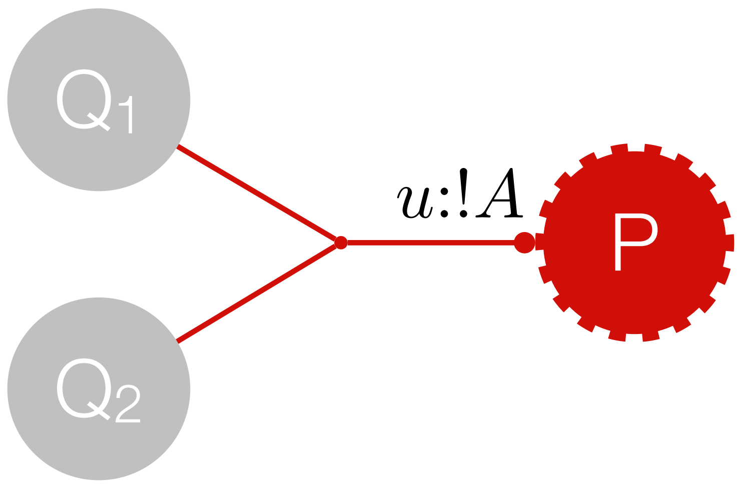

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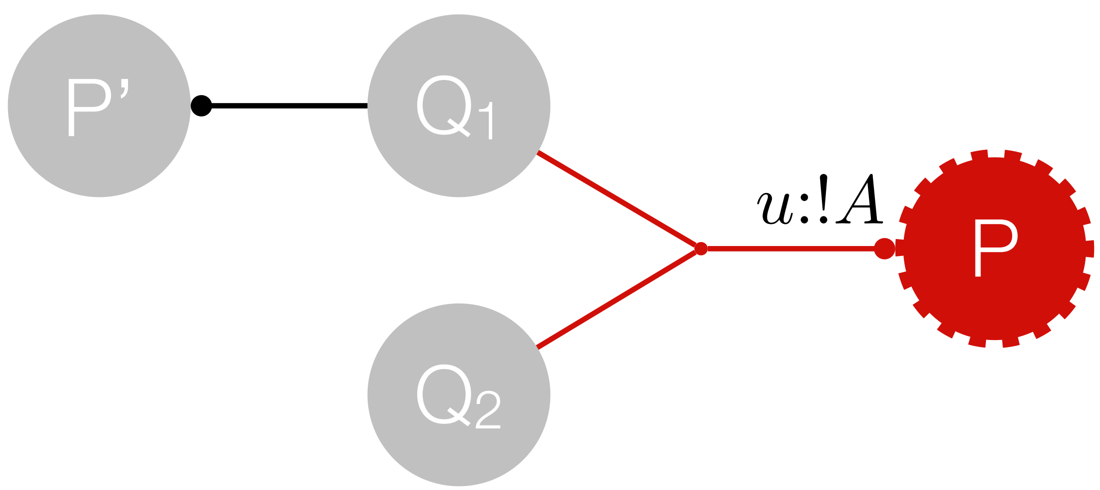

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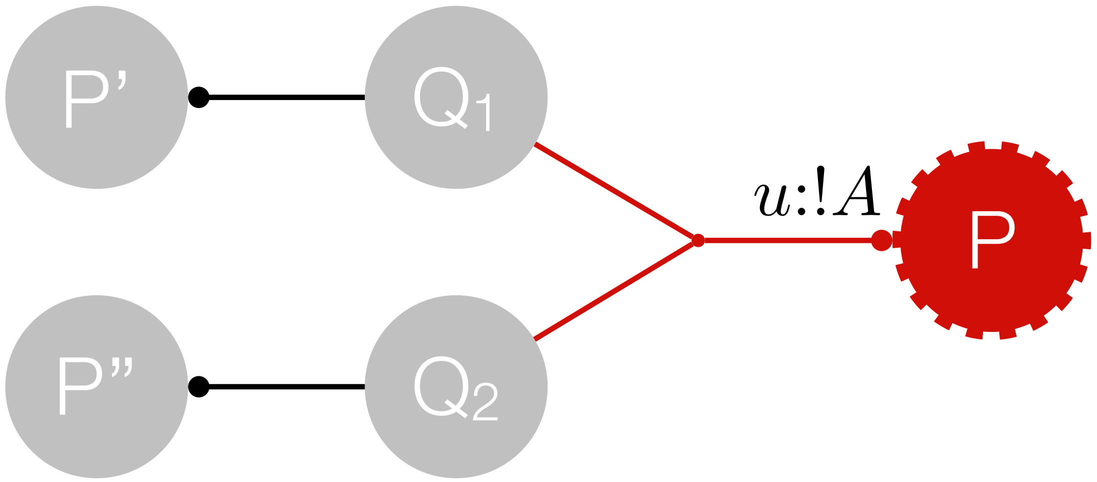

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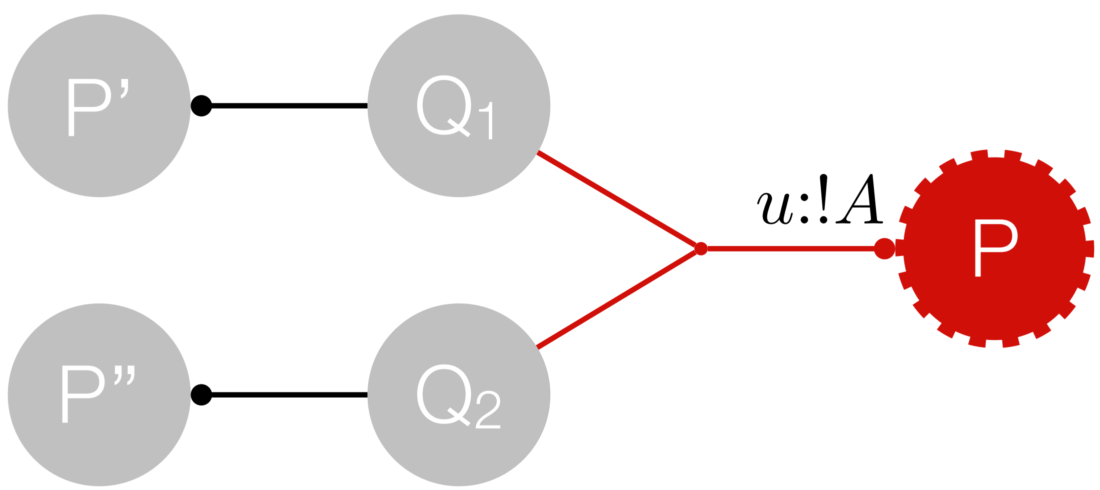
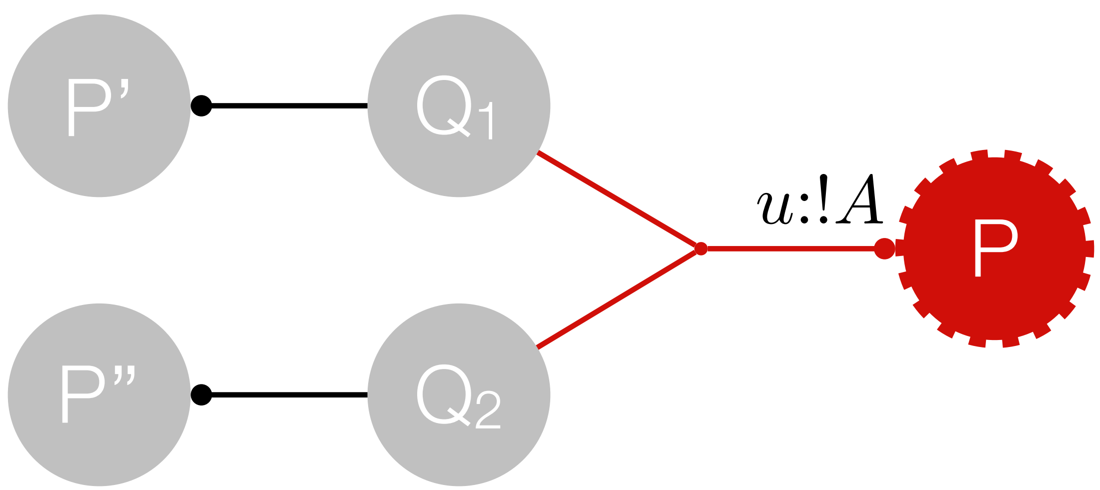


any communication of one client with its copy of P will not affect the private copies of P of other clients

Taking stock

Replication — clients are shielded from each others effects



any communication of one client with its copy of P will not affect the private copies of P of other clients

for some applications this copying semantics is appropriate

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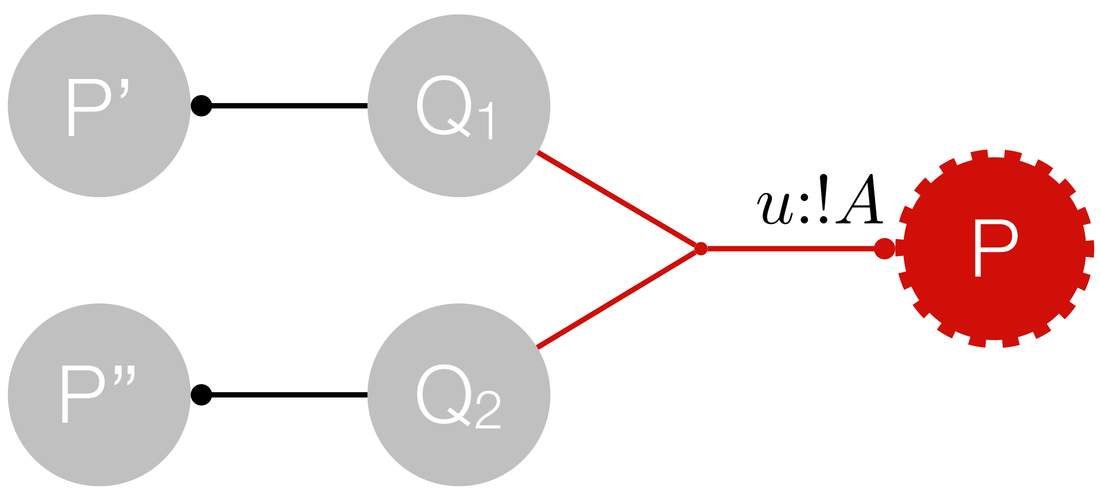

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- for some applications this copying semantics is appropriate
- other applications need a true sharing semantics

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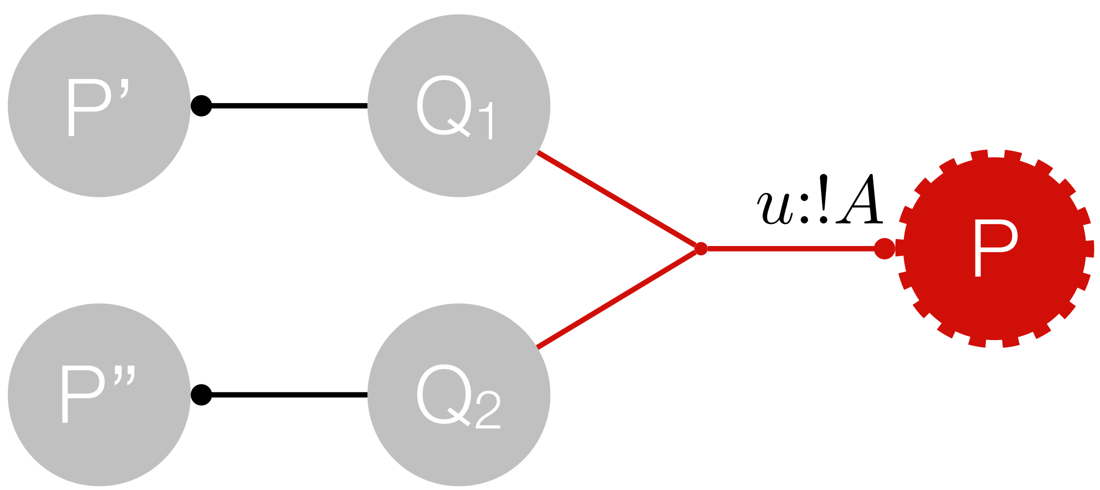

let's explore next!

- any communication of one client with its copy of P will not affect the private copies of P of other clients
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Manifest sharing

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Stephanie Balzer and Frank Pfenning. Manifest Sharing with Session Types. ICFP, 2017.

Copying versus sharing semantics

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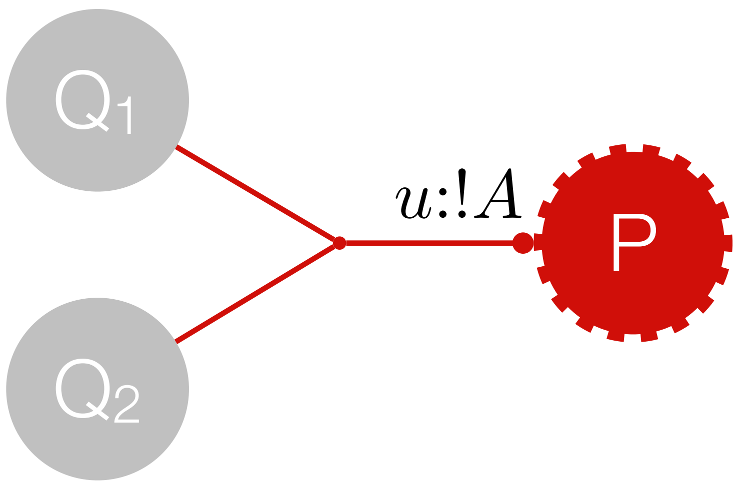
Copying semantics



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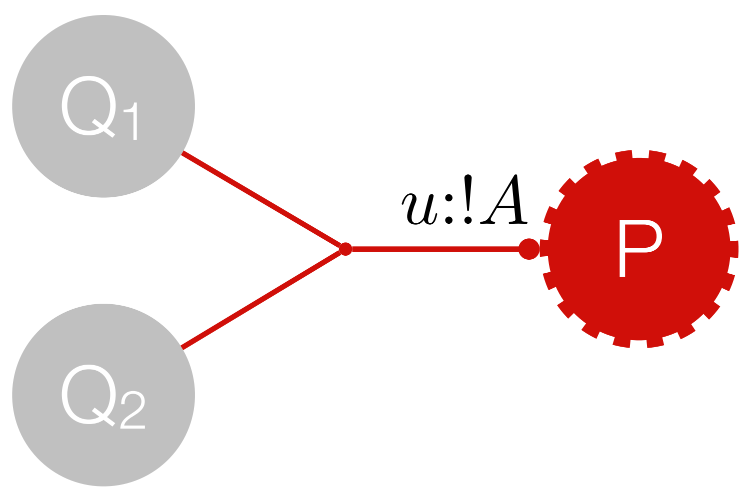
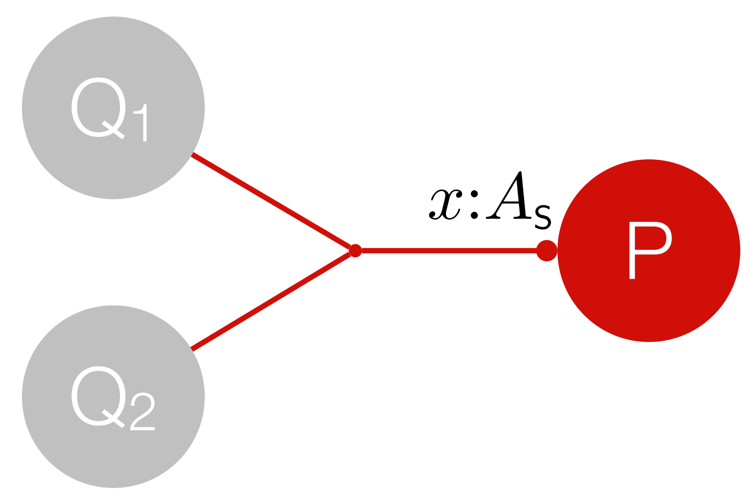


Sharing semantics



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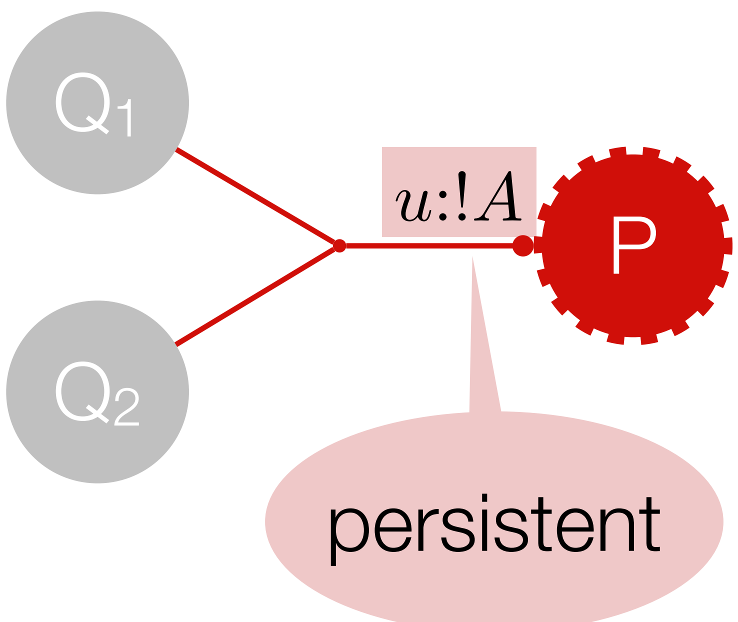

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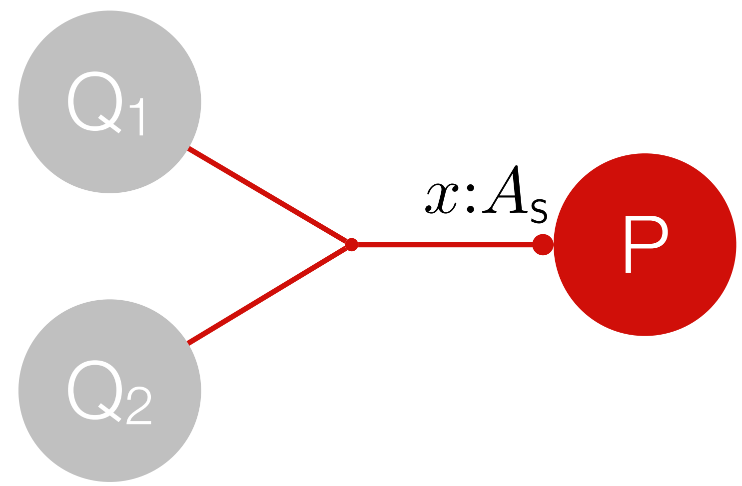

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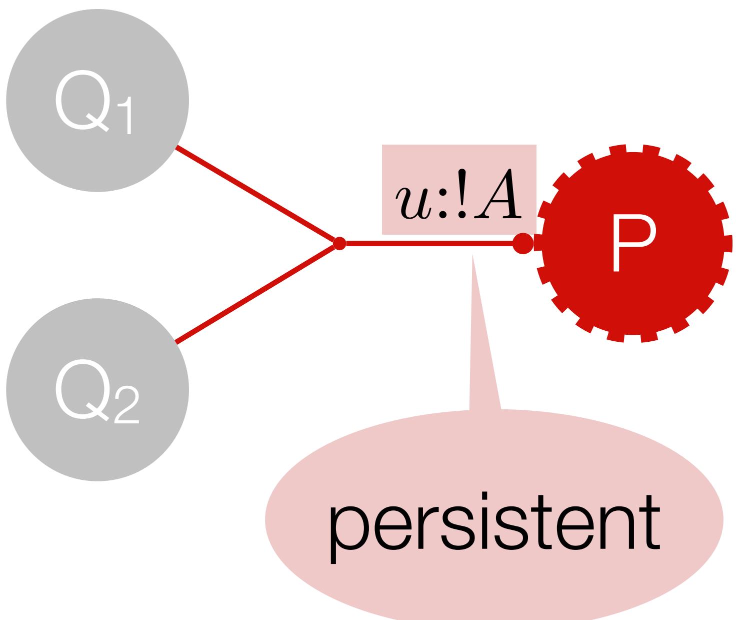

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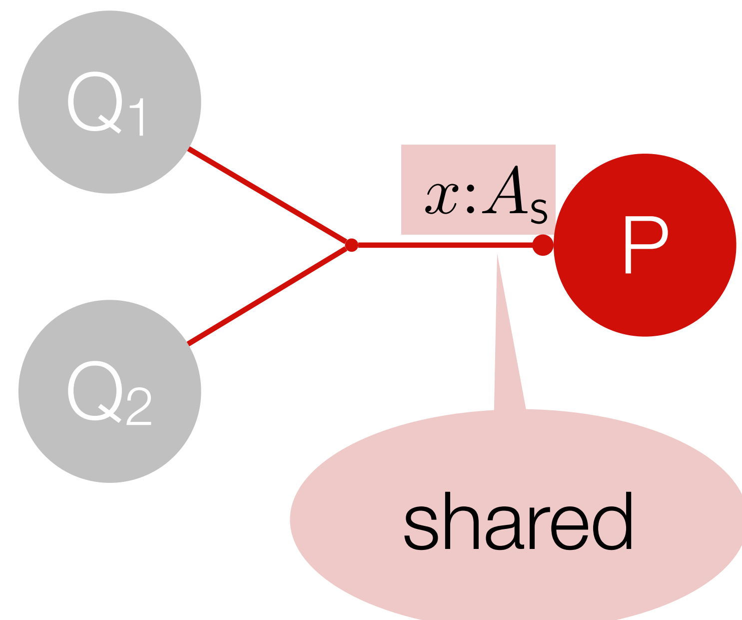

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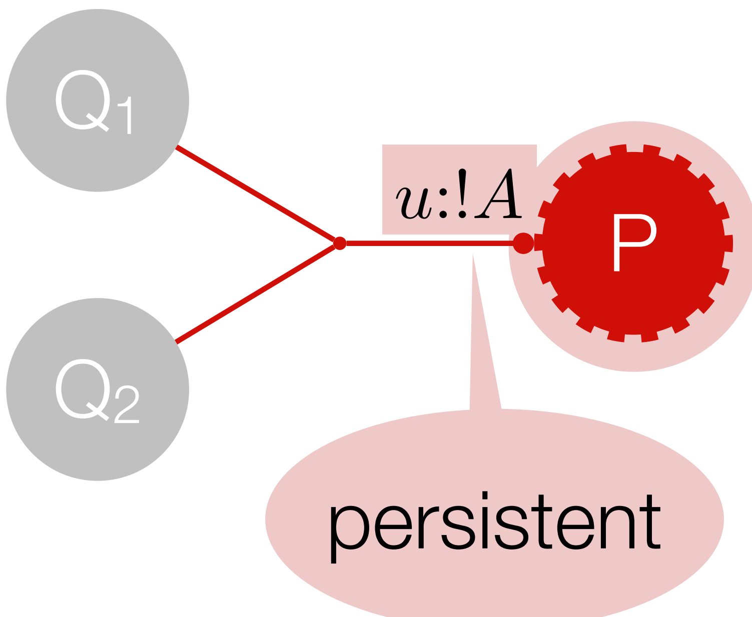

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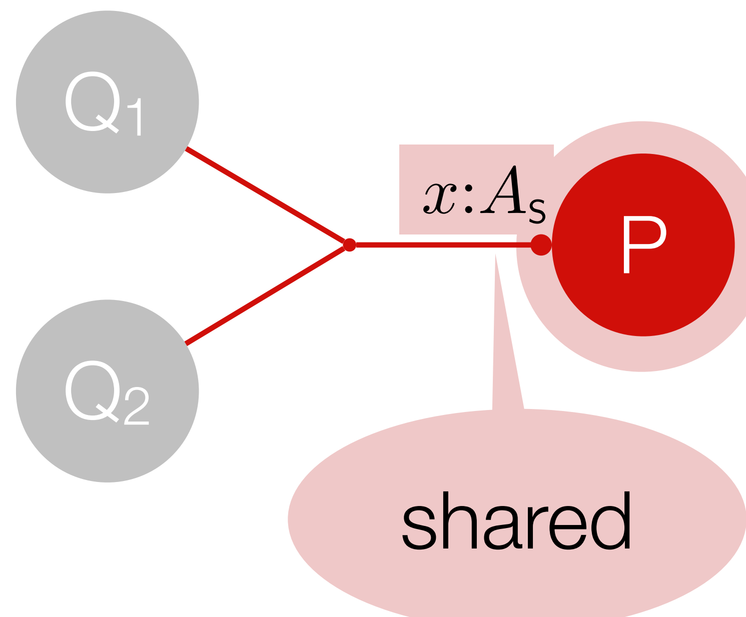

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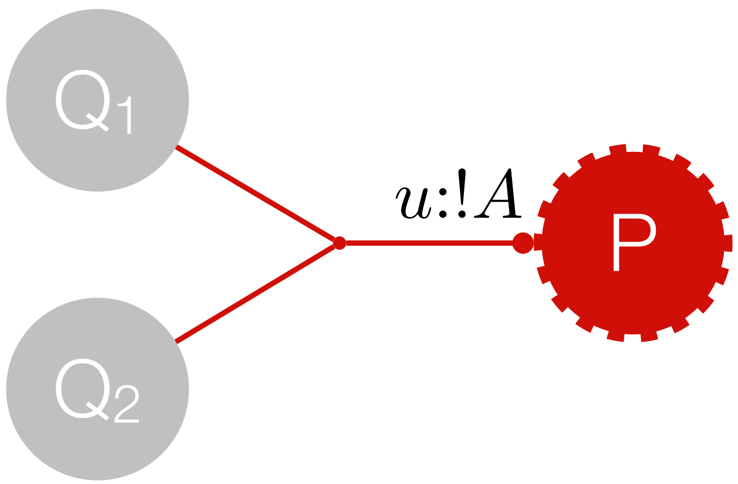

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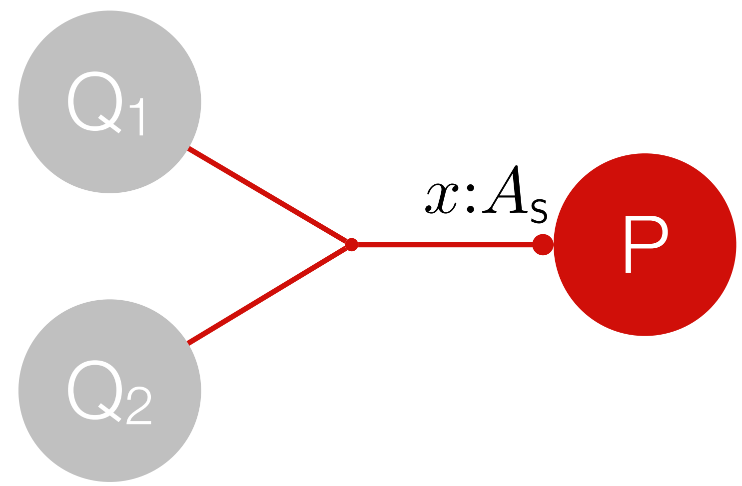

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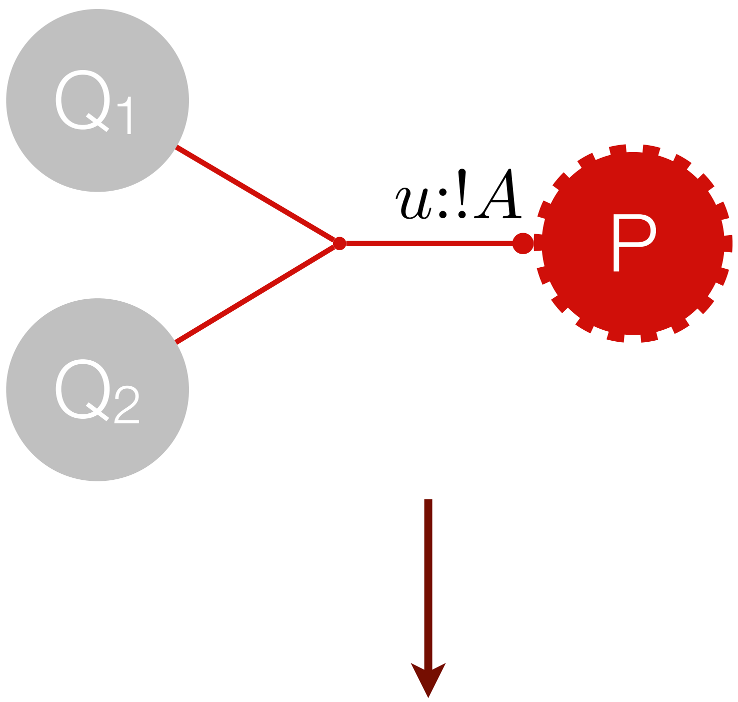

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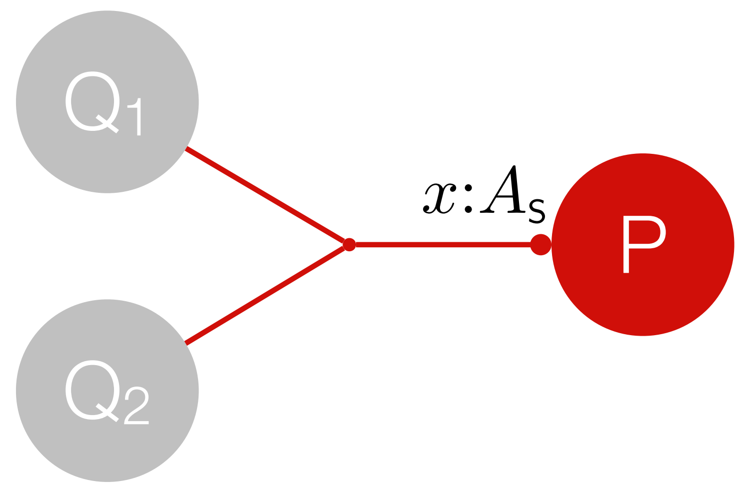

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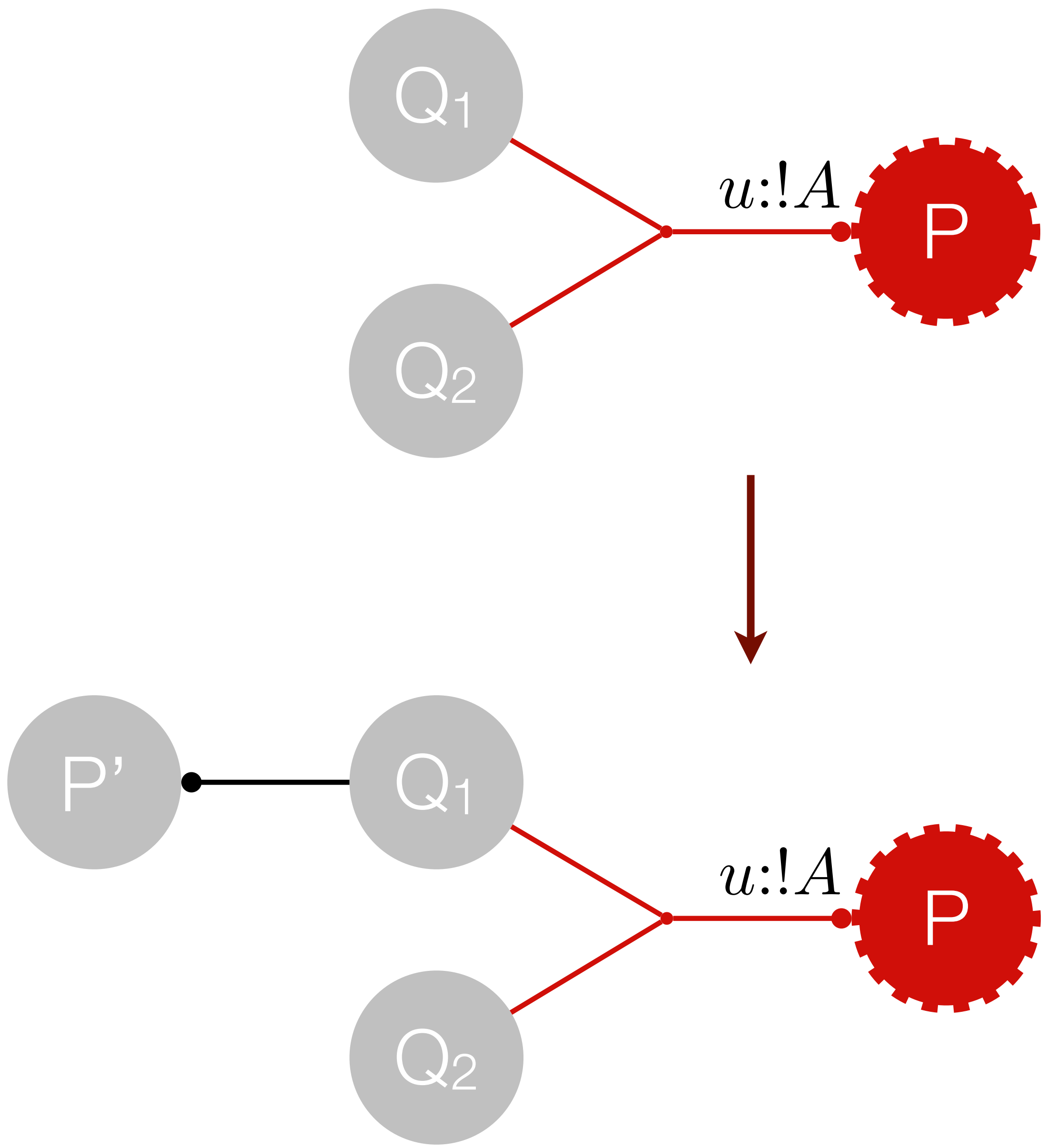

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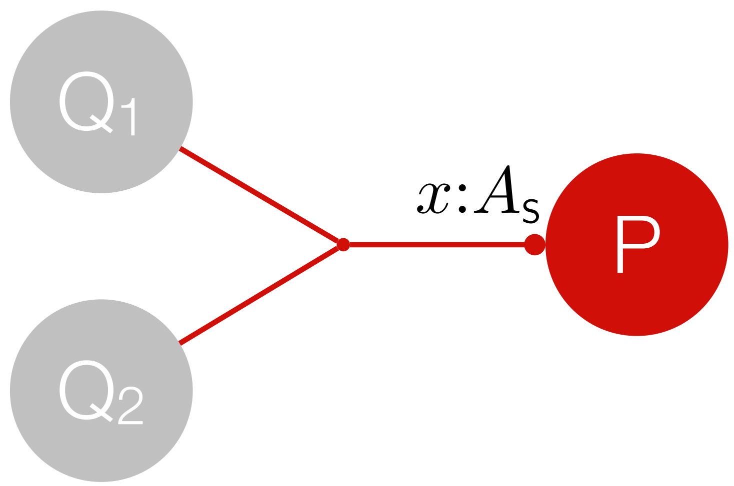

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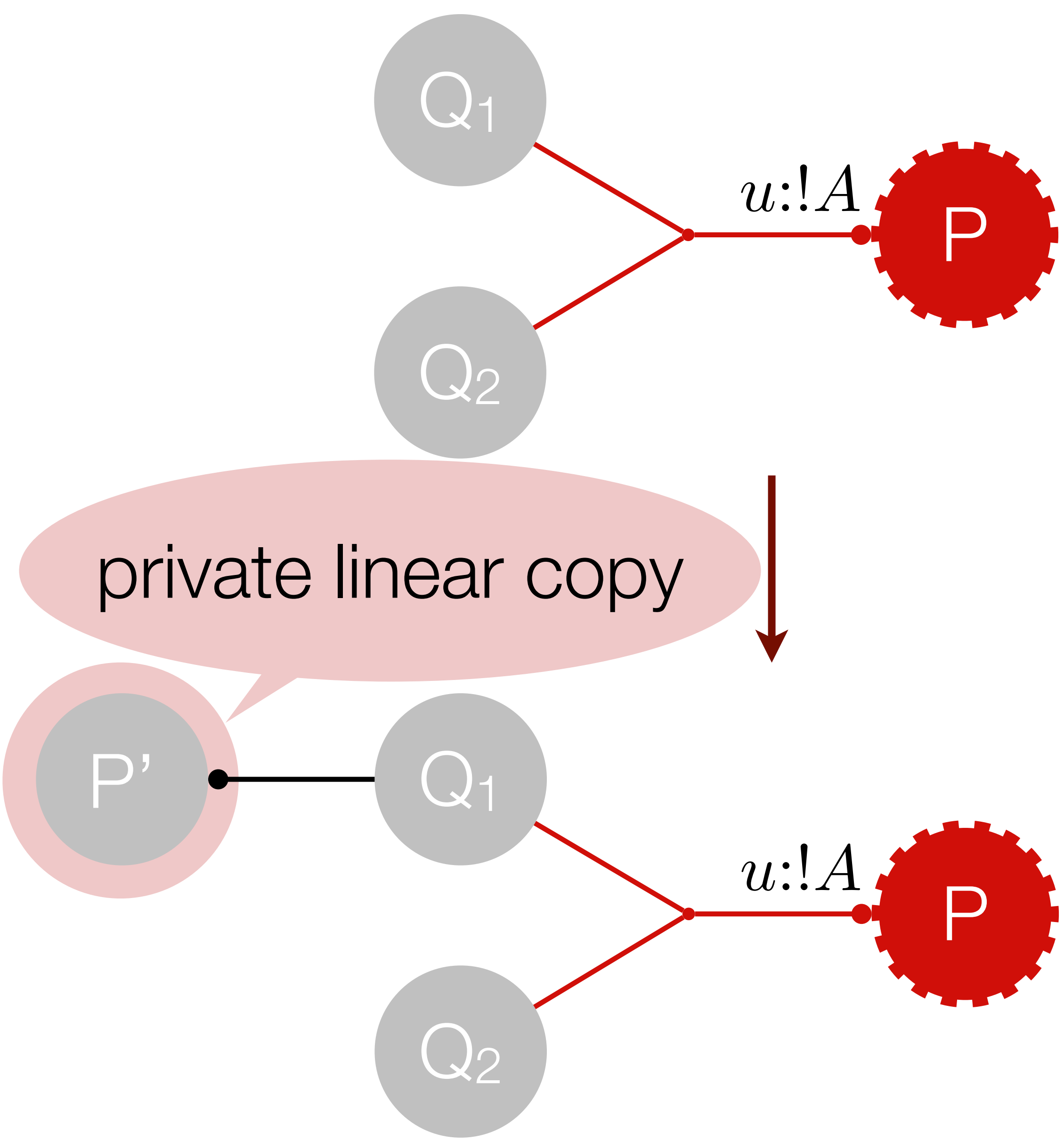

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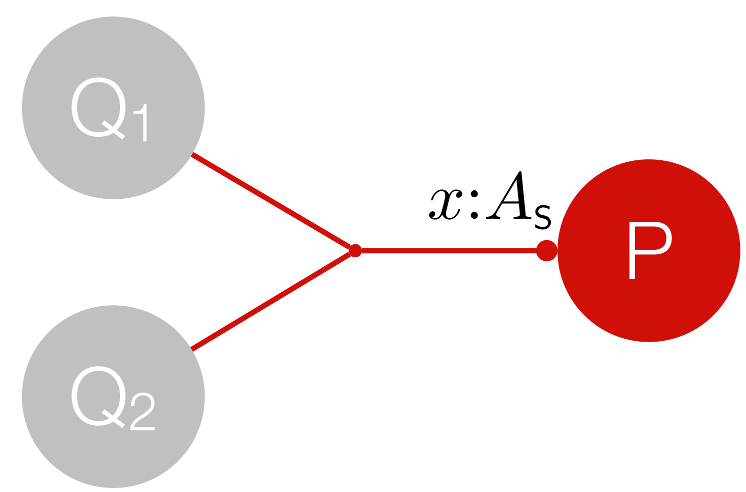

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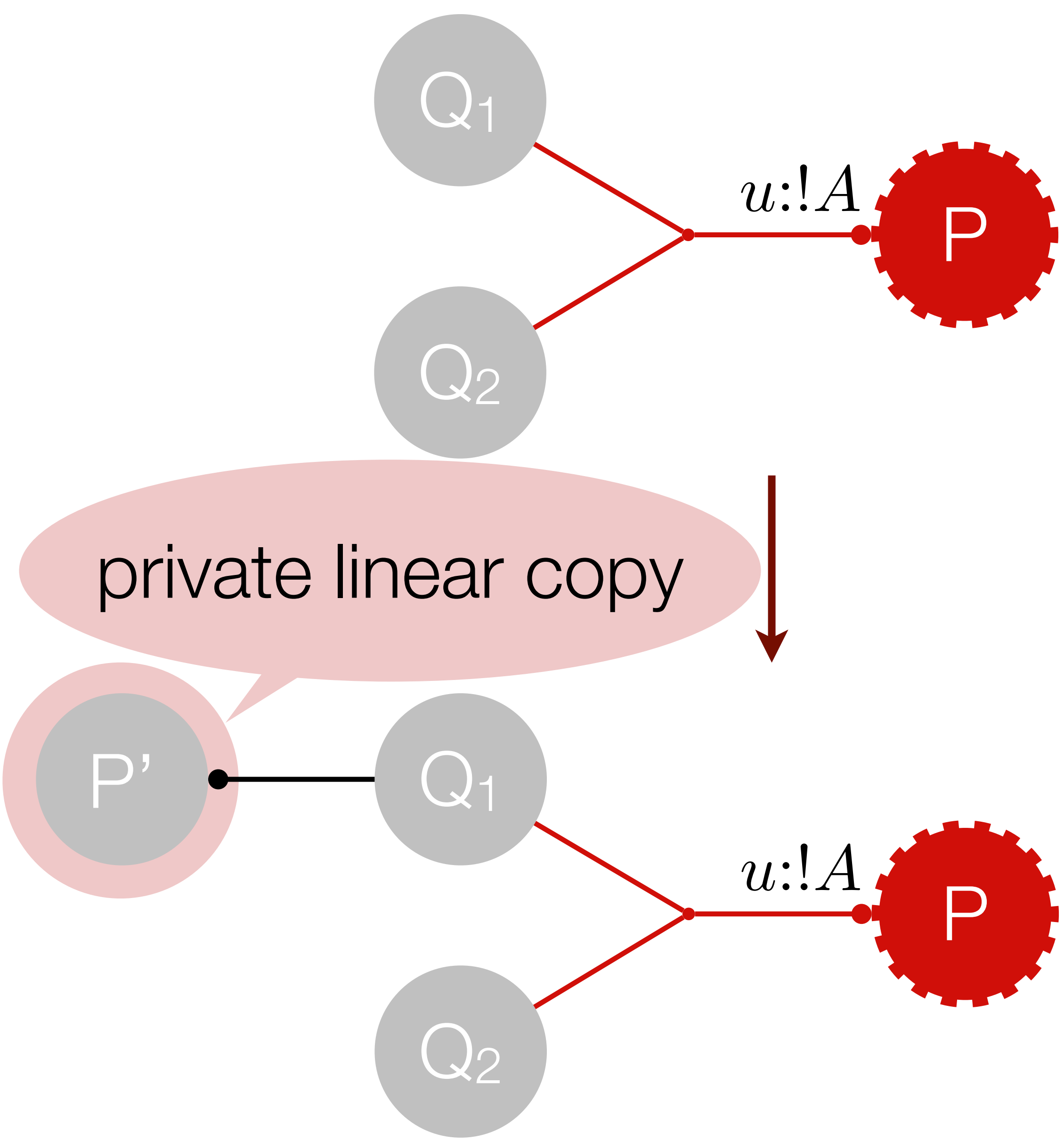

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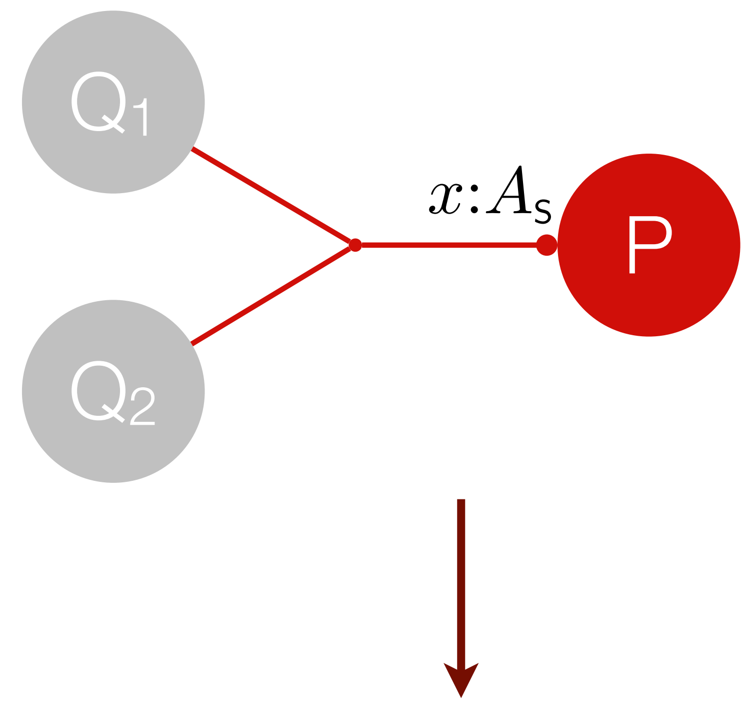

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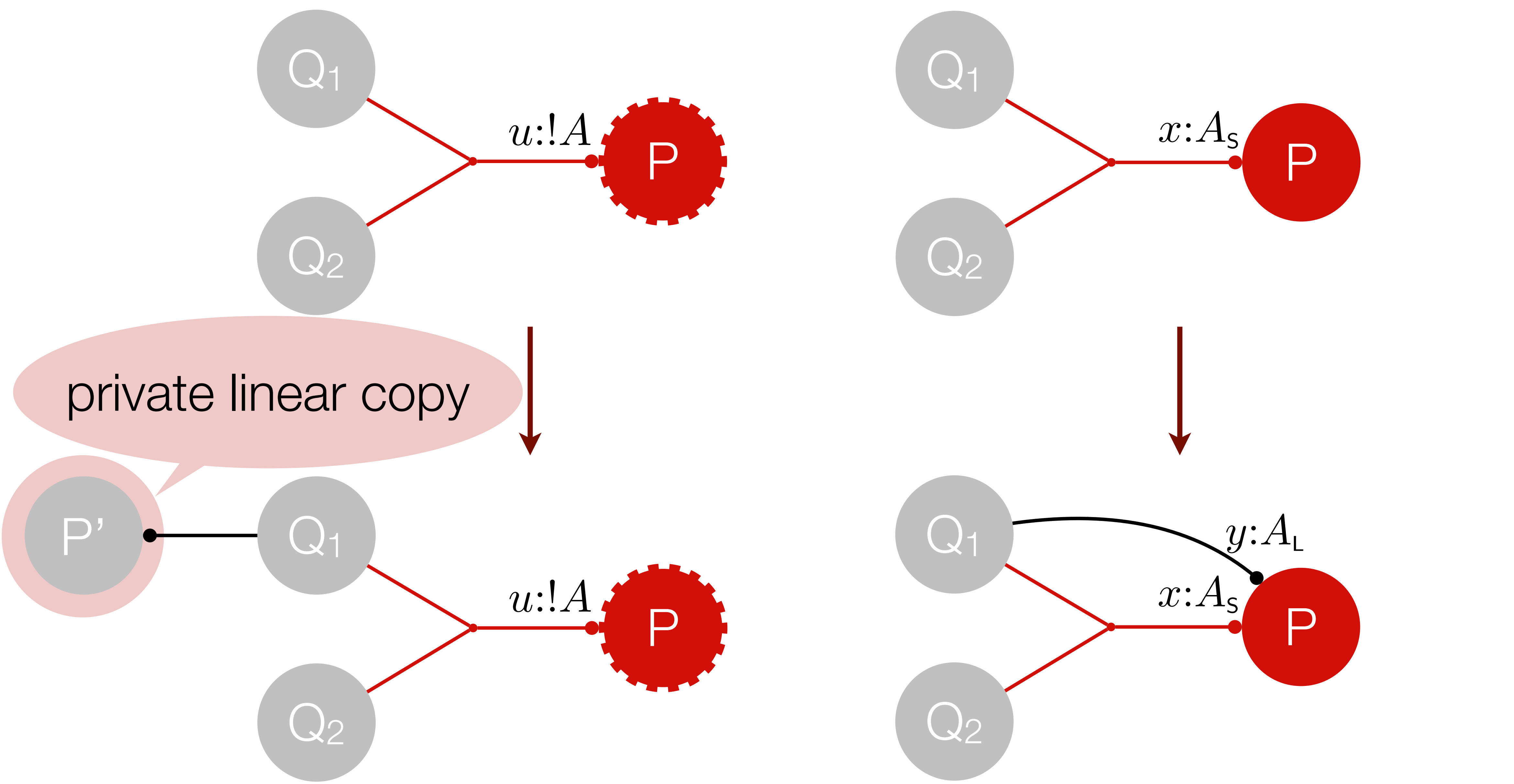

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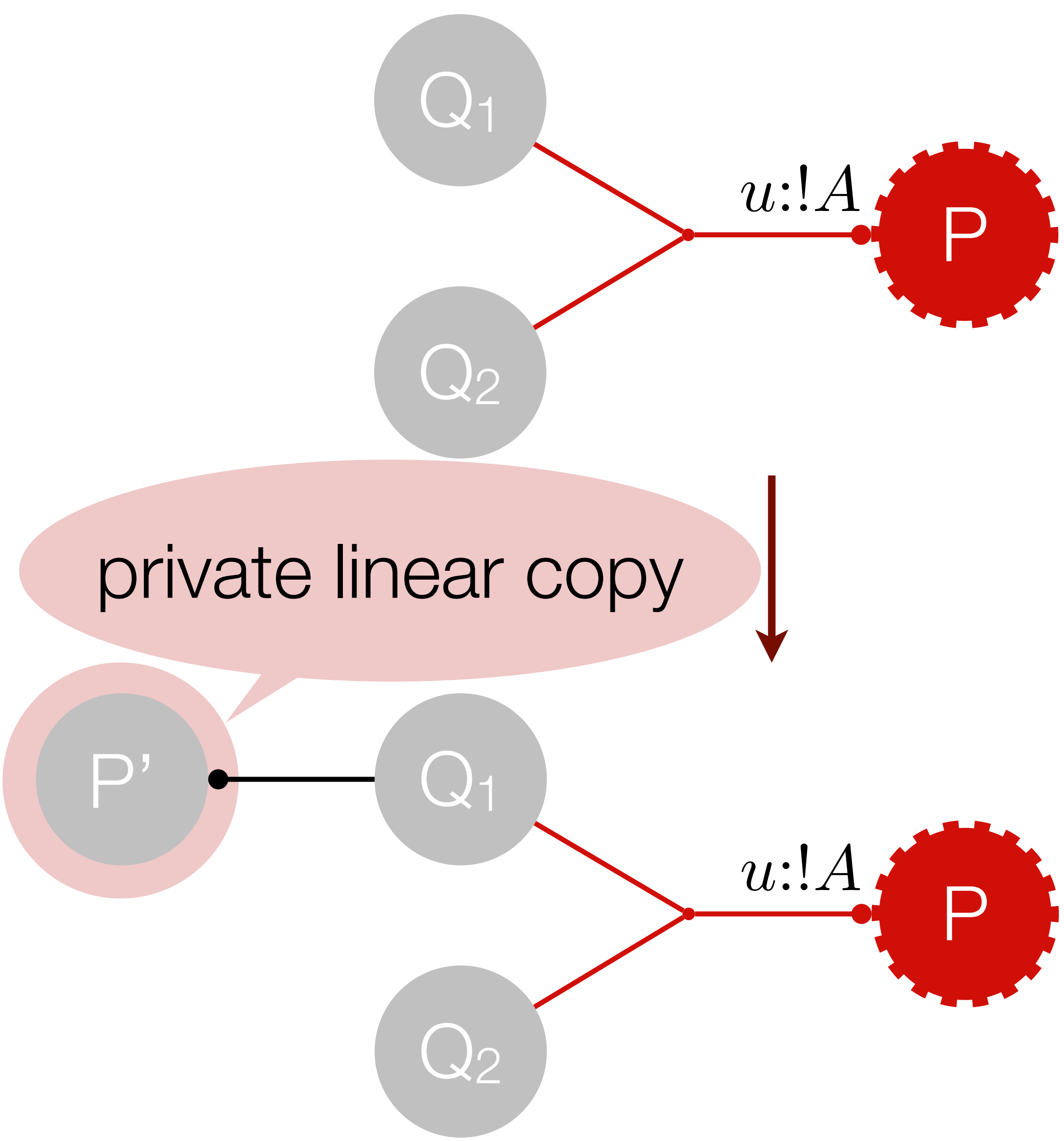

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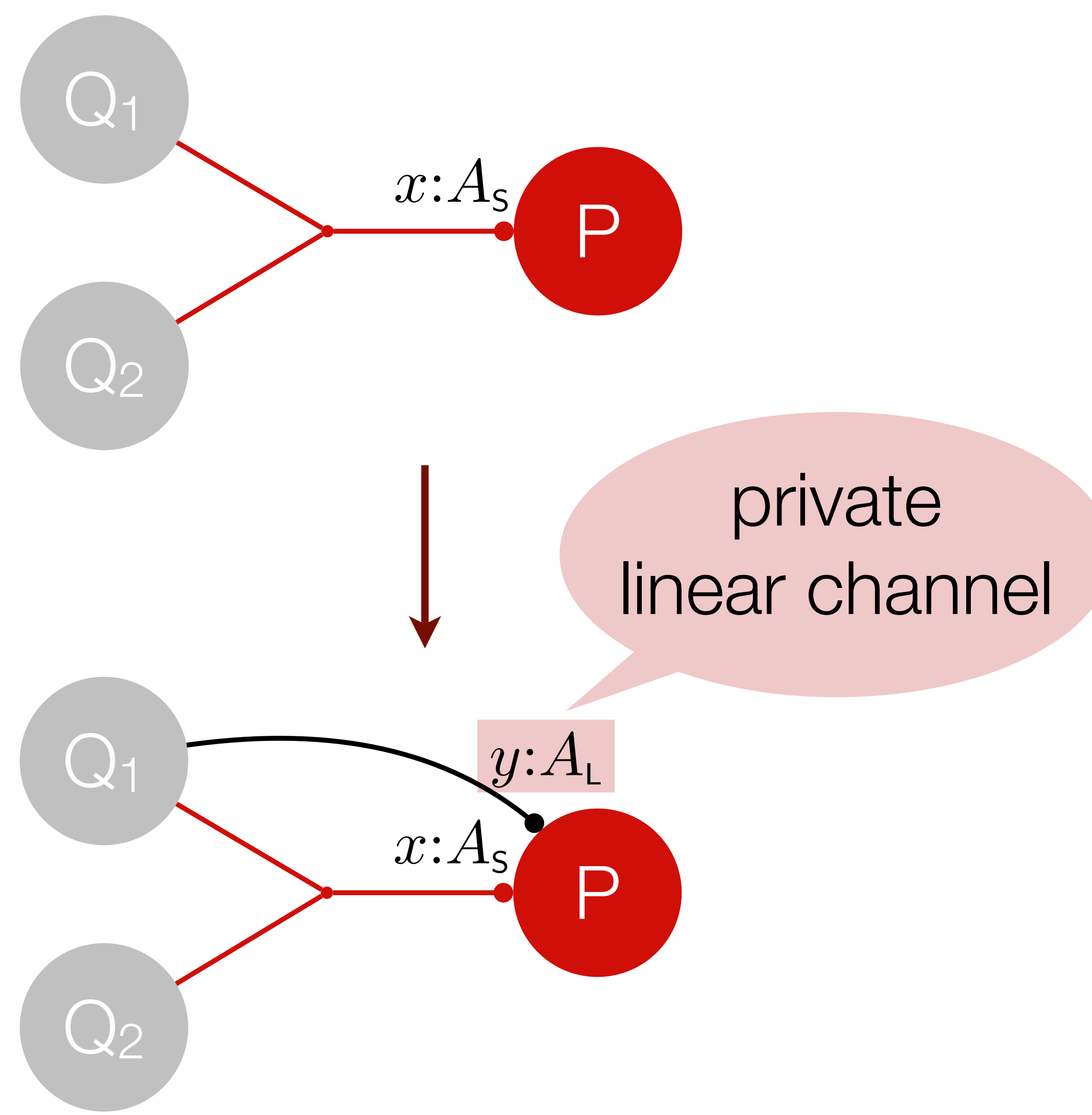

Key idea 1: acquire-release

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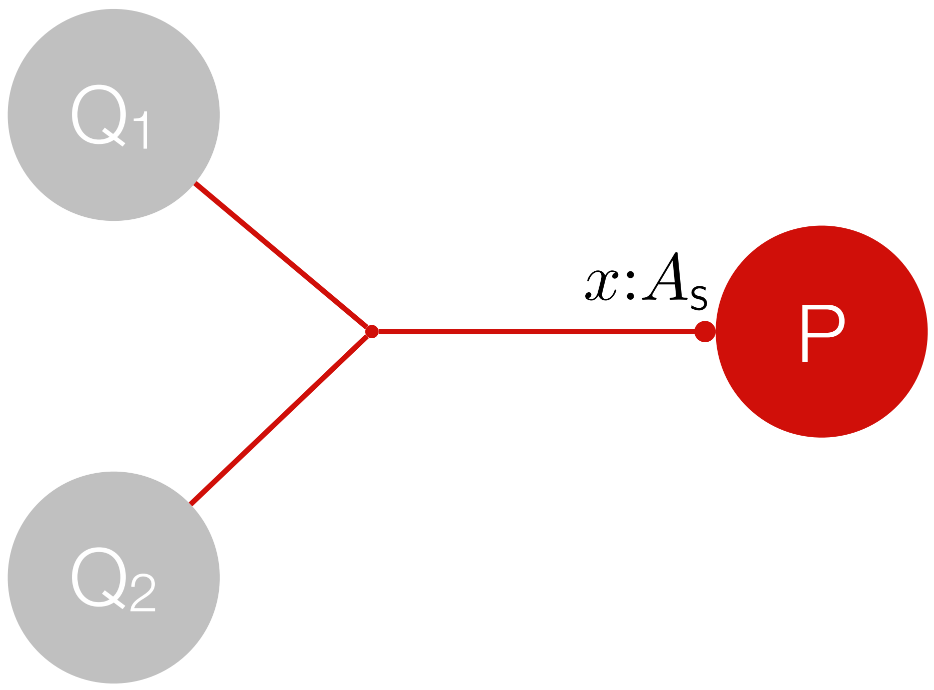

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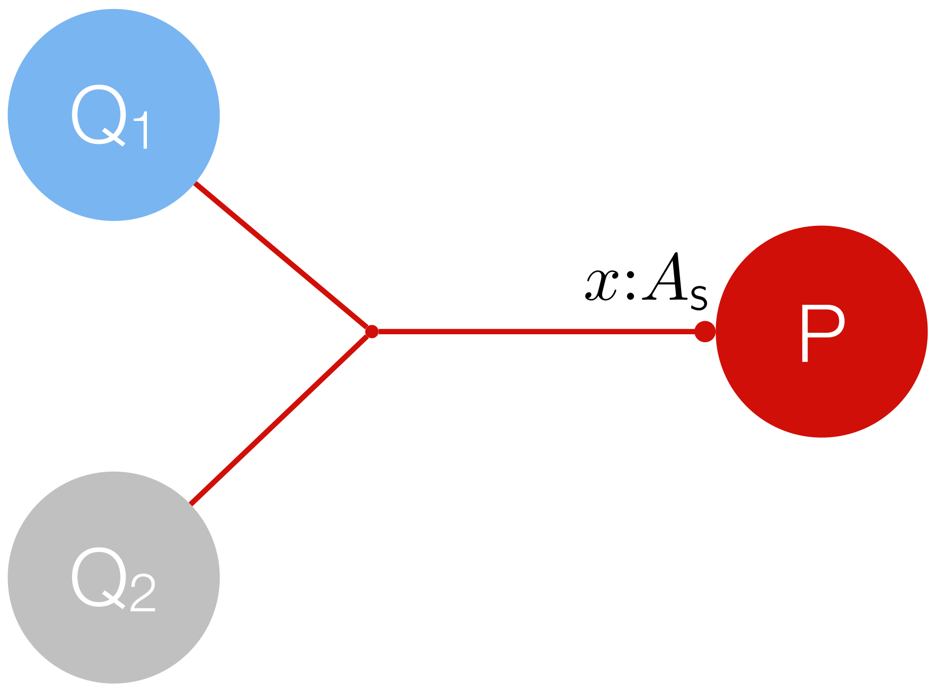

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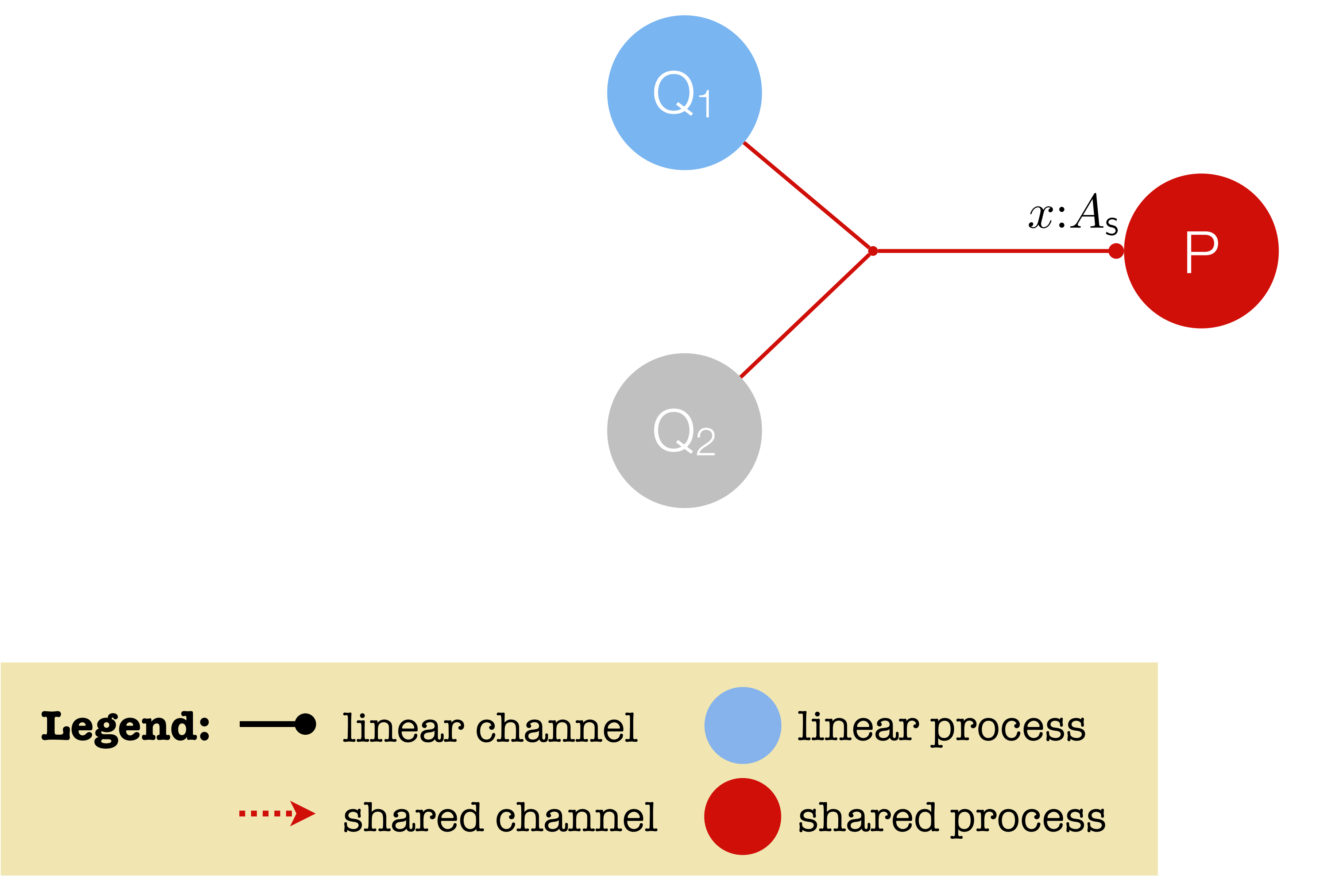

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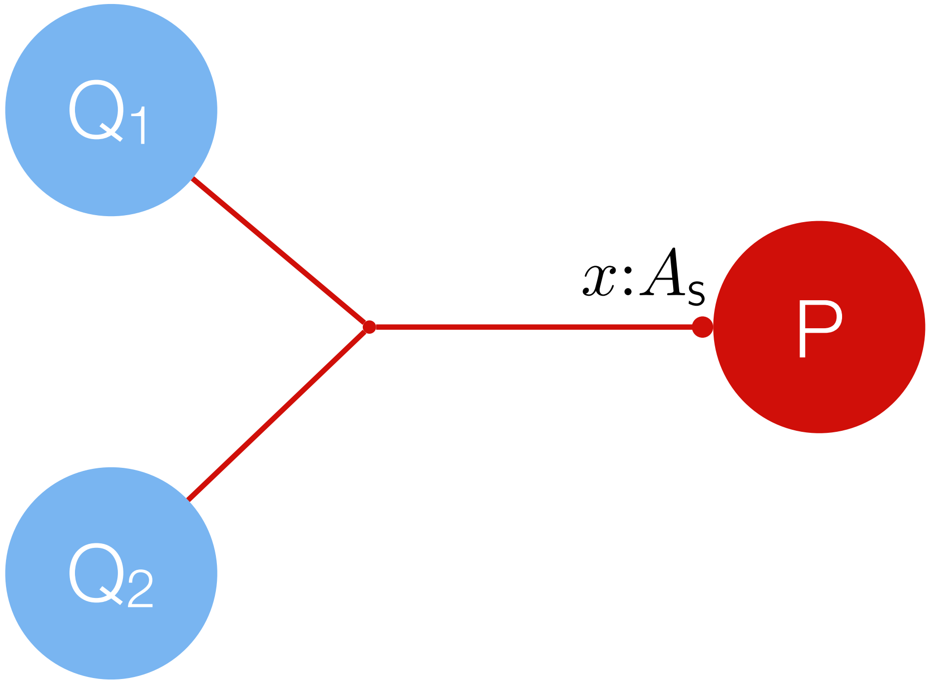

Legend: linear channel — linear process — shared channel — shared process

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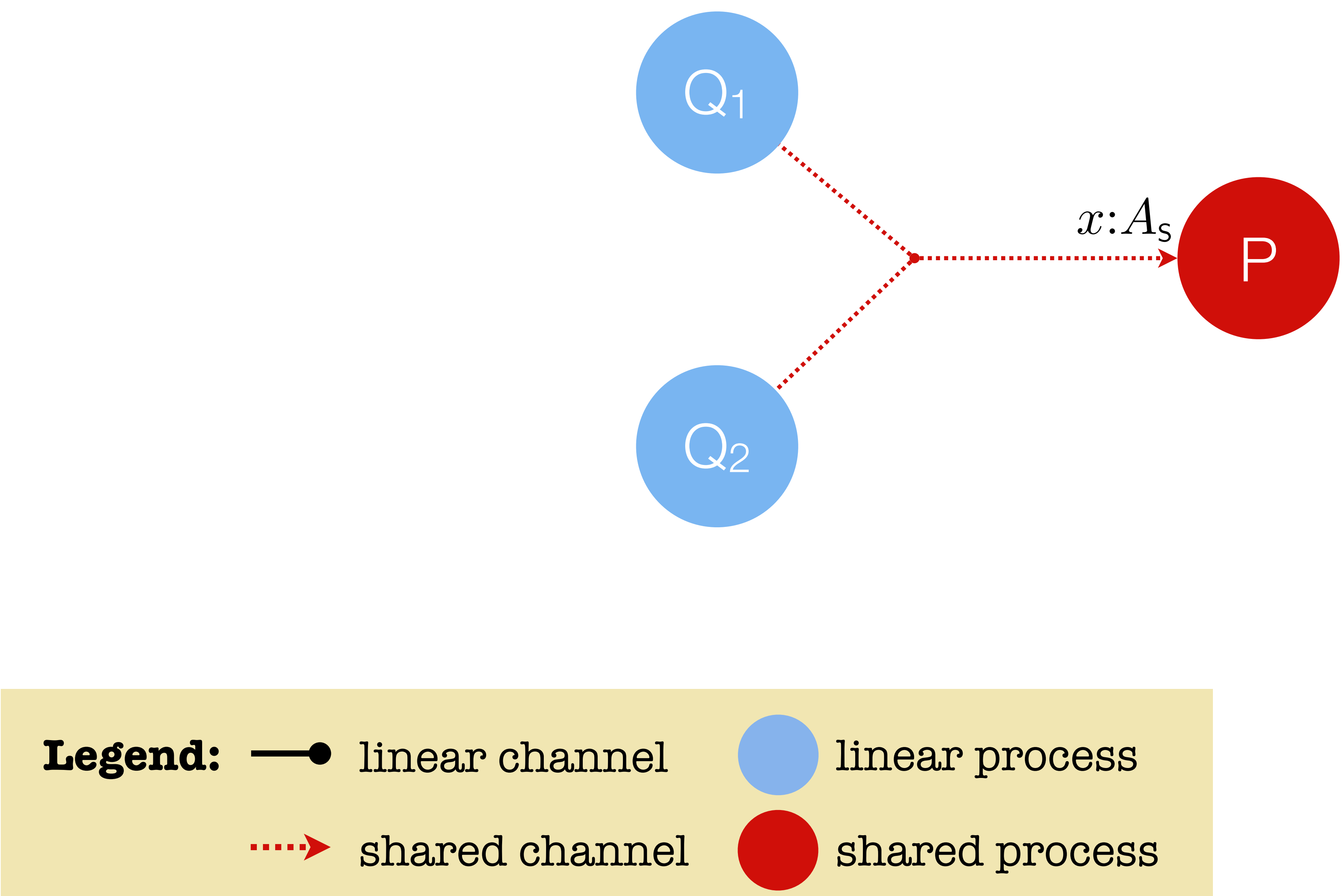

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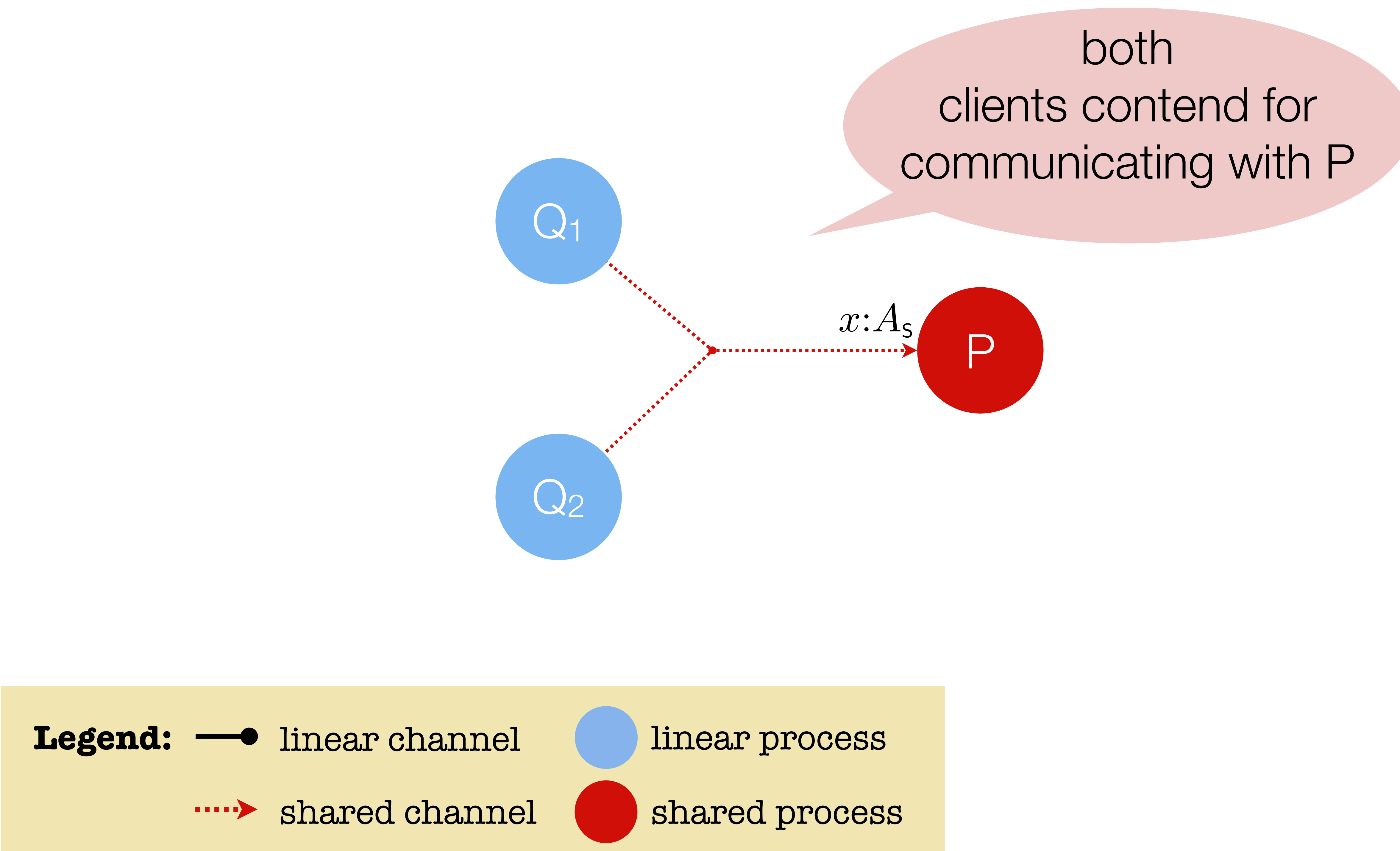

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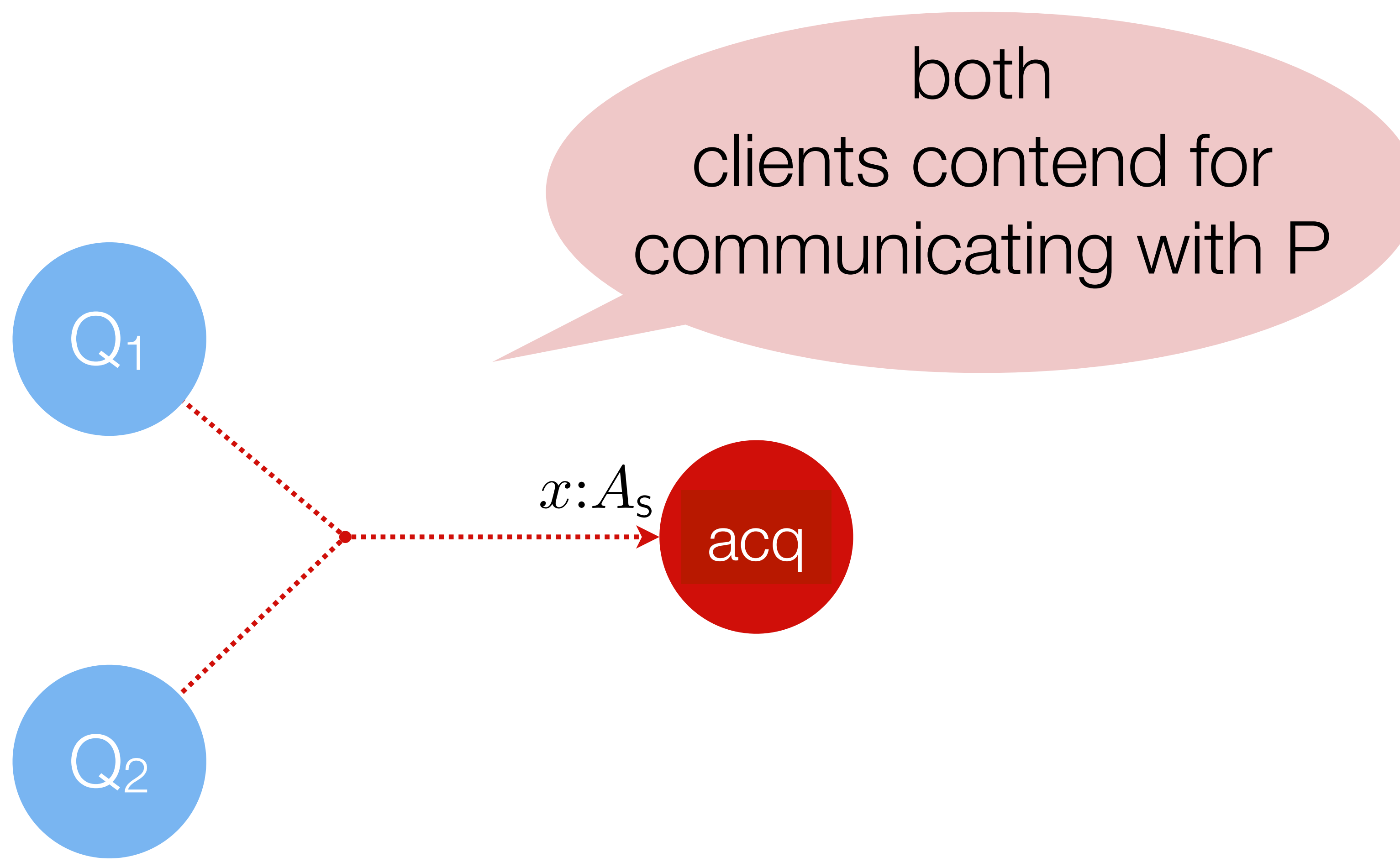

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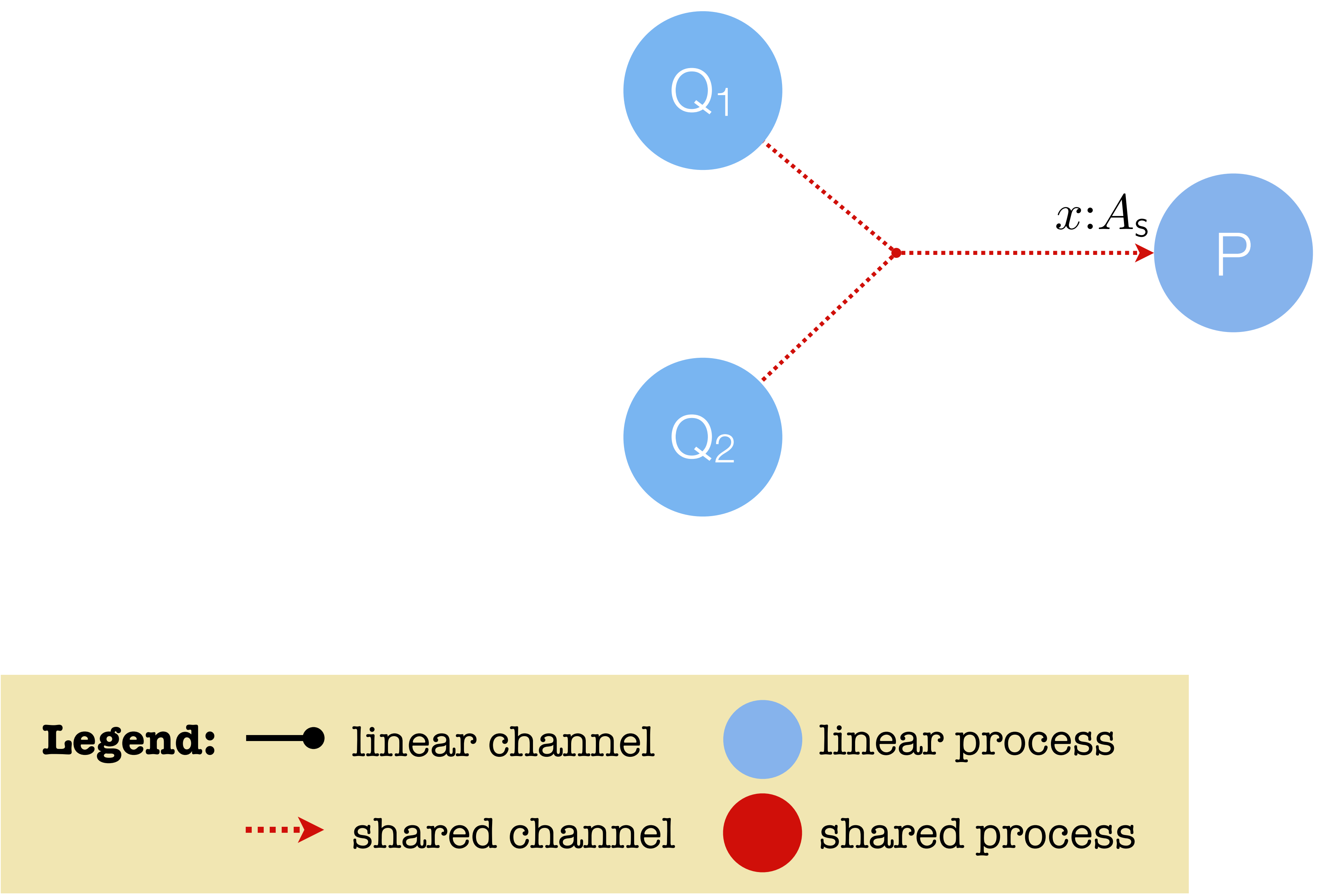

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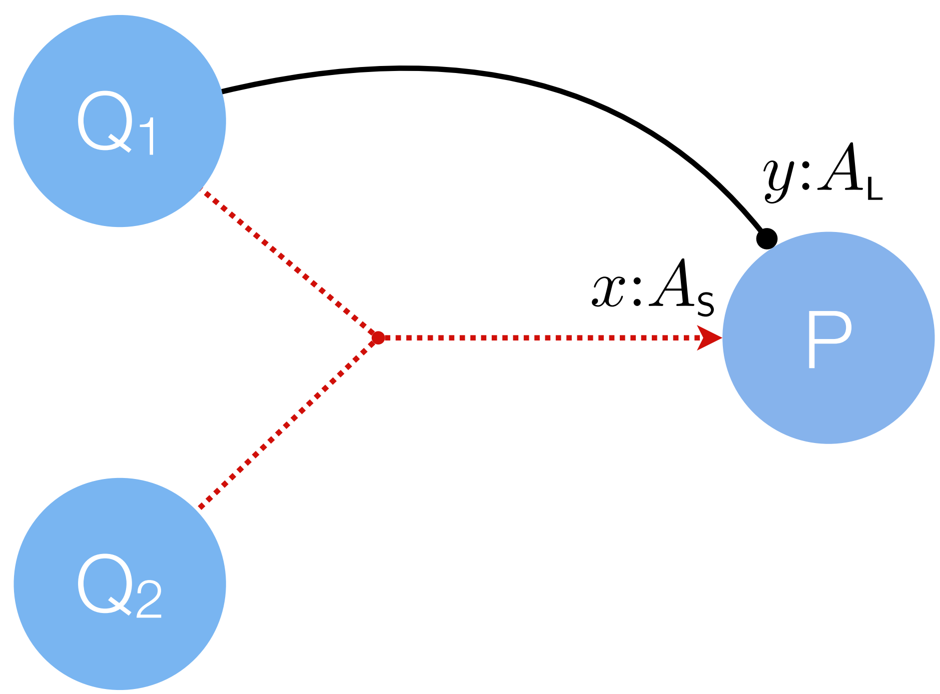

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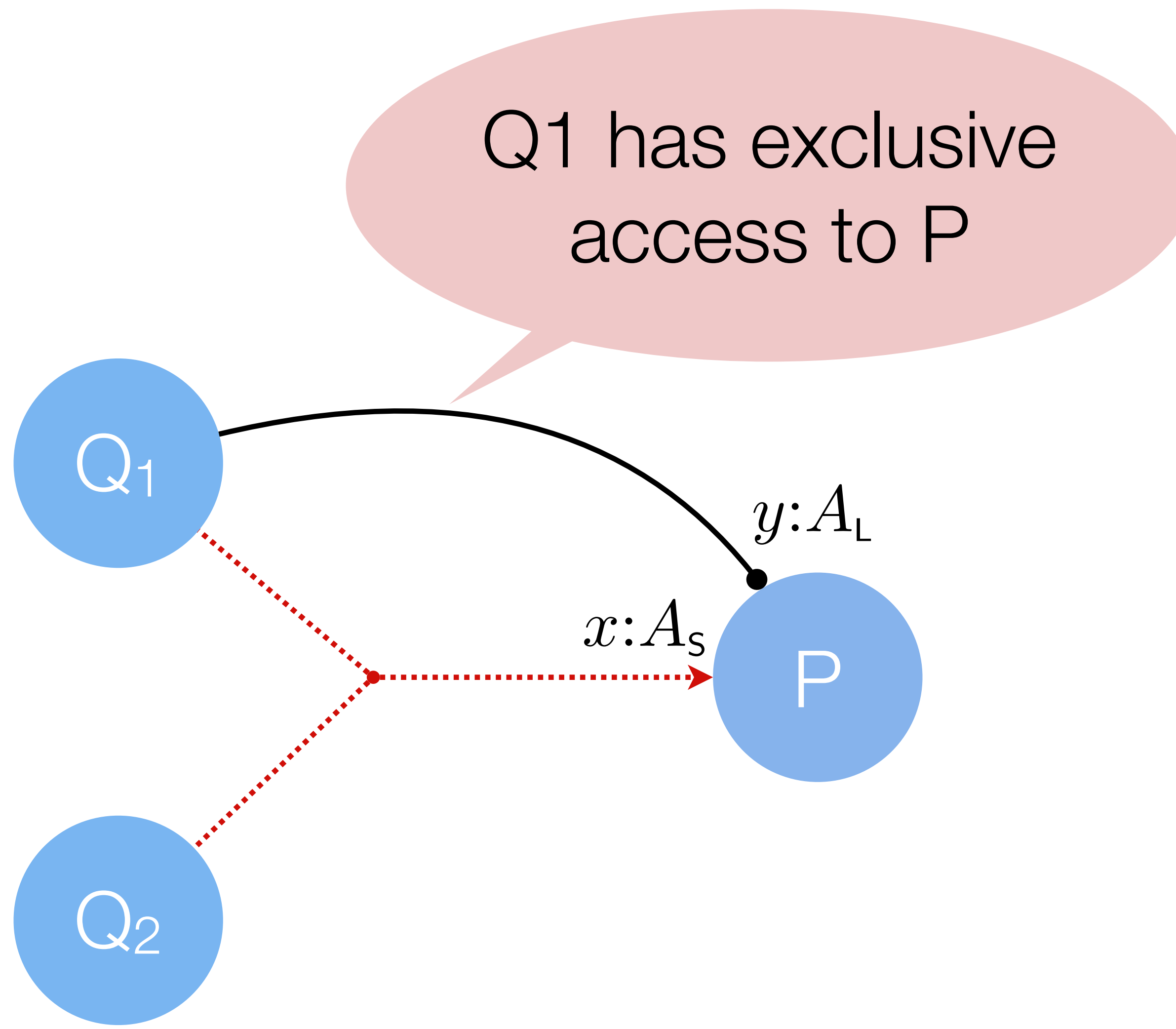

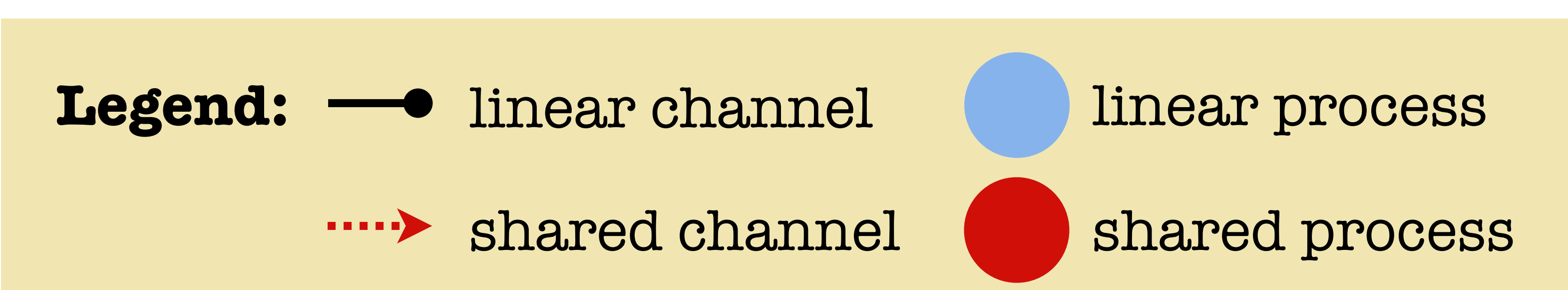

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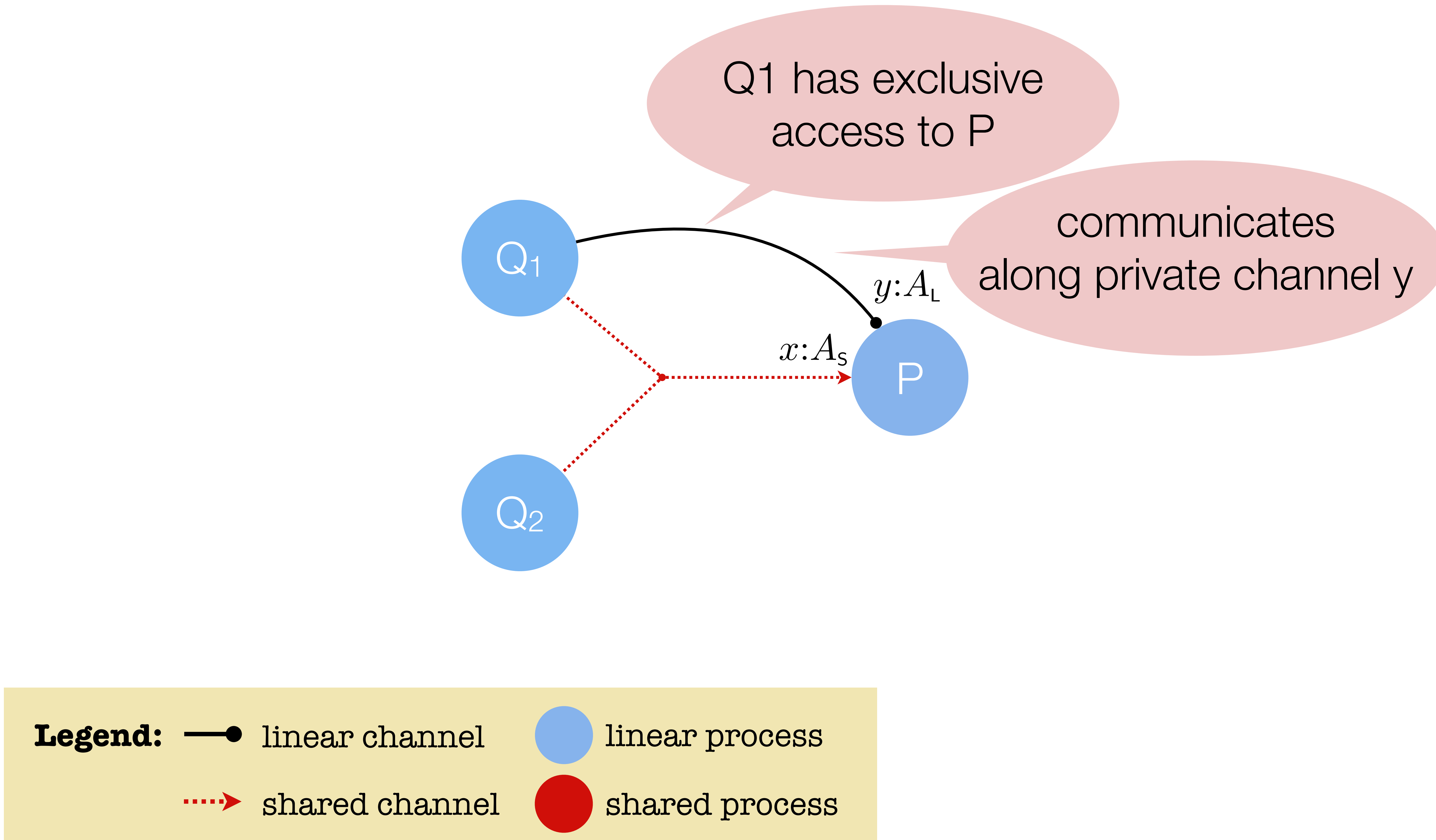

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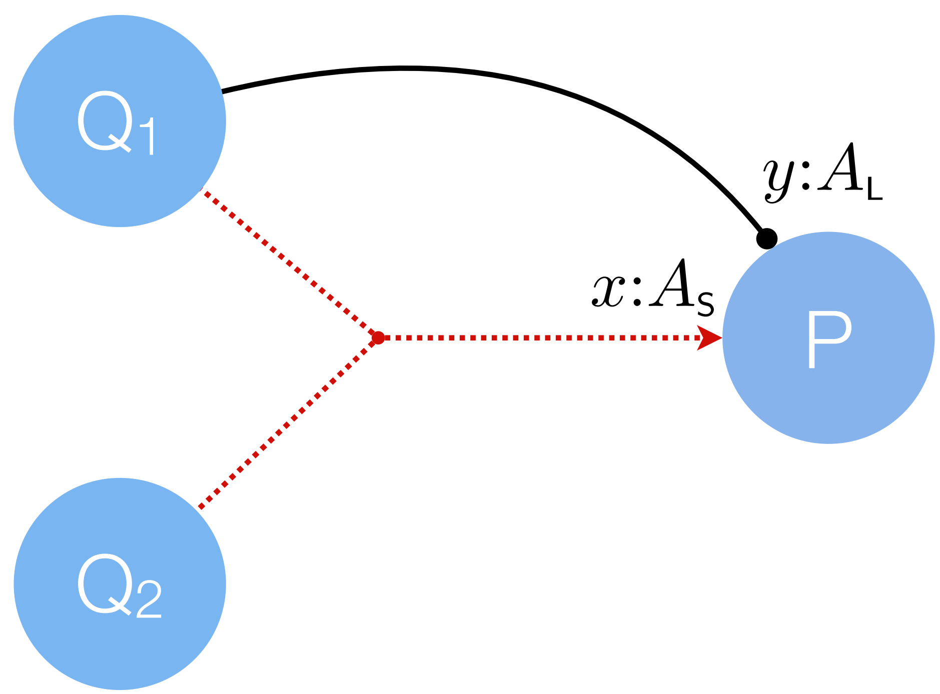

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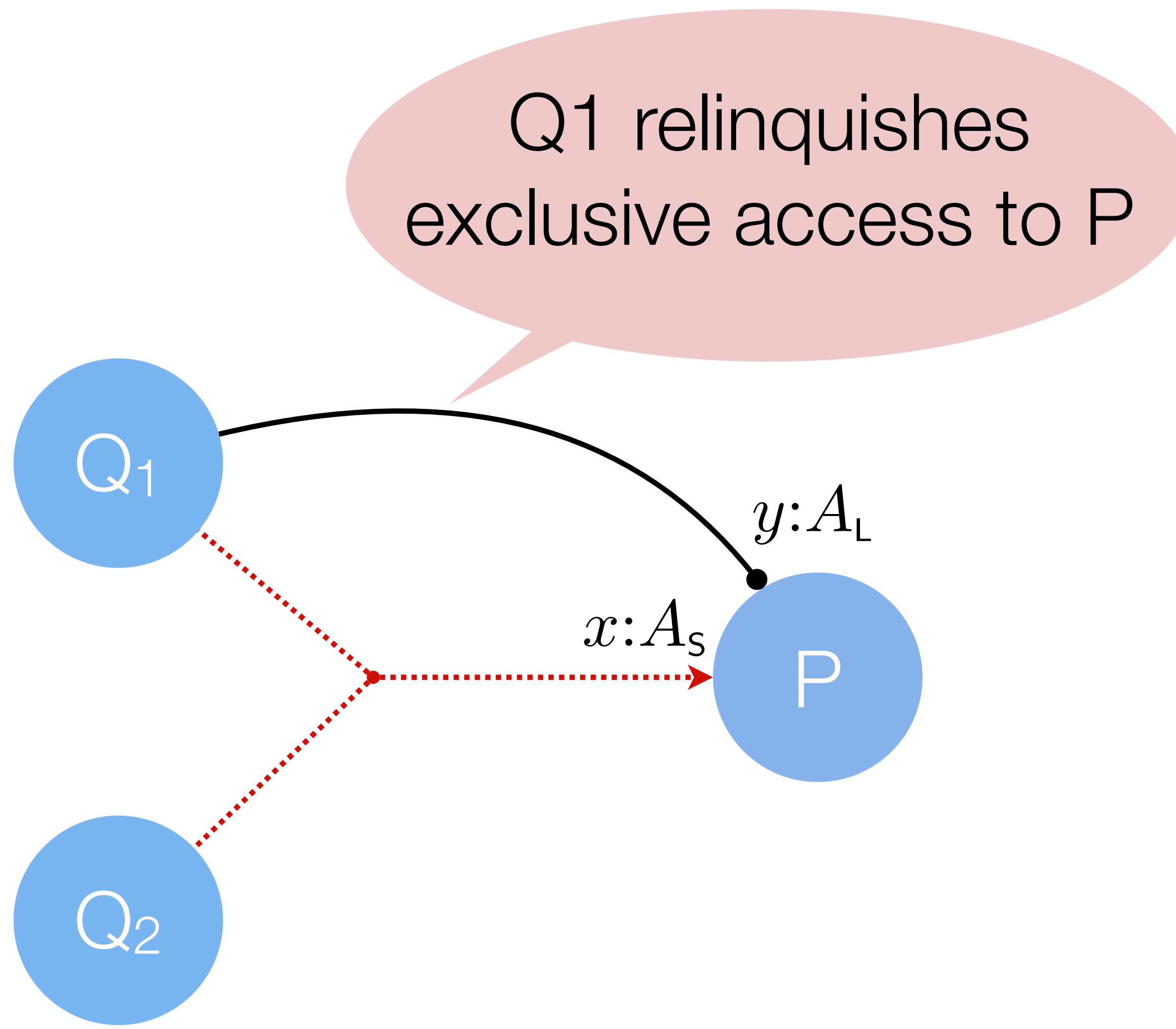

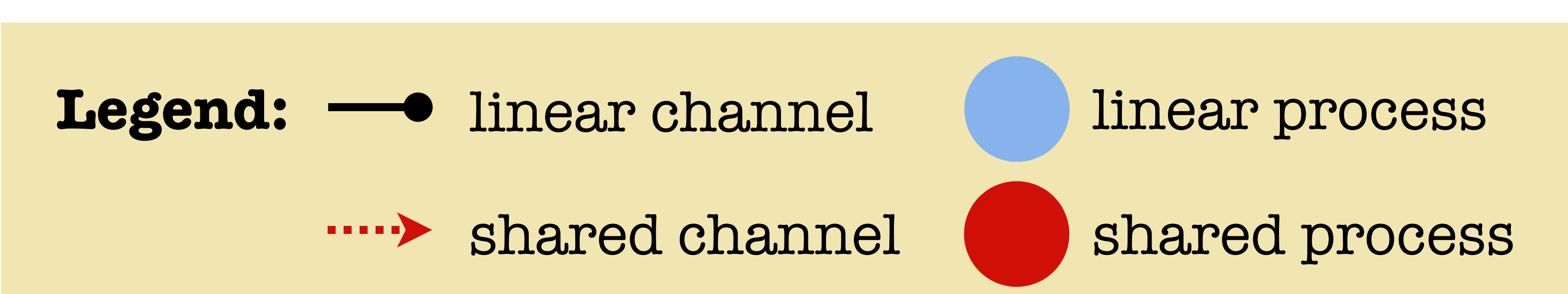

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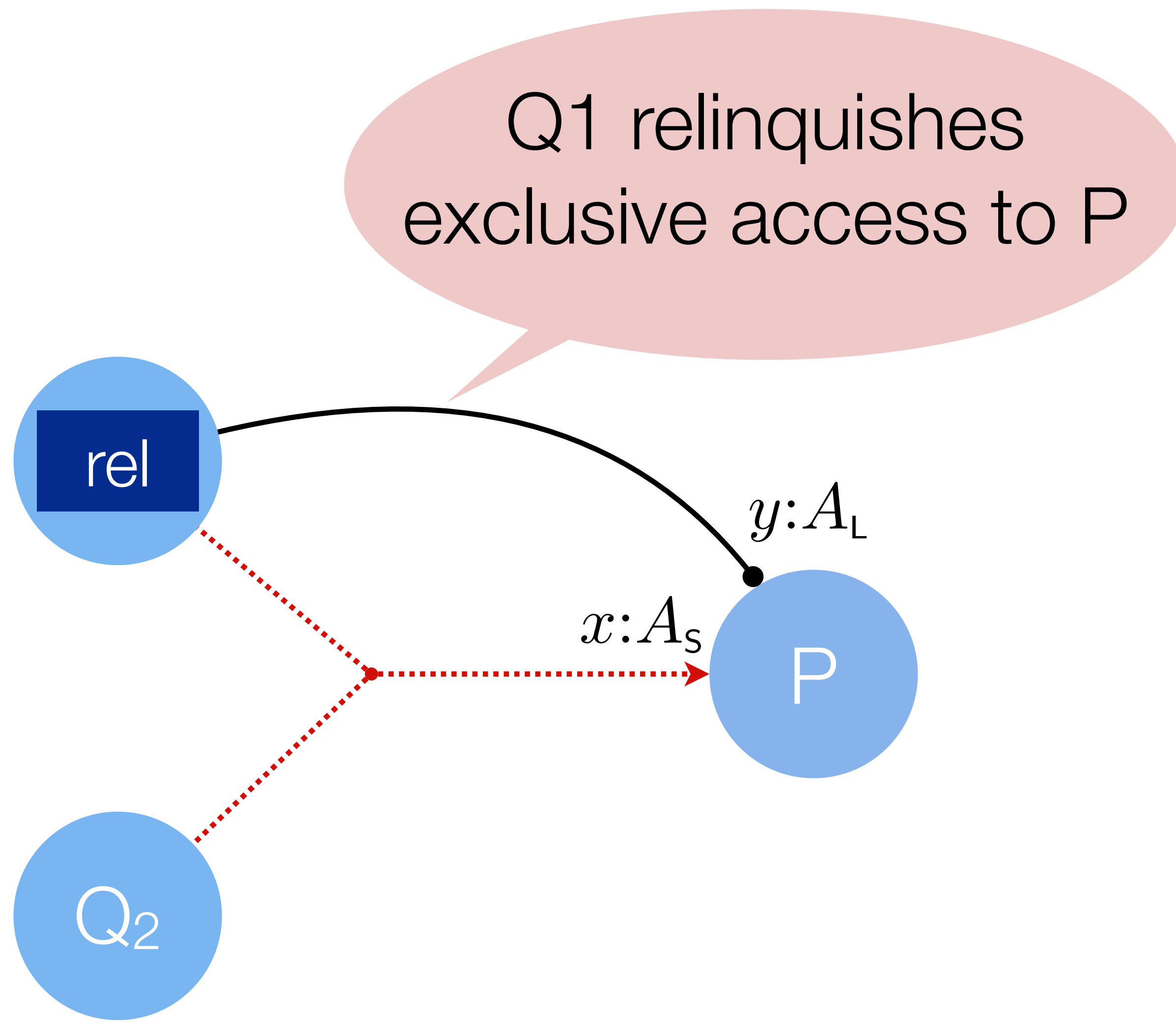

Legend: linear channel; shared channel; linear process; shared process

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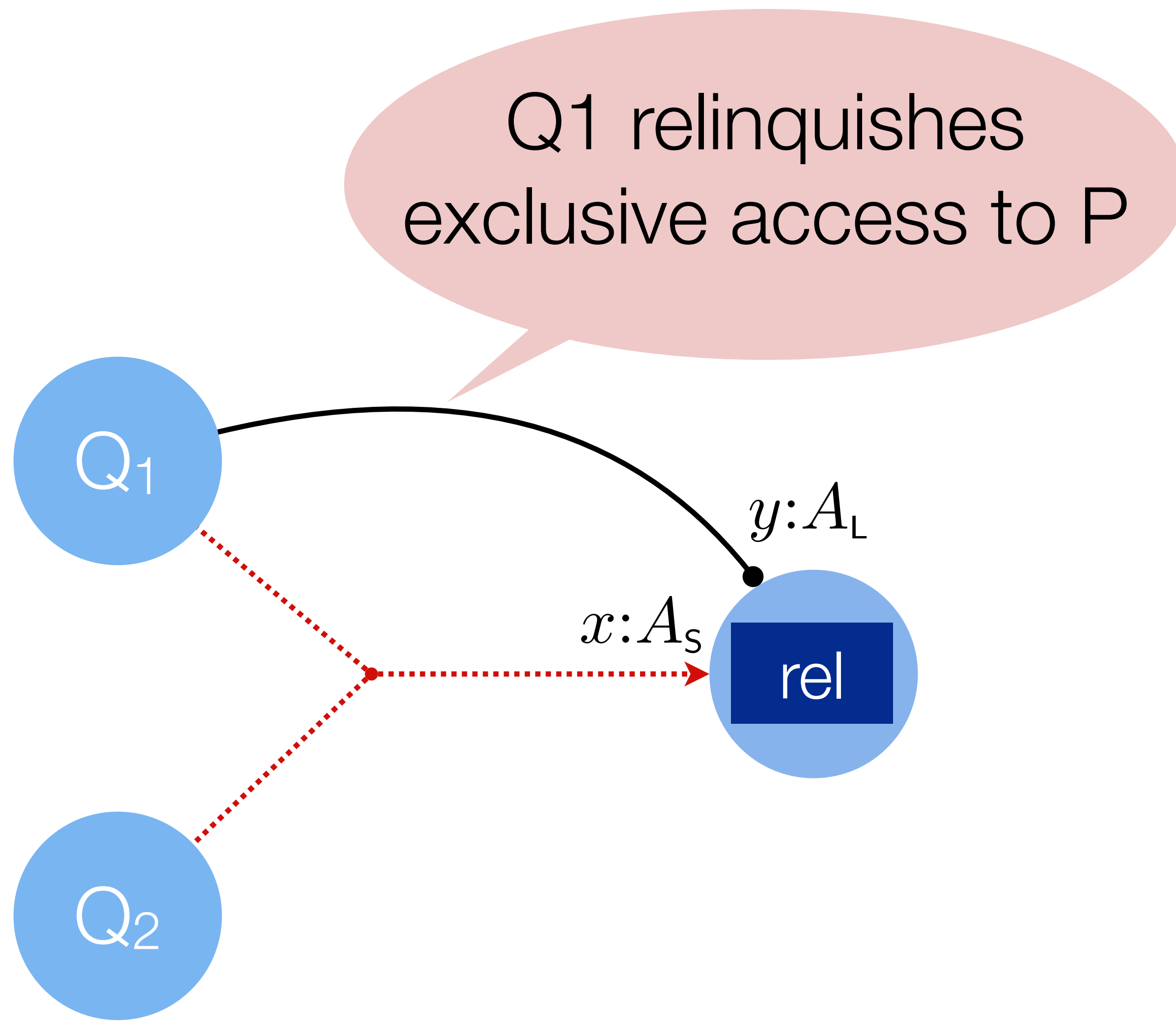

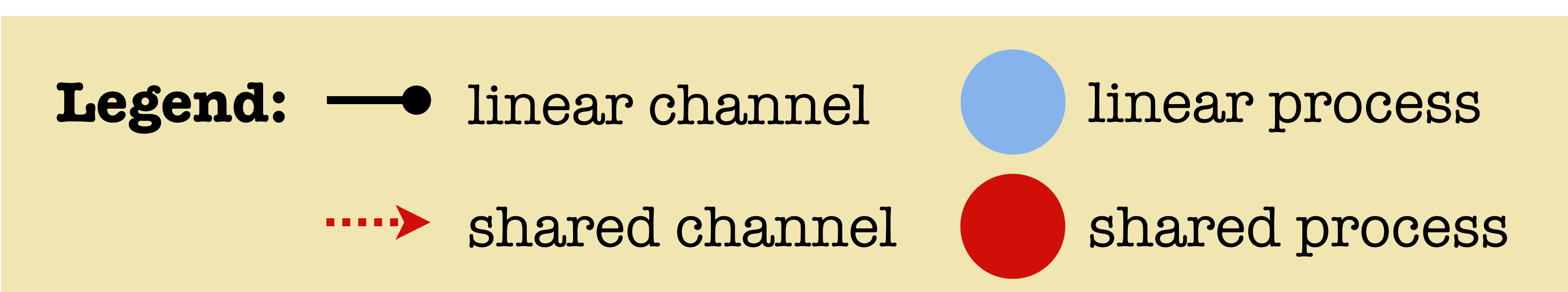

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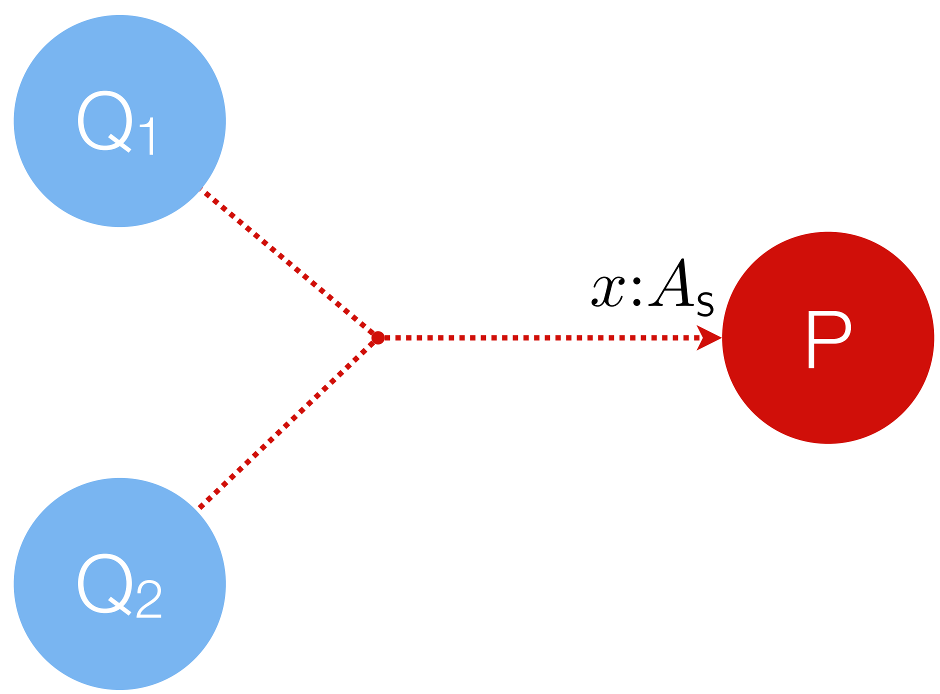

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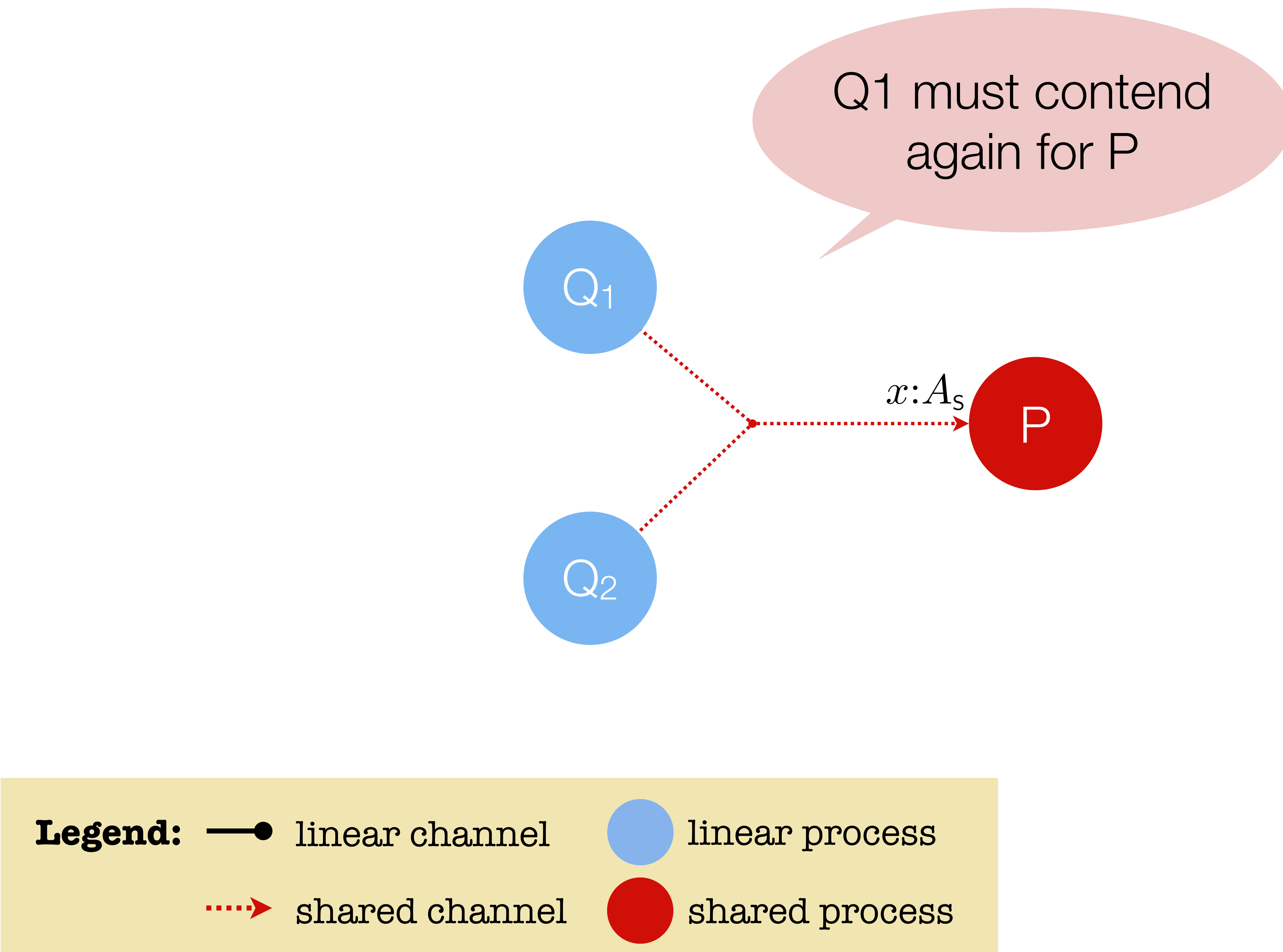

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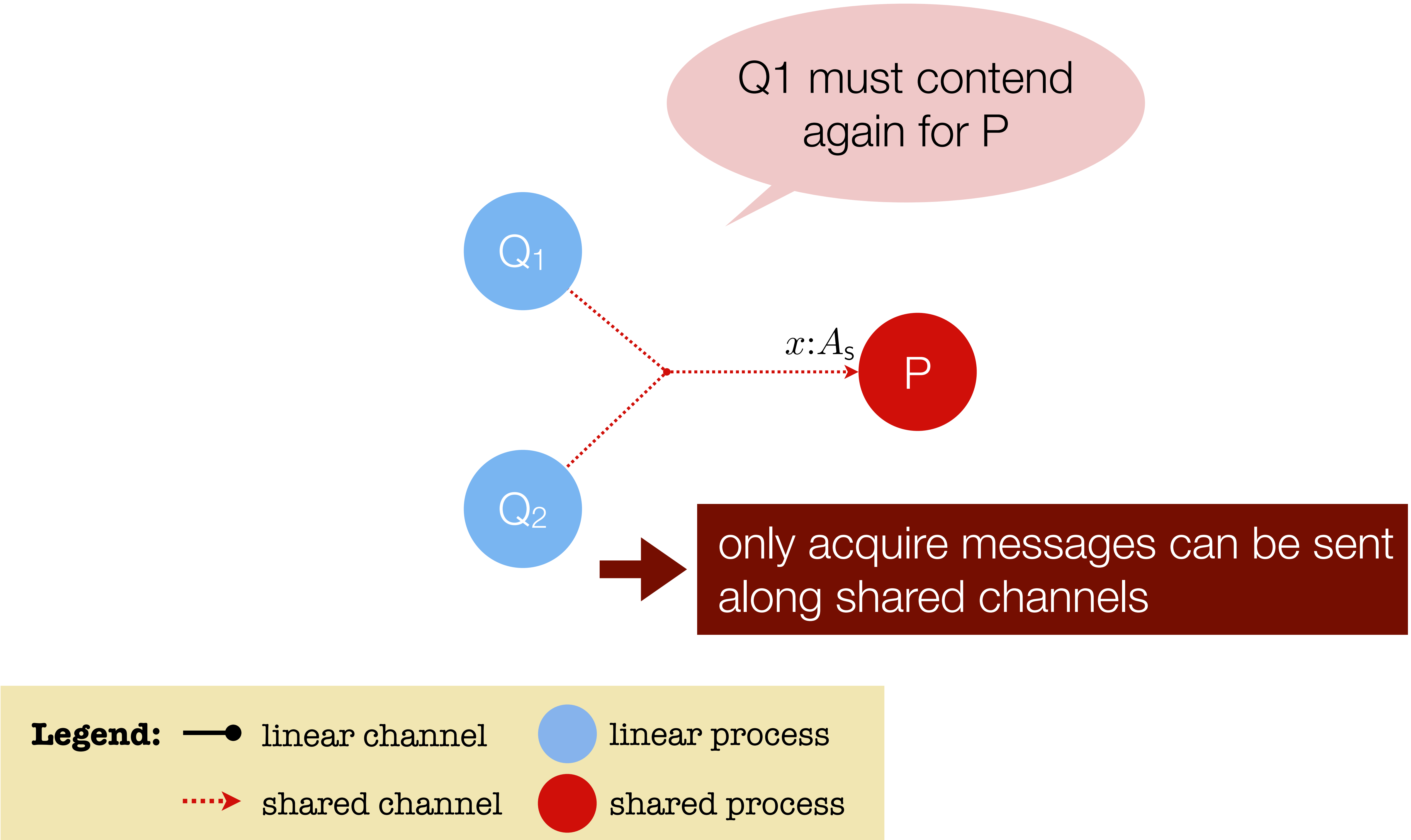

Key idea 2: manifest acquire-release in types

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$$A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}}$$

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Adjoint stratification of session types:

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weakening
contraction

+ ↑ -

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Adjoint stratification of session types:

- stratify session types into a linear and shared layer, s.t. S > L
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weakening
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+ (↑) −

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$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

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$$A_S \triangleq \uparrow_L^S A_L$$

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- support of sending shared channels along linear channels

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- support of sending shared channels along linear channels

Example: shared queue

What should be the type of a shared queue?

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weakening
contraction

+ ↑ -

$$A_S \triangleq \uparrow_L^S A_L$$

$$A_L, B_L \triangleq \oplus\{\overline{l : A_L}\} \mid A_L \otimes B_L \mid \mathbf{1} \mid \exists x{:}A_S.\, B_L \mid$$
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What should be the type of a shared queue?

weakening
contraction

+ (weakening, contraction) ↑ −

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$$\begin{aligned} \text{queue } A_S = \quad & \&\{\text{enq} : \Pi x{:}A_S.\quad \text{queue } A_S, \\ & \quad \text{deq} : \oplus\{\text{none} : \quad \text{queue } A_S,\ \text{some} : \exists x{:}A_S.\quad \text{queue } A_S\}\} \end{aligned}$$

Example: shared queue

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weakening
contraction

+ ↑ −

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$$\mathsf{queue}\ A_S = \&\{\mathsf{enq} : \Pi x{:}A_S.\ \mathsf{queue}\ A_S,$$
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Example: shared queue

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weakening
contraction

+ (↑) −

$$A_{\mathsf{S}} \triangleq {\uparrow}_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid$$
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What should be the type of a shared queue?

weakening
contraction

\+ (↑) −

$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$\begin{aligned} A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq\ & \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\ B_{\mathsf{L}} \mid \\ & \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\ B_{\mathsf{L}} \end{aligned}$$

$$\begin{aligned} \mathsf{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \&\{ & \mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}, \\ & \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}\}\} \end{aligned}$$

Example: shared queue

What should be the type of a shared queue?

weakening
contraction

+ ↑ -

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Takeaway:

Example: shared queue

What should be the type of a shared queue?

weakening
contraction

+

-

$$A_S \triangleq \uparrow_L^S A_L$$

$$A_L, B_L \triangleq \oplus\{\overline{l : A_L}\} \mid A_L \otimes B_L \mid \mathbf{1} \mid \exists x{:}A_S.\ B_L \mid$$
$$\&\{\overline{l : A_L}\} \mid A_L \multimap B_L \mid \downarrow_L^S A_S \mid \Pi x{:}A_S.\ B_L$$

$$\text{queue } A_S = \uparrow_L^S \&\{\text{enq} : \Pi x{:}A_S.\ \downarrow_L^S \text{queue } A_S,$$
$$\text{deq} : \oplus\{\text{none} : \downarrow_L^S \text{queue } A_S,\ \text{some} : \exists x{:}A_S.\ \downarrow_L^S \text{queue } A_S\}\}$$

Takeaway:

up-shift is an acquire

Example: shared queue

What should be the type of a shared queue?

weakening
contraction

+ (up) / - (down)

$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}}$$

$$\mathsf{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}\}\}$$

Takeaway:

- up-shift is an acquire
- down-shift is a release

Key idea 3: equi-synchronizing

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Is mutual exclusion enough for restoring preservation?

Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

$$\mathsf{queue}\ A_{\mathsf{S}} = {\uparrow}_{\mathsf{L}}^{\mathsf{S}}\ \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\ \mathsf{queue}\ A_{\mathsf{S}},\\ \mathsf{deq} : \oplus\{\mathsf{none} : {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\ \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\ \mathsf{queue}\ A_{\mathsf{S}}\}\}$$

Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

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Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

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Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

$$\mathsf{queue}\ A_{\mathsf{S}} = {\uparrow}^{\mathsf{S}}_{\mathsf{L}}\ \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ {\downarrow}^{\mathsf{S}}_{\mathsf{L}}\ \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{deq} : \oplus\{\mathsf{none} : {\downarrow}^{\mathsf{S}}_{\mathsf{L}}\ \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ {\downarrow}^{\mathsf{S}}_{\mathsf{L}}\ \mathsf{queue}\ A_{\mathsf{S}}\}\}$$

Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

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process is released
back to same type previously
acquired

Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

$$\begin{aligned}\mathsf{queue}\ A_{\mathsf{S}} = {\uparrow}_{\mathsf{L}}^{\mathsf{S}}\ \&\{&\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\ \mathsf{queue}\ A_{\mathsf{S}},\\ &\mathsf{deq} : \oplus\{\mathsf{none} : {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\ \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\ \mathsf{queue}\ A_{\mathsf{S}}\}\}\end{aligned}$$

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Is mutual exclusion enough for restoring preservation?

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Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

$$\mathsf{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}\}\}$$

$$\mathsf{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \uparrow_{\mathsf{L}}^{\mathsf{S}} \mathbf{1},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}\}\}$$

Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

$$\mathsf{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}\}\}$$

$$\mathsf{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}}\uparrow_{\mathsf{L}}^{\mathsf{S}} \mathbf{1},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}\}\}$$

Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

$$\mathsf{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}\}\}$$

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Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

$$\mathsf{queue}\ A_\mathsf{S} = \uparrow_\mathsf{L}^\mathsf{S} \&\{\mathsf{enq} : \Pi x{:}A_\mathsf{S}.\ \downarrow_\mathsf{L}^\mathsf{S} \mathsf{queue}\ A_\mathsf{S},\ \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_\mathsf{L}^\mathsf{S} \mathsf{queue}\ A_\mathsf{S},\ \mathsf{some} : \exists x{:}A_\mathsf{S}.\ \downarrow_\mathsf{L}^\mathsf{S} \mathsf{queue}\ A_\mathsf{S}\}\}$$

$$\mathsf{queue}\ A_\mathsf{S} = \uparrow_\mathsf{L}^\mathsf{S} \&\{\mathsf{enq} : \Pi x{:}A_\mathsf{S}.\ \downarrow_\mathsf{L}^\mathsf{S} \mathsf{queue}\ A_\mathsf{S},\ \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_\mathsf{L}^\mathsf{S}\uparrow_\mathsf{L}^\mathsf{S} \mathbf{1},\ \mathsf{some} : \exists x{:}A_\mathsf{S}.\ \downarrow_\mathsf{L}^\mathsf{S} \mathsf{queue}\ A_\mathsf{S}\}\}$$

process is released back to different type

Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

$$\begin{aligned}\mathsf{queue}\ A_{\mathsf{S}} = {\uparrow}^{\mathsf{S}}_{\mathsf{L}}\ \&\{&\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ {\downarrow}^{\mathsf{S}}_{\mathsf{L}}\ \mathsf{queue}\ A_{\mathsf{S}},\\ &\mathsf{deq} : \oplus\{\mathsf{none} : {\downarrow}^{\mathsf{S}}_{\mathsf{L}}\ \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ {\downarrow}^{\mathsf{S}}_{\mathsf{L}}\ \mathsf{queue}\ A_{\mathsf{S}}\}\}\end{aligned}$$

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process is released back to different type

next client to acquire encounters protocol violation!

Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

$$\begin{aligned}\mathsf{queue}\ A_{\mathsf{S}} = {\uparrow}_{\mathsf{L}}^{\mathsf{S}}\ \&\{&\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\ \mathsf{queue}\ A_{\mathsf{S}},\\ &\mathsf{deq} : \oplus\{\mathsf{none} : {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\ \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\ \mathsf{queue}\ A_{\mathsf{S}}\}\}\end{aligned}$$

$$\begin{aligned}\mathsf{queue}\ A_{\mathsf{S}} = {\uparrow}_{\mathsf{L}}^{\mathsf{S}}\ \&\{&\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\ \mathsf{queue}\ A_{\mathsf{S}},\\ &\mathsf{deq} : \oplus\{\mathsf{none} : {\downarrow}_{\mathsf{L}}^{\mathsf{S}}{\uparrow}_{\mathsf{L}}^{\mathsf{S}}\ \mathbf{1},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\ \mathsf{queue}\ A_{\mathsf{S}}\}\}\end{aligned}$$

Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

$$\mathsf{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \& \{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}\}\}$$

$$\mathsf{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \& \{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \uparrow_{\mathsf{L}}^{\mathsf{S}} \mathbf{1},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}\}\}$$

→ equi-synchronizing: type wellformedness condition guaranteeing that any release is back to type at which previously acquired

Key idea 3: equi-synchronizing

Is mutual exclusion enough for restoring preservation?

$$\text{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \& \{\text{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \text{queue}\ A_{\mathsf{S}},\ \text{deq} : \oplus\{\text{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \text{queue}\ A_{\mathsf{S}},\ \text{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \text{queue}\ A_{\mathsf{S}}\}\}$$

$$\text{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \& \{\text{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \text{queue}\ A_{\mathsf{S}},\ \text{deq} : \oplus\{\text{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \uparrow_{\mathsf{L}}^{\mathsf{S}} \mathbf{1},\ \text{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \text{queue}\ A_{\mathsf{S}}\}\}$$

→ equi-synchronizing: type wellformedness condition guaranteeing that any release is back to type at which previously acquired

→ acquire-release and equi-synchronizing guarantee preservation

Typing judgments

Typing judgments

weakening
contraction

+

$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid$$
$$\&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}}$$

-

Typing judgments

weakening
contraction

+

$$
\begin{aligned}
A_{\mathsf{S}} &\triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}} \\
A_{\mathsf{L}}, B_{\mathsf{L}} &\triangleq \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid \\
&\quad\ \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}}
\end{aligned}
$$

-

$\Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}})$	shared process P, providing session of type A_{S} along x_{S}, using channels in Γ
$\Gamma;\ \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}})$	linear process P, providing session of type A_{L} along x_{L}, using channels in Γ and Δ
Γ	shared (structural) context
Δ	linear context

Typing judgments

weakening
contraction

\+ ↑ -

$$A_S \triangleq {\uparrow}_L^S A_L$$

$$A_L, B_L \triangleq \oplus\{\overline{l : A_L}\} \mid A_L \otimes B_L \mid \mathbf{1} \mid \exists x{:}A_S.\, B_L \mid \\ \&\{\overline{l : A_L}\} \mid A_L \multimap B_L \mid {\downarrow}_L^S A_S \mid \Pi x{:}A_S.\, B_L$$

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Typing judgments

weakening
contraction

$+$ / $-$

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Typing judgments

weakening
contraction

+ ↑ −

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Typing judgments

weakening
contraction

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$$A_L, B_L \triangleq \oplus\{\overline{l : A_L}\} \mid A_L \otimes B_L \mid \mathbf{1} \mid \exists x{:}A_S.\, B_L \mid$$
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Typing judgments

weakening
contraction

+ ↑ −

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$$\&\{\overline{l : A_L}\} \mid A_L \multimap B_L \mid {\downarrow}_L^S A_S \mid \Pi x{:}A_S.\, B_L$$

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Acquire

Acquire

$$\frac{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}};\ \Delta, x_{\mathsf{L}} : A_{\mathsf{L}} \vdash_{\Sigma} Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}};\ \Delta \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{acquire}\ x_{\mathsf{S}}\ ; Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}\ (\mathrm{T}\text{-}\uparrow_{\mathsf{L}}^{\mathsf{S}}\mathrm{L})$$

Acquire

$$\frac{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}} ;\ \Delta, x_{\mathsf{L}} : A_{\mathsf{L}} \vdash_{\Sigma} Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}} ;\ \Delta \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{acquire}\ x_{\mathsf{S}}\ ; Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}\ (\mathrm{T}\text{-}\uparrow_{\mathsf{L}}^{\mathsf{S}}\mathrm{L})$$

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$$\frac{\Gamma;\ \cdot \vdash_{\Sigma} P_{x_{\mathsf{L}}} :: (x_{\mathsf{L}} : A_{\mathsf{L}})}{\Gamma \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{accept}\ x_{\mathsf{S}}\ ; P_{x_{\mathsf{L}}} :: (x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}})} \ (\text{T-}\uparrow_{\mathsf{L}}^{\mathsf{S}}\text{R})$$

Acquire

$$\frac{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}};\ \Delta, x_{\mathsf{L}} : A_{\mathsf{L}} \vdash_{\Sigma} Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}};\ \Delta \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{acquire}\ x_{\mathsf{S}}\ ; Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}\ (\mathrm{T}\text{-}\uparrow_{\mathsf{L}}^{\mathsf{S}}\mathrm{L})$$

$$\frac{\Gamma;\ \cdot \vdash_{\Sigma} P_{x_{\mathsf{L}}} :: (x_{\mathsf{L}} : A_{\mathsf{L}})}{\Gamma \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{accept}\ x_{\mathsf{S}}\ ; P_{x_{\mathsf{L}}} :: (x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}})}\ (\mathrm{T}\text{-}\uparrow_{\mathsf{L}}^{\mathsf{S}}\mathrm{R})$$

Acquire

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$$\frac{\Gamma;\ \cdot \vdash_{\Sigma} P_{x_{\mathsf{L}}} :: (x_{\mathsf{L}} : A_{\mathsf{L}})}{\Gamma \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{accept}\ x_{\mathsf{S}}\ ; P_{x_{\mathsf{L}}} :: (x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}})} \ (\mathrm{T}\text{-}{\uparrow_{\mathsf{L}}^{\mathsf{S}}}\mathrm{R})$$

$$(\mathrm{D}\text{-}{\uparrow_{\mathsf{L}}^{\mathsf{S}}}) \quad \mathsf{proc}(a_{\mathsf{S}}, x_{\mathsf{L}} \leftarrow \mathsf{accept}\ a_{\mathsf{S}}\ ; P_{x_{\mathsf{L}}}), \mathsf{proc}(c_{\mathsf{L}}, x_{\mathsf{L}} \leftarrow \mathsf{acquire}\ a_{\mathsf{S}}\ ; Q_{x_{\mathsf{L}}})$$
$$\longrightarrow \mathsf{unvail}(a_{\mathsf{S}}), \mathsf{proc}(a_{\mathsf{L}}, [a_{\mathsf{L}}/x_{\mathsf{L}}]\, P_{x_{\mathsf{L}}}), \mathsf{proc}(c_{\mathsf{L}}, [a_{\mathsf{L}}/x_{\mathsf{L}}]\, Q_{x_{\mathsf{L}}})$$

Release

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$$\frac{\Gamma, x_{\mathsf{S}} : A_{\mathsf{S}};\ \Delta \vdash_{\Sigma} Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma;\ \Delta, x_{\mathsf{L}} : \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \vdash_{\Sigma} x_{\mathsf{S}} \leftarrow \mathsf{release}\ x_{\mathsf{L}}\ ; Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}\ (\mathrm{T}\text{-}\downarrow_{\mathsf{L}}^{\mathsf{S}}\mathrm{L})$$

Release

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Release

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$$\frac{\Gamma \vdash_{\Sigma} P_{x_{\mathsf{S}}} :: (x_{\mathsf{S}} : A_{\mathsf{S}})}{\Gamma;\ \cdot \vdash_{\Sigma} x_{\mathsf{S}} \leftarrow \mathsf{detach}\ x_{\mathsf{L}}\ ; P_{x_{\mathsf{S}}} :: (x_{\mathsf{L}} : \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}})} \ (\text{T-}\downarrow_{\mathsf{L}}^{\mathsf{S}}\text{R})$$

Release

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$$\begin{array}{ll} (\text{D-}{\downarrow_{\mathsf{L}}^{\mathsf{S}}}) & \mathsf{proc}(a_{\mathsf{L}}, x_{\mathsf{S}} \leftarrow \mathsf{detach}\ a_{\mathsf{L}}\ ; P_{x_{\mathsf{S}}}), \mathsf{proc}(c_{\mathsf{L}}, x_{\mathsf{S}} \leftarrow \mathsf{release}\ a_{\mathsf{L}}\ ; Q_{x_{\mathsf{S}}}), \\ & \mathsf{unvail}(a_{\mathsf{S}}) \\ & \longrightarrow\ \mathsf{proc}(a_{\mathsf{S}}, [a_{\mathsf{S}}/x_{\mathsf{S}}]\, P_{x_{\mathsf{S}}}), \mathsf{proc}(c_{\mathsf{L}}, [a_{\mathsf{S}}/x_{\mathsf{S}}]\, Q_{x_{\mathsf{S}}}) \end{array}$$

Let's implement a shared queue in SILL$_S$

Taking stock

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We have a session type system that allows shared and linear channels to coexist and guarantees:

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  - data-race-freedom (low-level and high-level)

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next time!