

Session-Typed Concurrent Programming — Handout OPLSS 2021

Stephanie Balzer
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Intuitionistic Linear Logic Session Types

Typing Rules

$$\frac{\Delta \vdash P :: (x : B)}{\Delta, y : A \vdash \text{send } x y; P :: (x : A \otimes B)} \otimes_R \quad \frac{\Delta, x : B, y : A \vdash Q_y :: (z : C)}{\Delta, x : A \otimes B \vdash y \leftarrow \text{rcv } x; Q_y :: (z : C)} \otimes_L$$

$$\frac{\Delta, y : A \vdash P_y :: (x : B)}{\Delta \vdash y \leftarrow \text{rcv } x; P_y :: (x : A \multimap B)} \multimap_R \quad \frac{\Delta, x : B \vdash Q :: (z : C)}{\Delta, x : A \multimap B, y : A \vdash \text{send } x y; Q :: (z : C)} \multimap_L$$

$$\frac{\Delta \vdash P :: (x : A)}{\Delta \vdash x.\text{inl}; P :: (x : A \oplus B)} \oplus_{R1} \quad \frac{\Delta \vdash P :: (x : B)}{\Delta \vdash x.\text{inr}; P :: (x : A \oplus B)} \oplus_{R2}$$

$$\frac{\Delta, x : A \vdash Q_1 :: (z : C) \quad \Delta, x : B \vdash Q_2 :: (z : C)}{\Delta, x : A \oplus B \vdash \text{case } x \text{ of } (Q_1, Q_2) :: (z : C)} \oplus_L$$

$$\frac{\Delta \vdash P_1 :: (x : A) \quad \Delta \vdash P_2 :: (x : B)}{\Delta \vdash \text{case } x \text{ of } (P_1, P_2) :: (x : A \& B)} \&_R$$

$$\frac{\Delta, x : A \vdash Q :: (z : C)}{\Delta, x : A \& B \vdash x.\text{inl}; Q :: (z : C)} \&_{L1} \quad \frac{\Delta, x : B \vdash Q :: (z : C)}{\Delta, x : A \& B \vdash x.\text{inr}; Q :: (z : C)} \&_{L2}$$

$$\frac{}{\cdot \vdash \text{close } x :: (x : \mathbf{1})} \mathbf{1}_R \quad \frac{\Delta \vdash Q :: (z : C)}{\Delta, x : \mathbf{1} \vdash \text{wait } x; Q :: (z : C)} \mathbf{1}_L$$

$$\frac{\Delta_1 \vdash P :: (x : A) \quad \Delta_2, x : A \vdash Q :: (z : C)}{\Delta_1, \Delta_2 \vdash x \leftarrow P; Q :: (z : C)} \text{Cut}$$

$$\frac{}{y : A \vdash \text{fwd } x y :: (x : A)} \text{Id}$$