Effect typing

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Effect polymorphism

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Example: choice and failure

$$\begin{aligned} \mathsf{maybeFail} &: \forall e.A!(e \uplus \{\mathsf{fail} : a.1 \twoheadrightarrow a\}) \Rightarrow \mathsf{Maybe}\ A!e \\ \mathsf{allChoices} &: \forall e.A!(e \uplus \{\mathsf{choose} : 1 \twoheadrightarrow \mathsf{Bool}\}) \Rightarrow \mathsf{List}\ A!e \end{aligned}$$

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With explicit type applications we may write:

$$\textbf{handle}\ (\textbf{handle}\ \mathsf{drunkTosses}\ 2\ \textbf{with}\ \mathsf{maybeFail}\ \{\mathsf{choose} : 1 \twoheadrightarrow \mathsf{Bool}\})\ \textbf{with}\ \mathsf{allChoices}\ \emptyset$$

or

$$\textbf{handle}\ (\textbf{handle}\ \mathsf{drunkTosses}\ 2\ \textbf{with}\ \mathsf{allChoices}\ \{\mathsf{fail} : a.1 \twoheadrightarrow a\})\ \textbf{with}\ \mathsf{maybeFail}\ \emptyset$$

Effect polymorphism via row polymorphism

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For effect handlers labels are either operation names or effect names

Rémy-style row polymorphism

Rows as maps from labels to type-level maybes — each label is either present with type A (Pre(A)) or absent (Abs)

Duplicate labels disallowed

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Example:

$$\begin{aligned}
&\mathsf{maybeFail} : \forall (e : \mathsf{Row}_{\{\mathsf{fail}\}}), (p : \mathsf{Presence}).\\
&\qquad A!(\{\mathsf{fail} : (a : \mathsf{Type}).1 \twoheadrightarrow a; e\}) \Rightarrow \mathsf{Maybe}\ A!\{\mathsf{fail} : p; e\}\\
&\mathsf{allChoices} : \forall (e : \mathsf{Row}_{\{\mathsf{choose}\}}), (p : \mathsf{Presence}).\\
&\qquad A!(e \uplus \{\mathsf{choose} : 1 \twoheadrightarrow \mathsf{Bool}; e\}) \Rightarrow \mathsf{List}\ A!\{\mathsf{choose} : p; e\}
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Leijen-style row polymorphism

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Handler composition with row polymorphism

Instantiating an effect variable supports handler composition

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Rémy style (explicit instantiation):

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Example: abstracting over an exception handler

Rémy style:

$$\mathrm{catch} : (1 \to b!\{\mathsf{fail} : a.1 \twoheadrightarrow a; e\}) \to (1 \to b!\{\mathsf{fail} : p; e\}) \to b!\{\mathsf{fail} : p; e\}$$

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If h can itself fail then e is instantiated to $(\mathsf{fail} : a.1 \twoheadrightarrow a; e')$ for some e', which means the type of m is $(1 \to b!\{\mathsf{fail} : a.1 \twoheadrightarrow a, \mathsf{fail} : a.1 \twoheadrightarrow a; e'\})$

Invisible effect polymorphism

Key observation: for higher-order functions the effect variables almost always match up because we typically *use* the function arguments

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And further that empty polymorphic effects need not be written at all:

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We do now need to use explicit syntax to denote a closed row ($\emptyset$), but with row-based effect typing closed rows are uncommon

Effect pollution example

Handlers

$$\text{reads} : \text{List Nat} \to a!\{\text{get} : 1 \twoheadrightarrow \text{Nat}\} \Rightarrow a$$

$$\begin{aligned}\text{reads}\,([]) = \ & \textbf{return}\,x && \mapsto x \\ & \langle \text{get}\,() \to r \rangle && \mapsto \text{fail}\,()\end{aligned} \qquad \begin{aligned}\text{reads}\,(n :: ns) = \ & \textbf{return}\,x && \mapsto x \\ & \langle \text{get}\,() \to r \rangle && \mapsto r\;ns\;n\end{aligned}$$

$$\begin{aligned}&\text{maybeFail} : b!\{\text{fail} : a.1 \twoheadrightarrow a\} \Rightarrow \text{Maybe}\ b \\ &\text{maybeFail} = \textbf{return}\,x \quad \mapsto \text{Just}\,x \\ &\qquad\qquad\quad\ \langle \text{fail}\,() \to r \rangle \mapsto \text{Nothing}\end{aligned}$$

Effect pollution example

$$\text{bad} : \text{List}\ b \to (1 \to b!\{\text{get} : 1 \twoheadrightarrow \text{Nat}, \text{fail} : a.1 \twoheadrightarrow a\}) \to \text{Maybe}\ b$$
$$\text{bad}\ ns\ t = \textbf{handle}\ (\textbf{handle}\ t\ ()\ \textbf{with}\ \text{reads}\ ns)\ \textbf{with}\ \text{maybeFail}$$

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How can we encapsulate the use of fail as an intermediate effect?

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The aim is to define

$$\mathsf{good} : \mathsf{List}\ b \to (1 \to b!\{\mathsf{get} : 1 \twoheadrightarrow \mathsf{Nat}\}) \to \mathsf{Maybe}\ b$$

by composing reads and maybeFail such that

$$\mathsf{good}\ [1, 2]\ (\lambda().\mathsf{get}\ () + \mathsf{fail}\ ()) : \mathsf{Maybe\ Nat}!\{\mathsf{fail} : a.1 \twoheadrightarrow a\}$$

performs the fail operation.

Effect encapsulation

Two solutions to the effect pollution problem:

- Mask the intermediate effect (only works for Leijen-style row-typing)

  $$\text{good} : \text{List } b \to (1 \to b!\{\text{get} : 1 \twoheadrightarrow \text{Nat}\}) \to \text{Maybe } b$$
  $$\text{good } ns\ t = \textbf{handle } (\textbf{handle } (\langle \text{fail} \rangle\ (t\ ()))\ \textbf{with} \text{ reads } ns)\ \textbf{with} \text{ maybeFail}$$

  Frank, Koka, and Helium support this approach.
  [Biernacki, Piróg, Polesiuk, Sieczkowski, POPL 2018, "Handle with care"]
  [Convent, Lindley, McBride, McLaughlin, JFP 2019, "Doo bee doo bee doo"]

- Add support for fresh effects

  Helium and Links support this approach.
  [Biernacki, Piróg, Polesiuk, Sieczkowski, POPL 2019, "Abstracting algebraic effects"]

Effect masking

$$\frac{\Delta; \Gamma \vdash M : A!\{R\}}{\Delta; \Gamma \vdash \langle \mathsf{op} \rangle\ M : A!\{\mathsf{op} : B \twoheadrightarrow C; R\}}$$

Akin to weakening for effects

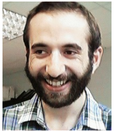
Doo bee doo bee doo

Shall I be pure or impure?

—Philip Wadler

A value is. A computation does.

—Paul Blain Levy



'To be is to do'—Socrates.
'To do is to be'—Sartre.
'Do be do be do'—Sinatra.

—anonymous graffiti, via Kurt Vonnegut

Frank

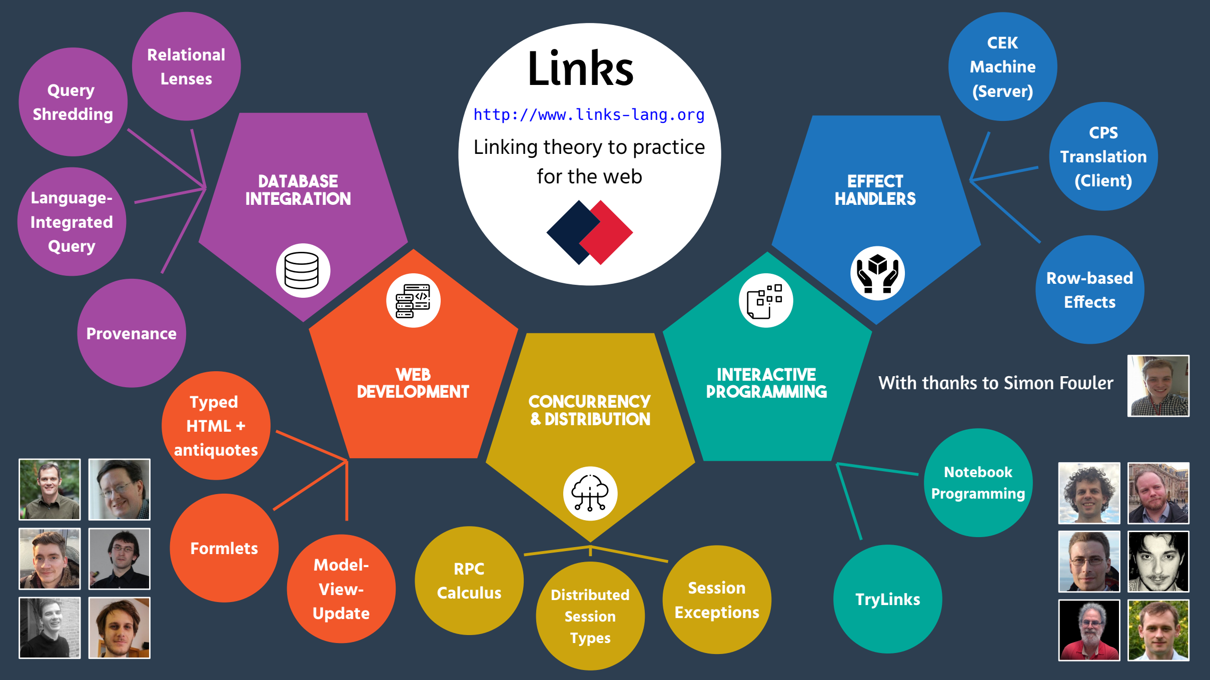
[Lindley, McBride, McLaughlin, POPL 2017, “Do be do be do”]
[Convent, Lindley, McBride, McLaughlin, JFP 2019, “Doo bee doo bee doo”]

Frank is an unequivocally effect handler oriented research programming language

Key features include:

- invisible effect polymorphism
- call-by-handling
- multihandlers
- adjustments
- adaptors (a generalisation of mask)

Probably a misfeature: unusual syntax


Links
http://www.links-lang.org
Linking theory to practice for the web
DATABASE INTEGRATION
Relational Lenses
Query Shredding
Language-Integrated Query
Provenance
WEB DEVELOPMENT
Typed HTML + antiquotes
Formlets
Model-View-Update
CONCURRENCY & DISTRIBUTION
RPC Calculus
Distributed Session Types
Session Exceptions
INTERACTIVE PROGRAMMING
Notebook Programming
TryLinks
EFFECT HANDLERS
CEK Machine (Server)
CPS Translation (Client)
Row-based Effects
With thanks to Simon Fowler

Handlers in Links and Frank (demo)

Effect typing scalability challenges

Effect encapsulation

Linearity

Generativity

Indexed effects

Equations