

FROM PROGRAM EQUIVALENCES TO PROGRAM METRICS

JUNE 23,
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OPL55

RELATION

$$R: X \rightarrow Y \quad R \subseteq X \times Y$$

$$S: Y \rightarrow Z$$

$$x(R;S)z \Leftrightarrow \exists y \in Y. \quad \begin{matrix} x R y \wedge \\ y S z \end{matrix}$$

$$\text{REL}(X, Y) = \{ R \mid R: X \rightarrow Y \}$$

 COMPLETE LATTICE

$$R: X \rightarrow Y$$

$$R^T = \{ (y, x) \mid (x, y) \in R \}$$

REFLEXIVITY FOR $R \in \text{REL}(X, X)$
 $I \subseteq R$

TRANSITIVITY FOR $R \in \text{REL}(X, X)$
 $R; R \subseteq R$

SIMMETRY FOR $R \in \text{REL}(X, X)$
 $R^T \subseteq R$

INTERMEZZO: λ^{ST}

$\sigma, \tau ::= \text{BOOL} \mid \text{UNIT} \mid \sigma \times \tau \mid \sigma \rightarrow \tau$ TYPES

$v, w ::= x \mid \langle \rangle \mid \text{true} \mid \text{false} \mid \langle v, w \rangle$

$\lambda x. e$

VALUES

$e, f ::= \text{return } v \mid \text{if } v \text{ then } e \text{ else } f$

$\text{proj}_e v \mid \text{proj}_r v \mid vw$

$\text{let } x = e \text{ in } f$ COMPUTATIONS

$\text{if } e \text{ then } f \text{ else } g \equiv \text{let } x = e \text{ in}$
 $\text{if } x \text{ then } F \text{ else } g$

$e[v/x]$ $w[v/x]$

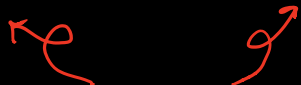


CAPTURE-AVOIDING
SUBSTITUTIONS

ENVIRONMENT $\Gamma =$ PARTIAL FUNCTION
FROM VARIABLES TO
TYPES

$\Gamma \vdash^v v : \sigma$

$\Gamma \vdash^{\wedge} e : \sigma$



JUDGMENTS FOR VALUES

AND COMPUTATIONS

$$\frac{\Gamma \vdash^\Delta e : \sigma \quad \Gamma, x : \sigma \vdash^\Delta f : \tau}{\Gamma \vdash^\Delta \text{let } x = e \text{ in } f : \tau}$$

$$\frac{\Gamma \vdash^\nu v : \sigma \rightarrow \tau \quad \Gamma \vdash^\nu w : \sigma}{\Gamma \vdash^\Delta vw : \tau}$$

$\Lambda_\sigma \leftarrow$ OPEN COMPUT.

$\Lambda_{\Gamma \vdash \sigma}$

$\Lambda^\bullet_\sigma \leftarrow$ CLOSED COMPUT.

OPEN VALUES $\rightarrow \mathcal{V}_\sigma$

$\mathcal{V}_{\Gamma \vdash \sigma}$

CLOSED VALUES $\rightarrow \mathcal{V}^\bullet_\sigma$

$$\llbracket - \rrbracket_\varepsilon^n : \prod_\sigma (\Lambda^\bullet_\sigma \rightarrow \mathcal{V}^\bullet_\sigma)$$

$$\llbracket e \rrbracket_\varepsilon^0 = \perp$$

$$\llbracket \text{return } v \rrbracket_\varepsilon^{n+1} = v$$

$$\llbracket \text{proj } e \langle v, w \rangle \rrbracket_\varepsilon^{n+2} = v$$

⋮

$$\llbracket \text{let } x = e \text{ in } f \rrbracket_\varepsilon^{n+1} = \begin{cases} \llbracket f[v/x] \rrbracket_\varepsilon^n & \text{if } \llbracket e \rrbracket_\varepsilon^n = v \\ \perp & \text{OTHERWISE} \end{cases}$$

LEMMA

THE SEQUENCE OF MAPS $(\llbracket - \rrbracket_{\varepsilon}^n)_{n \geq 0}$
IS AN ω -CHAIN IN THE ω -CPO

$$\Lambda_{\sigma}^{\circ} \rightarrow \mathcal{V}_{\sigma}^{\circ}$$

THERE IS THUS AN UPPER BOUND,
CALL IT $\llbracket - \rrbracket_{\varepsilon}$, NAMELY THE OPERATIONAL
SEMANTICS OF Λ^{ST}