PROGRAM SYNTHESIS

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HOW DOES SKETCH WORK?

```
harness void doublesketch(int x)
{
  int t = x * ??;
  assert t == x + x;
}
```

Search through all possible ways to fill the hole “??” such that the specification is satisfied:

Find c such that:

$$\forall x.\ \ (x * c = x + x)$$

Equivalently:

$$\exists c. \forall x.\ \ (x * c = x + x)$$

In general, every sketching problem can be converted to solving a formula of the form:

$$\exists c_1 c_2 \ldots \forall x_1 x_2.\ \varphi(c_1, c_2, .., x_1, x_2 \ldots)$$

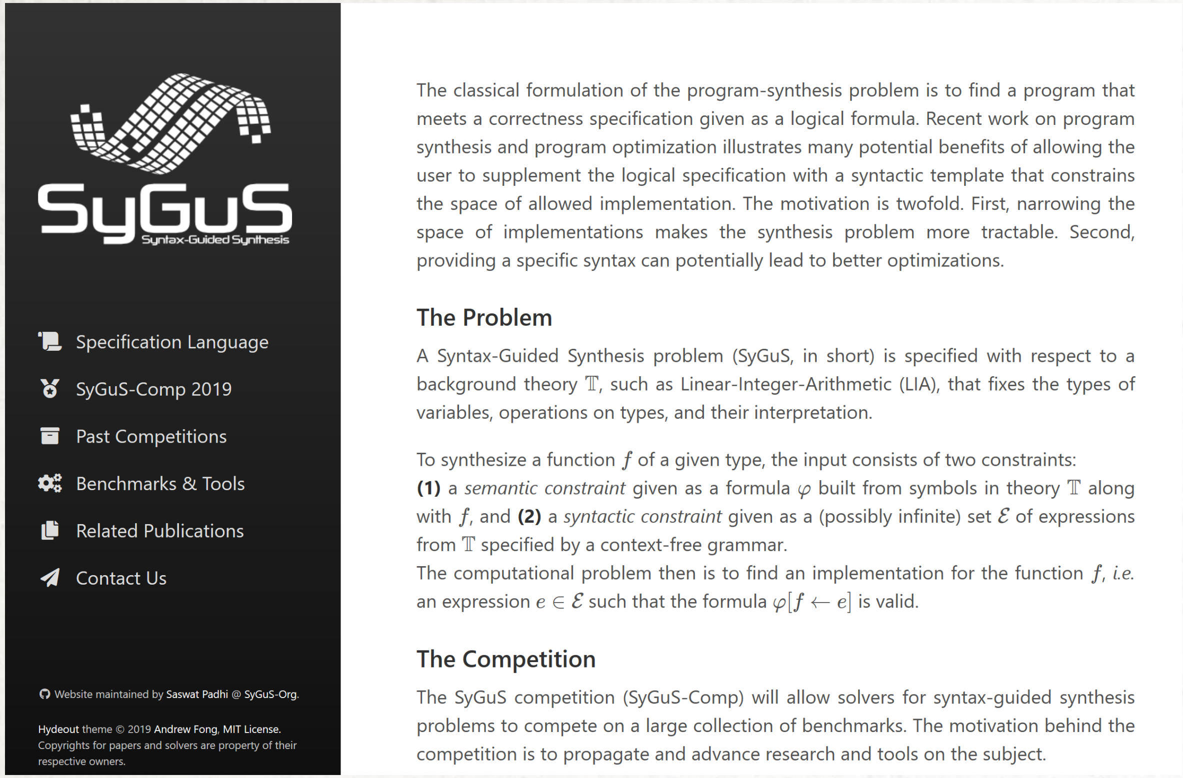
RECIPE FOR CONSTRAINT BASED SYNTHESIS

1. Convert the synthesis problem P to a formula φ
2. Solve φ using a constraint solver
3. Map solution from constraint solver back to the synthesis problem P

Types of constraint solvers:

- SAT Solvers (Boolean Satisfiability)
- SMT Solvers (Satisfiability Modulo Theories)
- Solvers for Quantified Formulas (QBF or CEGIS solvers)

SYGUS: SYNTAX GUIDED SYNTHESIS



SyGuS
Syntax-Guided Synthesis

- Specification Language
- SyGuS-Comp 2019
- Past Competitions
- Benchmarks & Tools
- Related Publications
- Contact Us

Website maintained by Saswat Padhi @ SyGuS-Org.

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The classical formulation of the program-synthesis problem is to find a program that meets a correctness specification given as a logical formula. Recent work on program synthesis and program optimization illustrates many potential benefits of allowing the user to supplement the logical specification with a syntactic template that constrains the space of allowed implementation. The motivation is twofold. First, narrowing the space of implementations makes the synthesis problem more tractable. Second, providing a specific syntax can potentially lead to better optimizations.

The Problem

A Syntax-Guided Synthesis problem (SyGuS, in short) is specified with respect to a background theory $\mathbb{T}$, such as Linear-Integer-Arithmetic (LIA), that fixes the types of variables, operations on types, and their interpretation.

To synthesize a function f of a given type, the input consists of two constraints: **(1)** a *semantic constraint* given as a formula φ built from symbols in theory $\mathbb{T}$ along with f, and **(2)** a *syntactic constraint* given as a (possibly infinite) set $\mathcal{E}$ of expressions from $\mathbb{T}$ specified by a context-free grammar.
The computational problem then is to find an implementation for the function f, *i.e.* an expression $e \in \mathcal{E}$ such that the formula $\varphi[f \leftarrow e]$ is valid.

The Competition

The SyGuS competition (SyGuS-Comp) will allow solvers for syntax-guided synthesis problems to compete on a large collection of benchmarks. The motivation behind the competition is to propagate and advance research and tools on the subject.

Syntax-guided synthesis; FMCAD'13

Alur, Bodik, Juniwal, Martin, Raghothaman, Seshia, Singh, Solar-Lezama, Torlak, Udupa

- Fix a background theory T: fixes types and operations
- Function to be synthesized: name f along with its type
  - General case: multiple functions to be synthesized
- Inputs to SyGuS problem:
  - Specification $\varphi(x, f(x))$
  - Typed formula using symbols in T + symbol f
- Set of expressions given by a context-free grammar G
  - Set of candidate expressions that use symbols in T
- Computational problem
  - Output e from grammar G, such that $\varphi\ [e \rightarrow f]$ is valid (in theory T)

SYNTAX-GUIDED PROGRAM SYNTHESIS

SLIDES BY RAJEEV ALUR AND THE EXCAPE: EXPEDITION TEAM

1. PROGRAMMING BY EXAMPLES (PBE)

Desired program P: bit-vector transformation that resets rightmost substring of contiguous 1's to 0's

1. P should be constructed from standard bit-vector operations

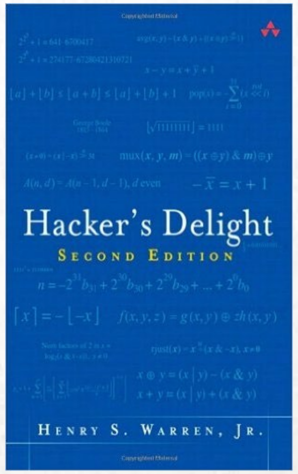
   |, &, ~, +, -, <<, >>, 0, 1, …

2. P specified using input-output examples

   00101 → 00100

   01010 → 01000

   10110 → 10000



Desired solution:

x & (1 + (x | (x-1))

FLASHFILL: PBE IN PRACTICE

REF: GULWANI (POPL 2011)

Input	Output
(425)-706-7709	425-706-7709
510.220.5586	510-220-5586
1 425 235 7654	425-235-7654
425 745-8139	425-745-8139

Wired: Excel is now a lot easier for people who aren't spreadsheet- and chart-making pros. The application's new Flash Fill feature recognizes patterns, and will offer auto-complete options for your data. For example, if you have a column of first names and a column of last names, and want to create a new column of initials, you'll only need to type in the first few boxes before Excel recognizes what you're doing and lets you press Enter to complete the rest of the column.

SCYTHE: SQL QUERIES FROM INPUT-OUTPUT EXAMPLES

WANG, CHEUNG, BODIK; SCYTHE.CS.WASHINGTON.EDU

Task: Collect the max vals below 50 for all oid groups in T2 and join them with T1.

T1

id	date	uid
1	12/25	1
2	11/21	3
4	12/24	2

T2

oid	val
1	30
1	10
1	10
2	50
2	10

Out

oid	date	uid	oid	MaxVal
1	12/25	1	1	30
4	12/24	2	2	10

Constants = { 50 }

AggrFunc = { Max, Min }

```
Select *
From (Select oid, Max(val)
      From  T2
      Where val < 50
      Group By oid) T3
Join T1
On T3.oid = T1.uid
```

2. PROGRAM OPTIMIZATION

Can regular programmers match experts in code performance?

- Improved energy performance in resource constrained settings
- Adoption to new computing platforms such as GPUs

Opportunity: Semantics-preserving code transformation

Possible Solution: Superoptimizing Compiler

- Structure of transformed code may be dissimilar to original

SUPEROPTIMIZATION ILLUSTRATION

Given a program P, find a “better” equivalent program P’

```
average (bitvec[32] x, y) {
  bitvec[64] x1 = x;
  bitvec[64] y1 = y;
  bitvec[64] z1 = (x1+y1)/2;
  bitvec[32] z = z1;
  return z
}
```

Find equivalent code without extension to 64 bit vectors

```
average (x, y) =
  (x and y) + [(x xor y) shift-right 1 ]
```

3. REPAIR/FEEDBACK FOR PROGRAMMING HOMEWORKS

SINGH ET AL (PLDI 2013)

```
1  def computeDeriv(poly):
2      deriv = []
3      zero = 0
4      if (len(poly)==1):
5          return deriv
6      for e in range(0,len(poly)):
7          if(poly[e]==0):
8              zero += 1
9          else:
10             deriv.append(poly[e]*e)
11
12     return deriv
```

Student Solution P
+ Reference Solution R
+ Error Model

The program requires **3** changes:

- In the return statement **return deriv** in **line 5**, replace **deriv** by **[0]**.
- In the comparison expression **(poly[e] == 0)** in **line 7**, change **(poly[e] == 0)** to **False**.
- In the expression **range(0, len(poly))** in **line 6**, replace **0** by **1**.

Find min no of edits to P so as to make it equivalent to R

4. AUTOMATIC INVARIANT GENERATION

```
SelectionSort(int A[],n) {
  i := 0;
  while(i < n−1) {
    v := i;
    j := i + 1;
    while (j < n) {
      if (A[j]<A[v])
        v := j ;
      j++;
    }
    swap(A[i], A[v]);
    i++;
  }
  return A;
}
```

Invariant: ?

Invariant: ?

post: $\forall k : 0 \leq k < n \Rightarrow A[k] \leq A[k+1]$

TEMPLATE-BASED AUTOMATIC INVARIANT GENERATION

```
SelectionSort(int A[],n) {
 i :=0;
 while(i < n−1) {
  v := i;
  j := i + 1;
  while (j < n) {
   if (A[j]<A[v])
    v := j ;
   j++;
  }
  swap(A[i], A[v]);
  i++;
 }
 return A;
}
```

Invariant:
∀k1,k2. ? ∧ ?

Invariant:
? ∧ ? ∧
(∀k1,k2. ? ∧ ?) ∧ (∀k. ? ∧ ?)

Constraint solver

post: $\forall k : 0 \le k < n \Rightarrow A[k] \le A[k+1]$

TEMPLATE-BASED AUTOMATIC INVARIANT GENERATION

```
SelectionSort(int A[],n) {
  i :=0;
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      if (A[j]<A[v])
        v := j ;
      j++;
    }
    swap(A[i], A[v]);
    i++;
  }
  return A;
}
```

Invariant:
$\forall k1,k2.\ 0 \le k1 < k2 < n \land$
$k1 < i \Rightarrow A[k1] \le A[k2]$

Invariant:
$i < j \land$
$i \le v < n \land$
$(\forall k1,k2.\ 0 \le k1 < k2 < n \land$
$k1 < i \Rightarrow A[k1] \le A[k2]) \land$
$(\forall k.\ i1 \le k < j \land$
$k \ge 0 \Rightarrow A[v] \le A[k])$

post: $\forall k : 0 \le k < n \Rightarrow A[k] \le A[k + 1]$

SYNTAX-GUIDED PROGRAM SYNTHESIS

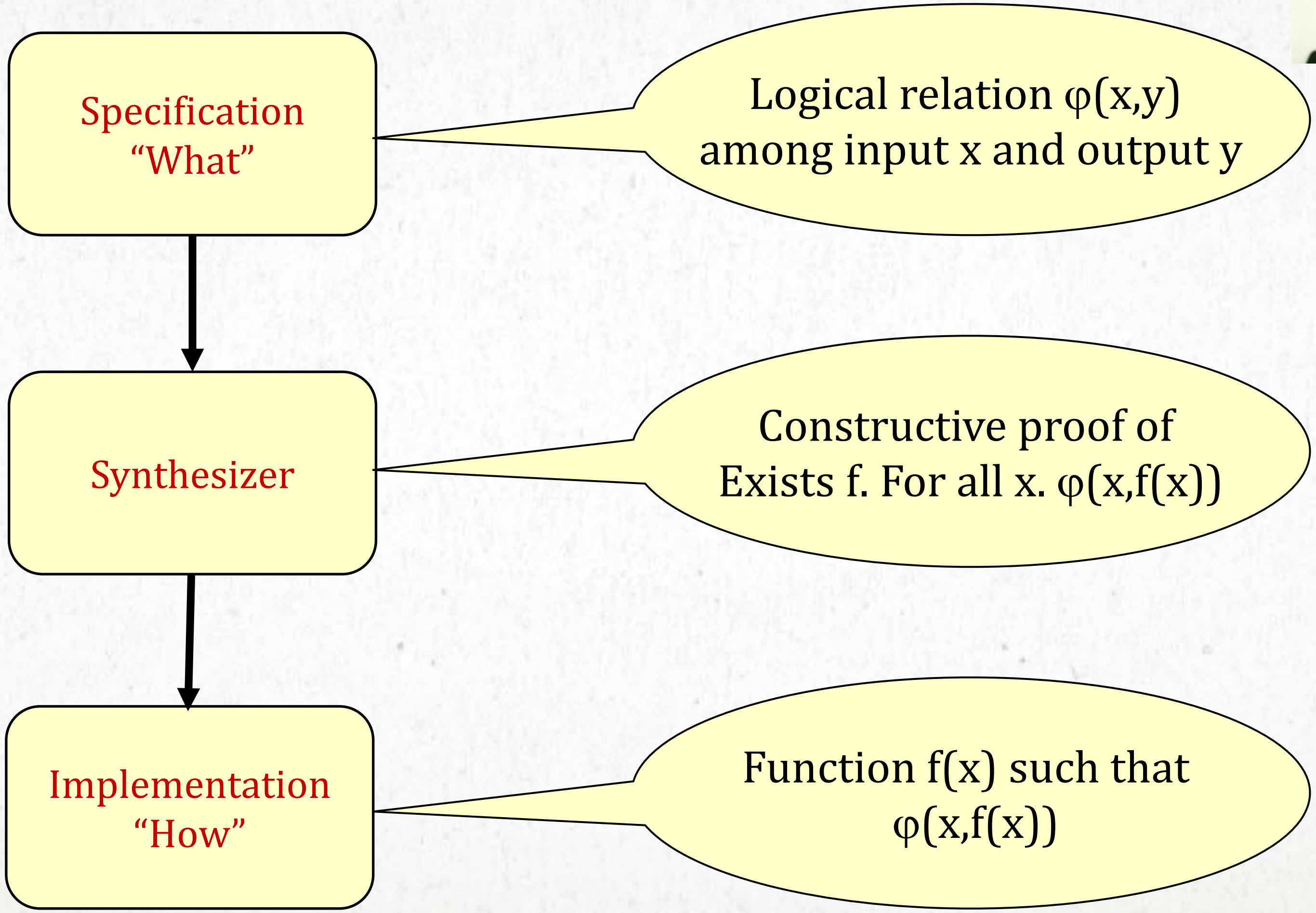
Rich variety of projects in programming systems and software engineering

1. Programming by examples
2. Program superoptimization
3. Automatic program repair
4. Template-guided invariant generation

Computational problem at the core of all these synthesis projects:

Find a program that meets given syntactic and semantic constraints

CLASSICAL PROGRAM SYNTHESIS



SYNTAX-GUIDED PROGRAM SYNTHESIS

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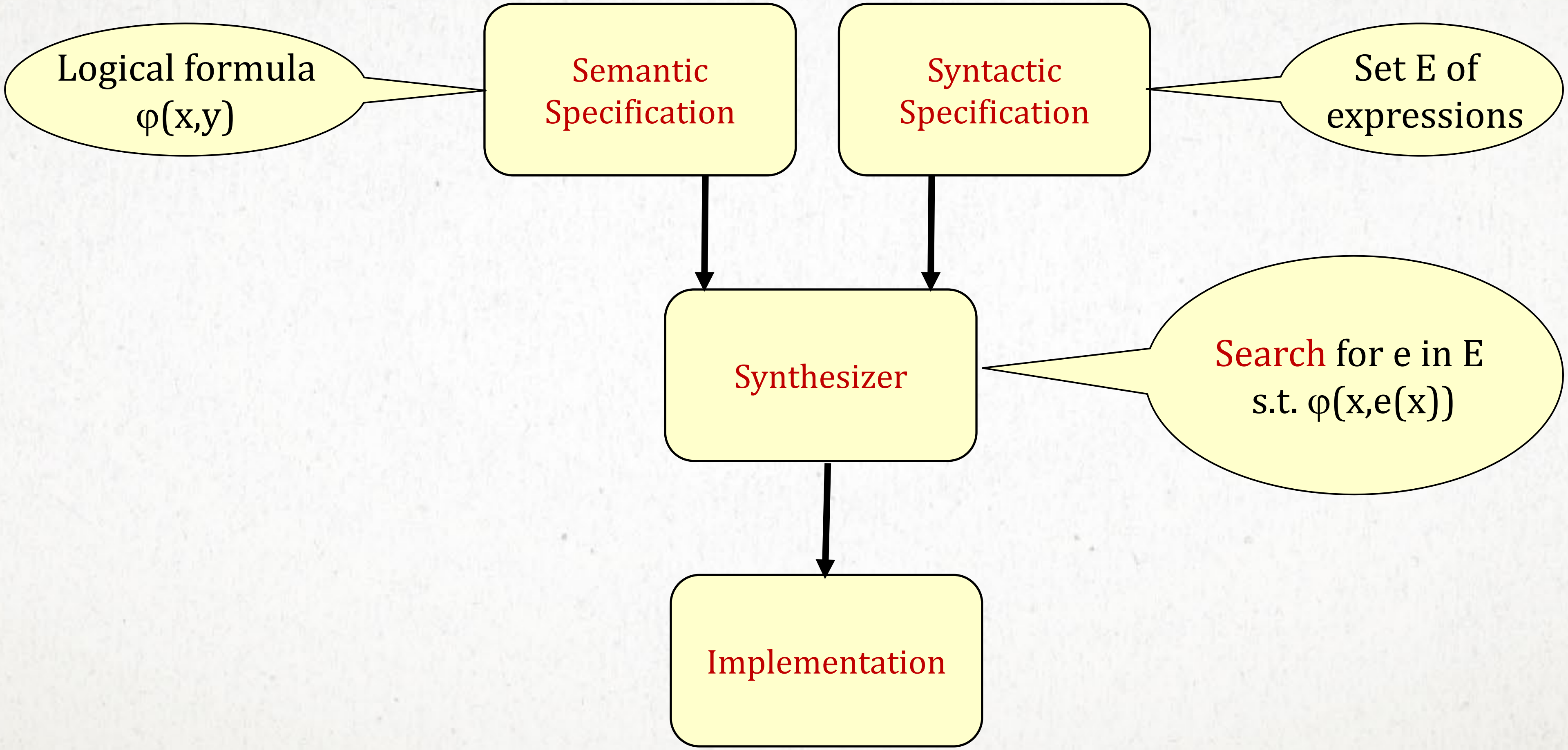

SYNTAX-GUIDED SYNTHESIS: FORMALIZATION

SYNTAX-GUIDED PROGRAM SYNTHESIS

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- Find a program snippet e such that
  1. e is in a set E of programs (syntactic constraint)
  2. e satisfies logical specification φ (semantic constraint)

- Core computational problem in many synthesis tools/applications

Can we formalize and standardize this computational problem?

Inspiration: Success of SMT solvers in formal verification

SMT: SATISFIABILITY MODULO THEORIES

- Computational problem: Find a satisfying assignment to a formula
  - Boolean + Int types, logical connectives, arithmetic operators
  - Bit-vectors + bit-manipulation operations in C
  - Boolean + Int types, logical/arithmetic ops + Uninterpreted functs

- "Modulo Theory": Interpretation for symbols is fixed
  - Can use specialized algorithms (e.g. for arithmetic constraints)

Little Engines of Proof

SAT; Linear arithmetic; Congruence closure

SYNTAX-GUIDED SYNTHESIS (SYGUS) PROBLEM

- Fix a background theory T: fixes types and operations

- Function to be synthesized: name f along with its type
  - General case: multiple functions to be synthesized

- Inputs to SyGuS problem:
  - Specification φ(x, f(x))
    Typed formula using symbols in T + symbol f
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    Set of candidate expressions that use symbols in T

- Computational problem:
  Output e in E such that φ[f/e] is valid (in theory T)

Syntax-guided synthesis; FMCAD'13

with Bodik, Juniwal, Martin, Raghothaman, Seshia, Singh, Solar-Lezama, Torlak, Udupa

SYGUS EXAMPLE 1

- Theory QF-LIA (Quantifier-free linear integer arithmetic)
  - Types: Integers and Booleans
  - Logical connectives, Conditionals, and Linear arithmetic
  - Quantifier-free formulas

- Function to be synthesized f (int x_1, x_2) : int

- Specification: $(x_1 \leq f(x_1, x_2))$ & $(x_2 \leq f(x_1, x_2))$

- Candidate Implementations: Linear expressions
  - LinExp := x_1 | x_2 | Const | LinExp + LinExp | LinExp - LinExp

- No solution exists

SYGUS EXAMPLE 2

- Theory QF-LIA

- Function to be synthesized: f (int x_1 , x_2) : int

- Specification: $(x_1 \leq f(x_1, x_2))$ & $(x_2 \leq f(x_1, x_2))$

- Candidate Implementations: Conditional expressions without +

  Term := x_1 | x_2 | Const | If-Then-Else (Cond, Term, Term)
  Cond := Term ≤ Term | Cond & Cond | ~ Cond | (Cond)

- Possible solution:
  If-Then-Else $(x_1 \leq x_2, x_2, x_1)$

FROM SMT-LIB TO SYNTH-LIB

www.sygus.org

```
(set-logic LIA)
(synth-fun max2 ((x Int) (y Int)) Int
    ((Start Int (x y 0 1
                (+ Start Start)
                (- Start Start)
                      (ite StartBool Start Start)))
      (StartBool Bool ((and StartBool StartBool)
                         (or StartBool StartBool)
                      (not StartBool)
                      (<= Start Start))))
 (declare-var x Int)
 (declare-var y Int)
 (constraint (≤  x (max2 x y)))
 (constraint (≤  y (max2 x y)))
 (constraint (or (= x (max2 x y)) (= y (max2 x y))))
 (check-synth)
```

Best SyGuS solver:
https://cvc5.github.io/

INVARIANT GENERATION AS SYGUS

```
bool x, y, z
int  a, b, c

while( Test ) {
   loop-body
   ....
}
```

- Goal: Find inductive loop invariant automatically
- Function to be synthesized

  Inv (bool x, bool z, int a, int b) : bool
- Compile loop-body into a logical predicate

  Update(x,y,z,a,b,c, x',y',z',a',b',c')
- Specification:

  (Inv & Update & Test') $\Rightarrow$ Inv'

  & Pre $\Rightarrow$ Inv & (Inv & ~Test $\Rightarrow$ Post)
- Template for set of candidate invariants

  Term := a | b | Const | Term + Term | If-Then-Else (Cond, Term, Term)

  Cond := x | z | Cond & Cond | ~ Cond | (Cond)

SOLVING SYGUS

SOLVING SYGUS

- Is SyGuS same as solving SMT formulas with quantifier alternation?

- SyGuS can sometimes be reduced to Quantified-SMT, but not always
  - Set E is all linear expressions over input vars x, y

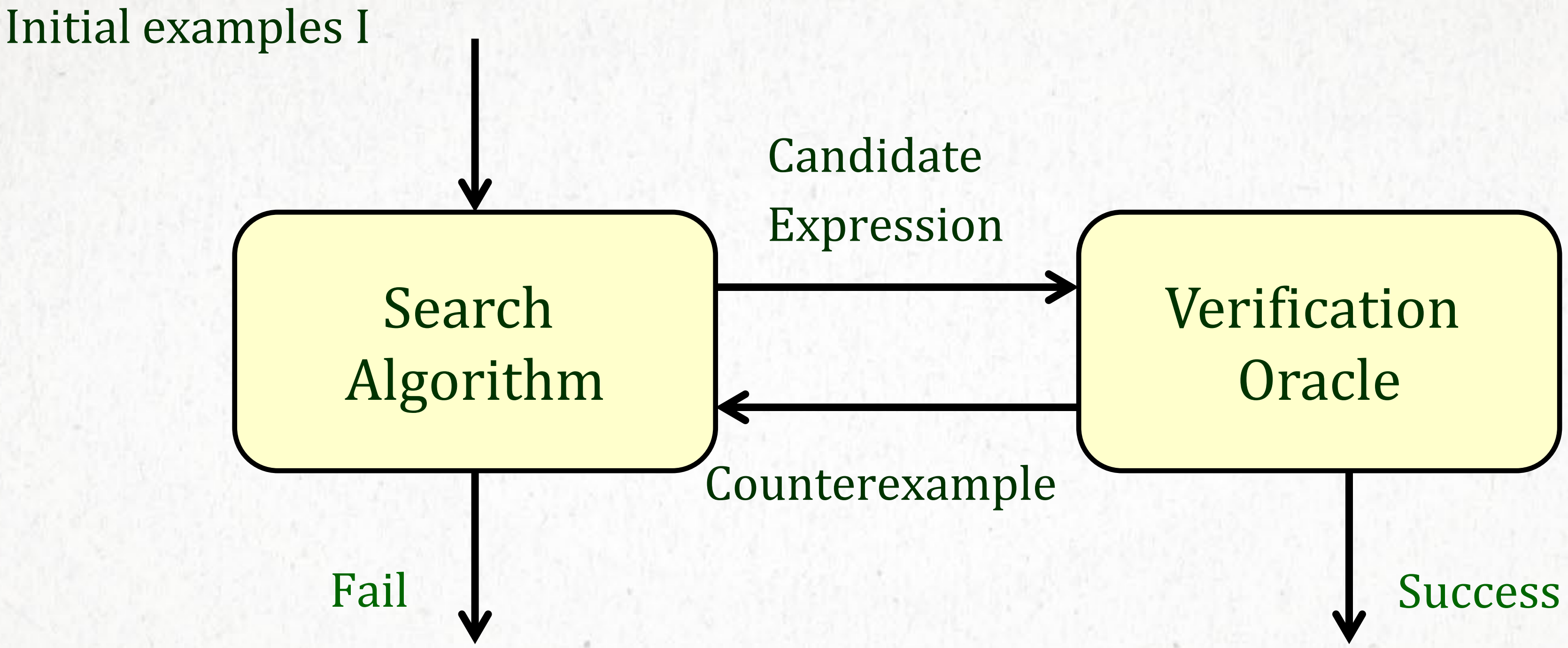
    SyGuS reduces to Exists a,b,c. Forall X. φ [f/ ax+by+c]
  - Set E is all conditional expressions

    SyGuS cannot be reduced to deciding a formula in LIA

- Syntactic structure of the set E of candidate implementations can be used effectively by a solver

- Existing work on solving Quantified-SMT formulas suggests solution strategies for SyGuS

SYGUS AS ACTIVE LEARNING



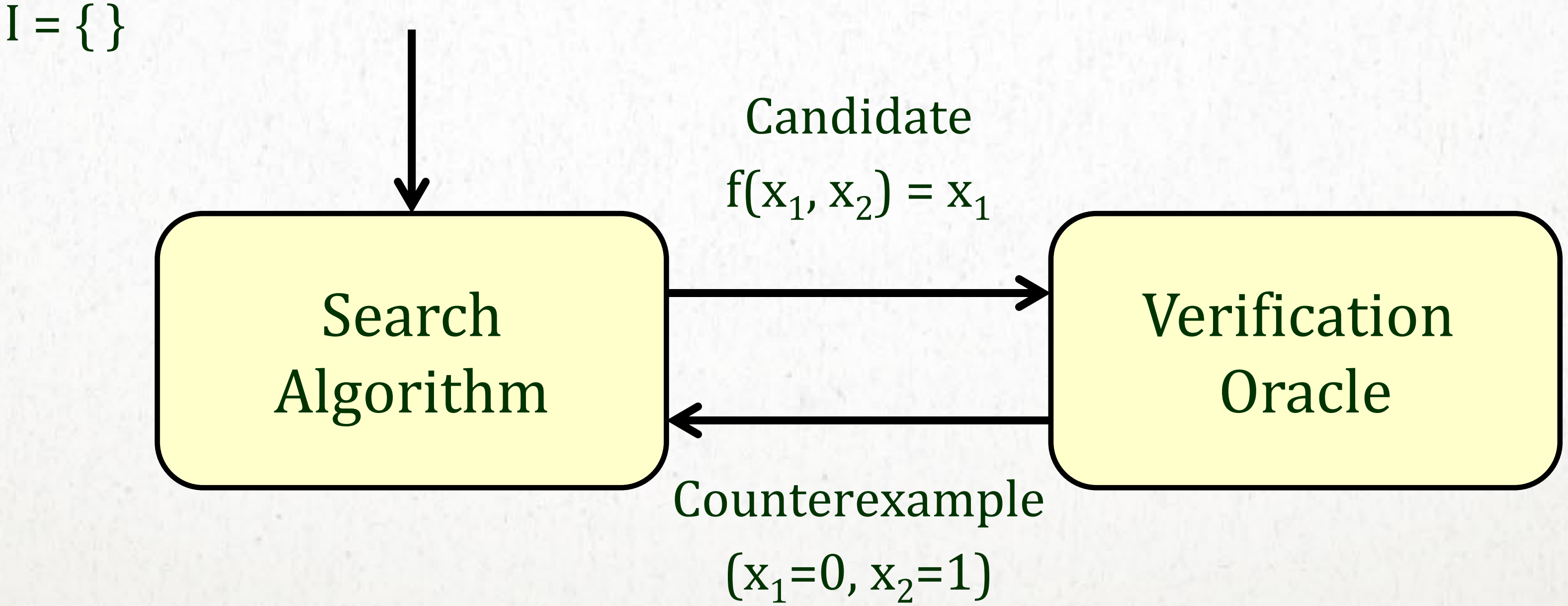
Concept class: Set E of expressions

Examples: Concrete input values

COUNTEREXAMPLE-GUIDED INDUCTIVE SYNTHESIS

SOLAR-LEZAMA ET AL (ASPLOS'06)

- Specification: $(x_1 \leq f(x_1, x_2))$ & $(x_2 \leq f(x_1, x_2))$
- Set E: All expressions built from x_1, x_2,0,1, Comparison, If-Then-Else



CEGIS EXAMPLE

- Specification: $(x_1 \leq f(x_1, x_2))$ & $(x_2 \leq f(x_1, x_2))$

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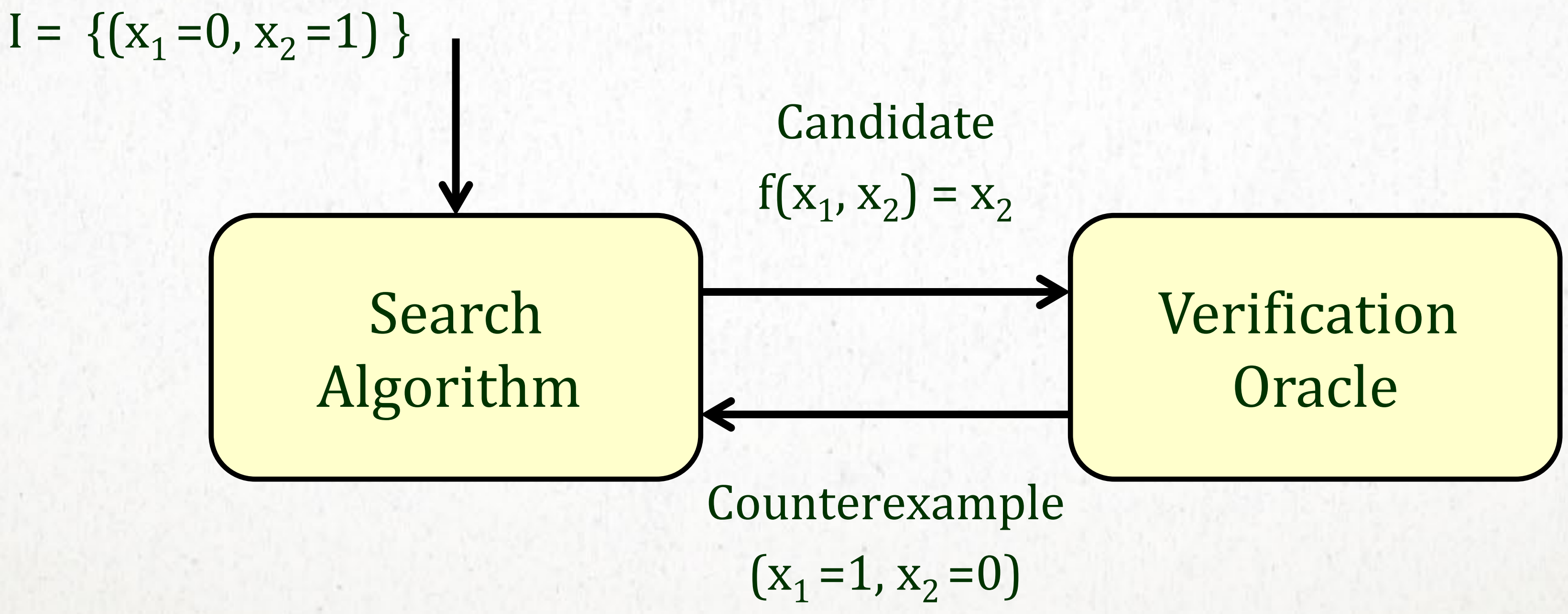

CEGIS EXAMPLE

- Specification: $(x_1 \leq f(x_1, x_2))$ & $(x_2 \leq f(x_1, x_2))$
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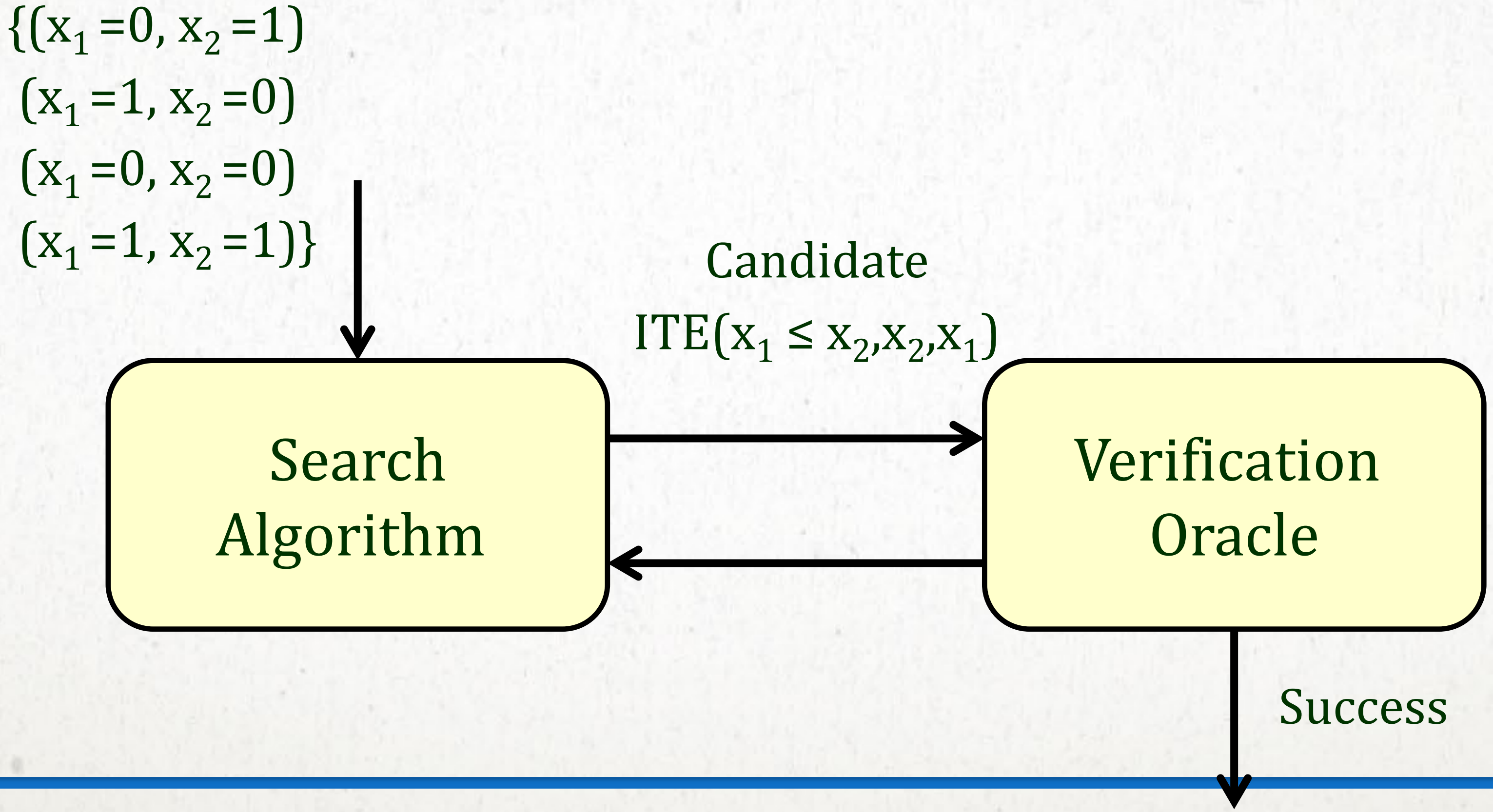


Counterexample-guided Inductive Synthesis (CEGIS)

Goal: Find f in E such that for all x in D, $\varphi(x, f)$ holds

I = { }; /* Interesting set of inputs */

Repeat

  Learn: Find f in E such that for all x in I, $\varphi(f, x)$ holds

  Verify: Find x in D such that $\varphi(f, x)$ does not hol

    If so, add x to I

    Else, return f

SYGUS SOLUTIONS

- CEGIS approach (Solar-Lezama et al, ASPLOS'08)

- Similar strategies for solving quantified formulas and invariant generation

- Initial learning strategies based on:
    1. Enumerative (search with pruning): Udupa et al (PLDI'13)
    2. Symbolic (solving constraints): Gulwani et al (PLDI'11)
    3. Stochastic (probabilistic walk): Schkufza et al (ASPLOS'13)

1. ENUMERATIVE SEARCH

- Given:
  - Specification $\varphi(x, f(x))$
  - Grammar for set E of candidate implementations
  - Finite set I of inputs

  Find an expression e(x) in E s.t. $\varphi(x,e(x))$ holds for all x in I

- Attempt 0: Enumerate expressions in E in increasing size till you find one that satisfies φ for all inputs in I

- Attempt 1: Pruning of search space based on:
  - Expressions e_1 and e_2 are equivalent
    - if $e_1(x)=e_2(x)$ on all x in I
  - Only one representative among equivalent subexpressions needs to be considered for building larger expressions

ILLUSTRATING PRUNING

- Spec: $(x_1 < f(x_1, x_2))$ & $(x_2 < f(x_1, x_2))$
- Grammar: $E := x_1 \mid x_2 \mid 0 \mid 1 \mid E + E$
- $I = \{ (x_1=0, x_2=1) \}$
- Find an expression f such that $(f(0,1) > 0)$ & $(f(0,1) > 1)$

x_1 x_2

$\cancel{0}$ $\cancel{1}$

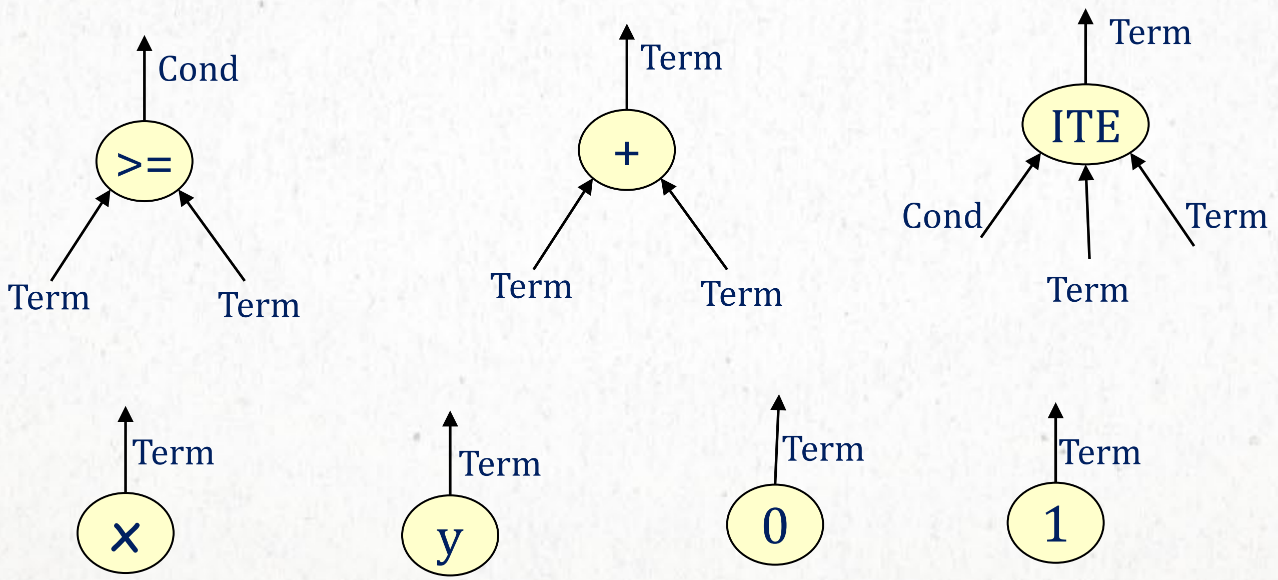
$\cancel{x_1 + x_1}$ $\cancel{x_1 + x_2}$ $x_2 + x_2$

$\cancel{x_2 + x_1}$

2. SYMBOLIC SEARCH

- Use a constraint solver for both synthesis and verification steps

- Each production in the grammar is thought of as a component.
  Input and Output ports of every component are typed.



- A well-typed loop-free program comprising these component corresponds to an expression DAG from the grammar.

SYMBOLIC ENCODING

- ❑ Start with a library consisting of some number of occurrences of each component.

n1	n2	n3	n4	n5	n6	n7	n8	n9	n10
$\times$	x	y	y	0	1	+	+	>=	ITE

- ❑ Synthesis Constraints:

  Shape is a DAG, Types are consistent

  Spec φ[f/e] is satisfied on every concrete input in I

- ❑ Use an SMT solver (Z3) to find a satisfying solution.

- ❑ If synthesis fails, try increasing the number of occurrences of components in the library in an outer loop

3. STOCHASTIC SEARCH

- Idea: Find desired expression e by probabilistic walk on graph where nodes are expressions and edges capture single-edits

- Metropolis-Hastings Algorithm: Given a probability distribution P over domain X, and an ergodic Markov chain over X, samples from X

- Fix expression size n. X is the set of expressions E_n of size n. $P(e) \propto Score(e)$ ("Extent to which e meets the spec φ")

- For a given set Examples, Score(e) = exp(- 0.5 Wrong(e)), where Wrong(e) = No of inputs in Examples for which ~ φ [f/e]

- Score(e) is large when Wrong(e) is small. Expressions e with Wrong(e) = 0 more likely to be chosen in the limit than any other expression

STOCHASTIC SEARCH

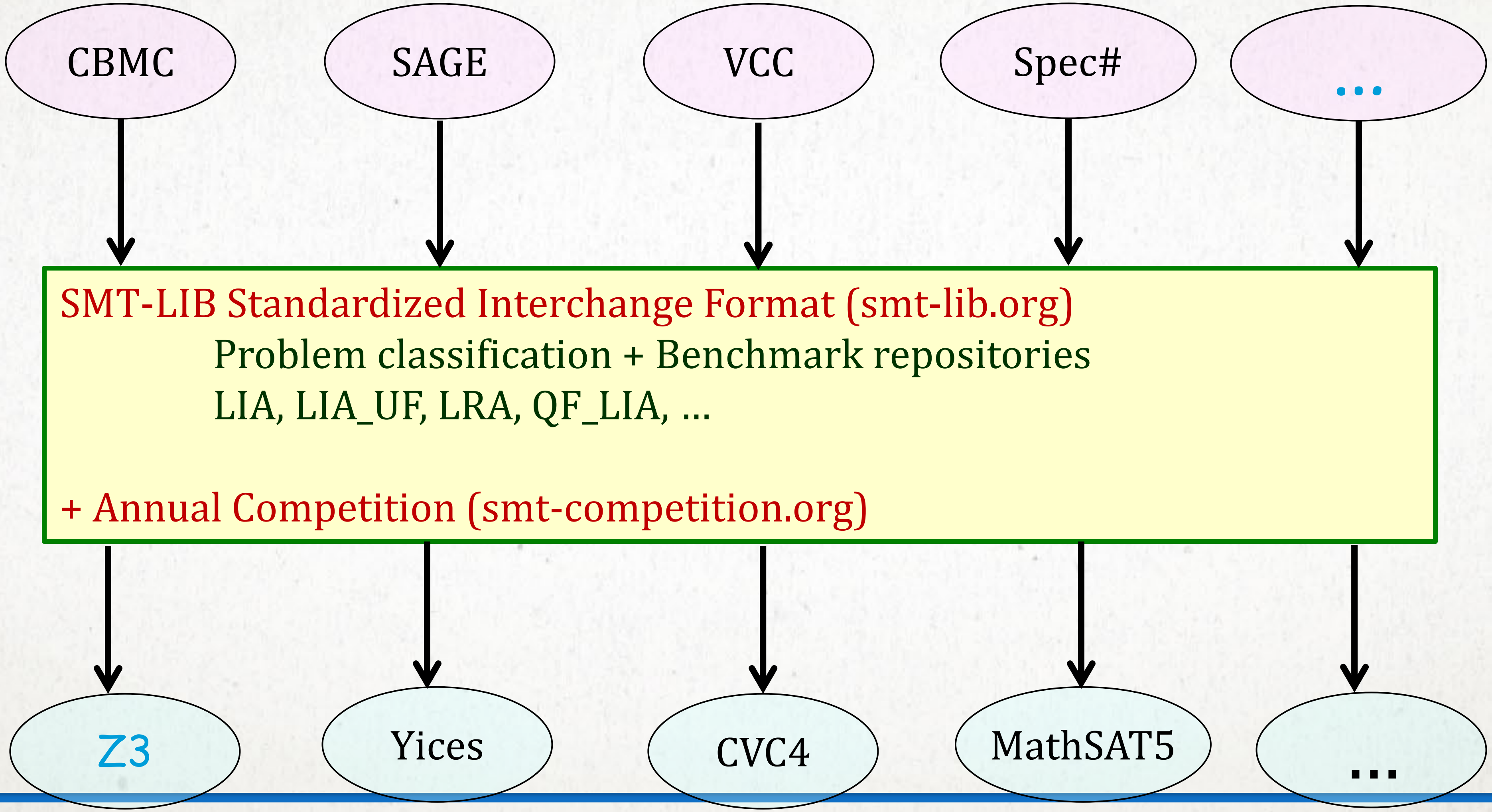
- Initial candidate expression e sampled uniformly from E_n
- When Score(e) = 1, return e
- Pick node v in parse tree of e uniformly at random. Replace subtree rooted at e with subtree of same size, sampled uniformly

- With probability min{ 1, Score(e')/Score(e) }, replace e with e'
- Outer loop responsible for updating expression size n

PART IV

SYGUS COMPETITION AND EVOLUTION

SMT SUCCESS STORY



SYGUS COMPETITION

Program optimization

Program repair

Programming by examples

Invariant generation

SYNTH-LIB Standardized Interchange Format
Problem classification + Benchmark repository
+ SyGuS-COMP (Competition for solvers) held since FLoC 2014

Techniques for Solvers:
Learning, Constraint solvers, Enumerative/stochastic search

Collaborators: D. Fisman, S. Padhi, A. Reynolds, R. Singh, A. Solar-Lezama, A. Udupa

SYGUS PROGRESS

www.sygus.org

- Over 2000 benchmarks
  - Hacker's delight
  - Invariant generation (based on verification competition SV-Comp)
  - FlashFill (programming by examples system from Microsoft)
  - Synthesis of attack-resilient crypto circuits
  - Program repair
  - Motion planning
  - ICFP programming competition
- Special tracks for competition
  - Invariant generation
  - Programming by examples
  - Conditional linear arithmetic
- New solution strategies and applications

SyGuS Conclusions

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- Problem definition
  - Syntactic constraint on space of allowed programs
  - Semantic constraint given by logical formula
- Solution strategies
  - Counterexample-guided inductive synthesis
  - Search in program space + Verification of candidate solutions
- Applications
  - Programming by examples
  - Program repair/optimization with respect to syntactic constraints
- Annual competition (SyGuS-comp)
  - Standardized interchange format + benchmarks repository

USING SYNTHESIS FOR MODULAR VERIFICATION

C
JOINT WORK WITH WILLIAM HALLAHAN AND RANJIT JHALA

VERIFICATION

```
map :: (a -> b) -> xs: [a] -> { ys:[b]| size xs == size ys }
map f [] = []
map f (x:xs) = f x: map f xs
```

```
8 | map f (x:xs) = map f xs
                   ^^^^^^^^^
 Inferred type
   VV : {v : [a] | size xs == size v
                && size v >= 0}
 not a subtype of Required type
   VV : {VV : [a] | size ?a == size VV}
 In Context
   xs : {v : [a] | size v >= 0}
```

William T. Hallahan, Anton Xue, Maxwell Troy Bland, Ranjit Jhala, and Ruzica Piskac. *Lazy Counterfactual Symbolic Execution*. PLDI 2019.

MODULAR VERIFICATION

```
add2 :: x:Int -> { y:Int | y == x + 2 }
add2 x = incr (incr x)
```

```
incr :: x:Int -> { y:Int | y > x }
```

```
Error: Liquid Type Mismatch

 5 | add2 x = incr (incr x)
                ^^^^^^^^^

  Inferred type
    VV : {v : Int | v > ?a}
```

add2 0 = 3
violating add2’s specification
if incr 0 = 2

G2

```
    x : Int
    ?a : {?a : Int | ?a > x}
```

To verify a caller, modular verifiers use callee’s specification

MODULAR VERIFICATION

```
add2 :: x:Int -> { y:Int | y == x + 2 }
add2 x = incr (incr x)
```

```
incr :: x:Int -> { y:Int | y > x }
```

Error: Liquid Type Mismatch

```
5 | add2 x = incr (incr x)
             ^^^^^^^^^^
```

Inferred type

add2 0 = 3
violating add2's specification
if incr 0 = 2

G2

In Context

x : Int

?a : {?a : Int | ?a > x}

To verify a caller, modular verifiers use callee's specification

MODULAR VERIFICATION

```
add2 :: x:Int -> { y:Int | y == x + 2 }
add2 x = incr (incr x)
```

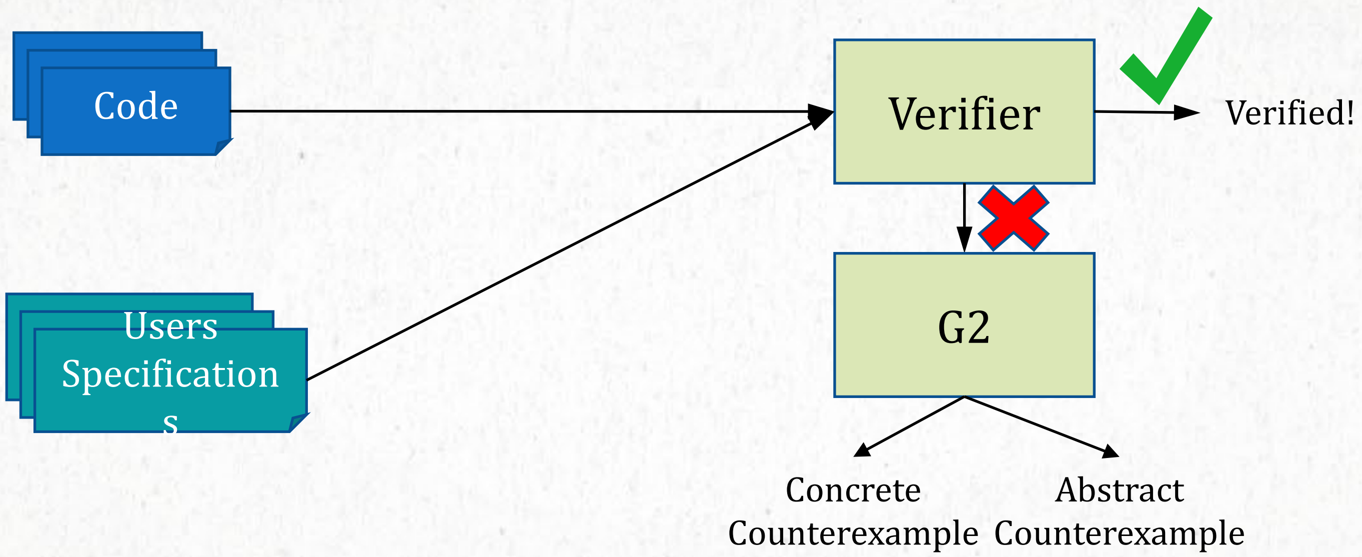
```
incr :: x:Int -> { y:Int | y = x + 1 }
incr x = x + 1
```

add2 0 = 3
violating add2’s specification
if incr 0 = 2

G2

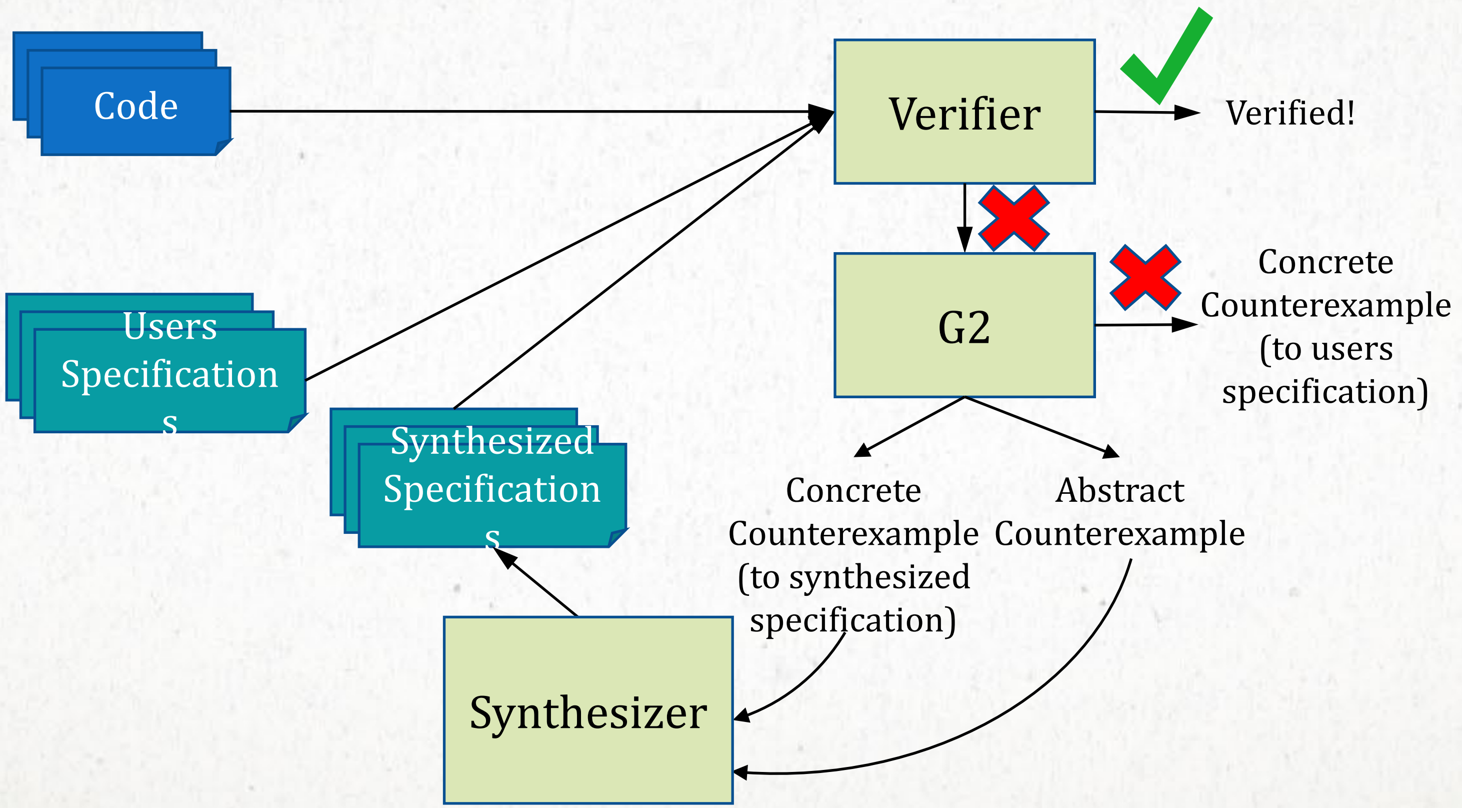
To verify a caller, modular verifiers use callee’s specification

OVERVIEW

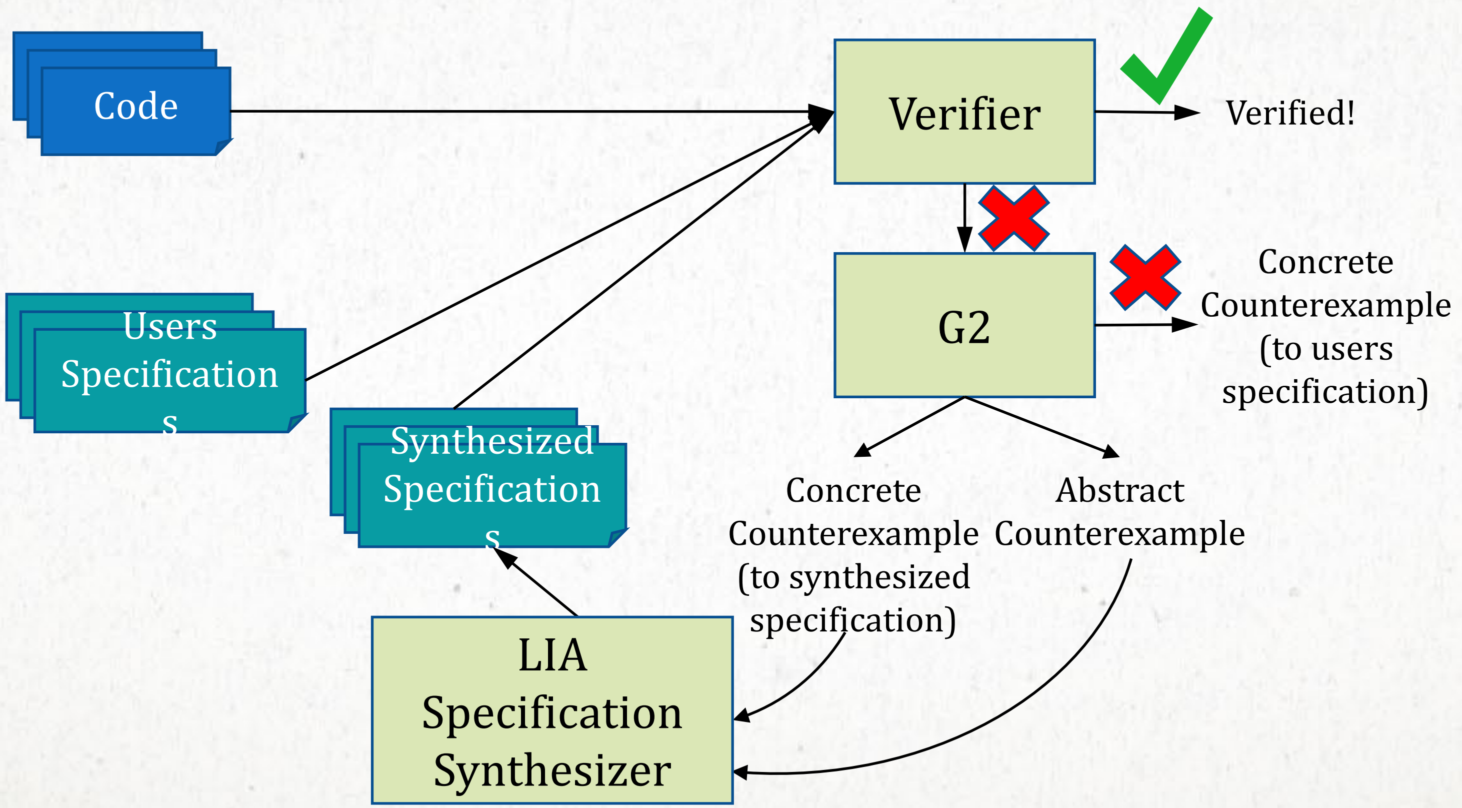


Code
Users
Specification
s
Verifier
Verified!
G2
Concrete
Counterexample
Abstract
Counterexample

OVERVIEW


Code
Users Specifications
Synthesized Specifications
Verifier
Verified!
G2
Concrete Counterexample (to users specification)
Concrete Counterexample (to synthesized specification)
Abstract Counterexample
Synthesizer

OVERVIEW



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LIA Specification Synthesizer

COUNTEREXAMPLES

Concrete Counterexample

```
map :: (a -> b) -> xs:[a] -> { ys:[b] | size xs == size ys}
map f [] = []
map f (x:xs) = map f xs
```

```
map id [1] = []
```

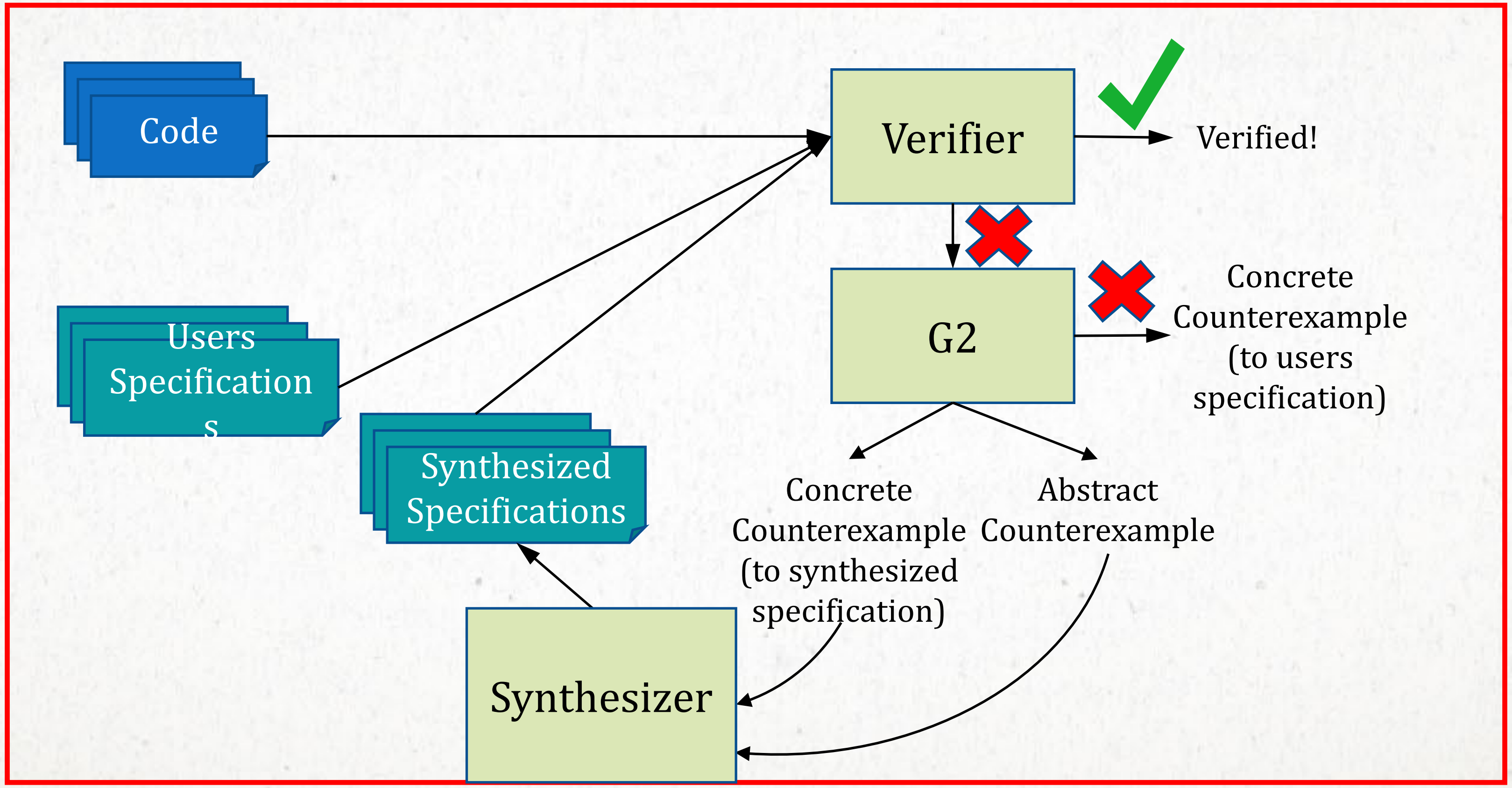
Abstract Counterexample

```
add2 :: x:Int -> { y:Int | y == x + 2 }
add2 x = incr (incr x)

incr :: x:Int -> { y:Int | y > x }
incr x = x + 1
```

```
add2 0 = 3
if  incr 0 = 2
```

OVERVIEW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EXAMPLE

```
concat :: x:[[a]] -> {v : [a] | size v = sumsize x}
concat [] = []
concat (xs:[]) = xs
concat (xs:(ys:xss)) = concat ((app xs ys):xss)

app :: x:[a] -> y:[a] ->   z:[a]
app [] [] = []
app xs [] = xs
app [] ys = ys
app (x:xs) ys  = x:app xs ys
```

Abstract counterexample:

```
concat [[], []] = [0]
if app [] [] = [0]
```

Real evaluation:

```
app [] [] = []
```

Synthesis

$pre_{app}([], []) \Rightarrow \neg post_{app}([], [], [0])$

$pre_{app}([], []) \Rightarrow post_{app}([], [], [])$

EXAMPLE

```
concat :: x:[[a]] -> {v : [a] | size v = sumsize x}
concat [] = []
concat (xs:[]) = xs
concat (xs:(ys:xss)) = concat ((app xs ys):xss)

app :: x:[a] -> y:[a] -> { z:[a] | size z == 0 }
app [] [] = []
app xs [] = xs
app [] ys = ys
app (x:xs) ys  = x:app xs ys
```

Abstract counterexample:

```
concat [[], []] = [0]
if app [] [] = [0]
```

Real evaluation:

```
app [] [] = []
```

Synthesis constraints:

$pre_{app}([], []) \Rightarrow \neg post_{app}([], [], [0])$

$pre_{app}([], []) \Rightarrow post_{app}([], [], [])$

EXAMPLE

```
concat :: x:[[a]] -> {v : [a] | size v = sumsize x}
concat [] = []
concat (xs:[]) = xs
concat (xs:(ys:xss)) = concat ((app xs ys):xss)

app :: x:[a] -> y:[a] -> { z:[a] | size z == 0 }
app [] [] = []
app xs [] = xs
app [] ys = ys
app (x:xs) ys  = x:app xs ys
```

Concrete counterexample:

```
app [0] [] = [0]
```

Synthesis constraints:

$pre_{app}([], []) \Rightarrow \neg post_{app}([], [], [0])$

$pre_{app}([], []) \Rightarrow post_{app}([], [], [])$

$pre_{app}([0], []) \Rightarrow post_{app}([0], [], [0])$

EXAMPLE

```
concat :: x:[[a]] -> {v : [a] | size v = sumsize x}
concat [] = []
concat (xs:[]) = xs
concat (xs:(ys:xss)) = concat ((app xs ys):xss)

app :: x:[a] -> y:[a] -> { z:[a] | size z == size x }
app [] [] = []
app xs [] = xs
app [] ys = ys
app (x:xs) ys  = x:app xs ys
```

Concrete counterexample:

app [0] [] = [0]

Synthesis constraints:

$pre_{app}([], []) \Rightarrow \neg post_{app}([], [], [0])$

$pre_{app}([], []) \Rightarrow post_{app}([], [], [])$

$pre_{app}([0], []) \Rightarrow post_{app}([0], [], [0])$

EXAMPLE

```
concat :: x:[[a]] -> {v : [a] | size v = sumsize x}
concat [] = []
concat (xs:[]) = xs
concat (xs:(ys:xss)) = concat ((app xs ys):xss)

app :: x:[a] -> y:[a] -> { z:[a] | size z == size x }
app [] [] = []
app xs [] = xs
app [] ys = ys
app (x:xs) ys  = x:app xs ys
```

Concrete counterexample:

app [0] [0] = [0, 0]

Synthesis constraints:

$pre_{app}([], []) \Rightarrow \neg post_{app}([], [], [0])$

$pre_{app}([], []) \Rightarrow post_{app}([], [], [])$

$pre_{app}([0], []) \Rightarrow post_{app}([0], [], [0])$

$pre_{app}([0], [0]) \Rightarrow post_{app}([0], [0], [0, 0])$

EXAMPLE

```
concat :: x:[[a]] -> {v : [a] | size v = sumsize x}
concat [] = []
concat (xs:[]) = xs
concat (xs:(ys:xss)) = concat ((app xs ys):xss)

app :: x:[a] -> y:[a] -> { z:[a] | size z == size x + size y}
app [] [] = []
app xs [] = xs
app [] ys = ys
app (x:xs) ys  = x:app xs ys
```

Concrete counterexample:

app [0] [0] = [0, 0]

Synthesis constraints:

$pre_{app}([], []) \Rightarrow \neg post_{app}([], [], [0])$

$pre_{app}([], []) \Rightarrow post_{app}([], [], [])$

$pre_{app}([0], []) \Rightarrow post_{app}([0], [], [0])$

$pre_{app}([0], [0]) \Rightarrow post_{app}([0], [0], [0])$

EXAMPLE

```
concat :: x:[[a]] -> {v : [a] | size v = sumsize x}
concat [] = []
concat (xs:[]) = xs
concat (xs:(ys:xss)) = concat ((app xs ys):xss)

app :: x:[a] -> y:[a] -> { z:[a] | size z == size x + size y}
app [] [] = []
app xs [] = xs
app [] ys = ys
app (x:xs) ys  = x:app xs ys
```

✓

Concrete counterexample:

app [0] [0] = [0, 0]

Synthesis constraints:

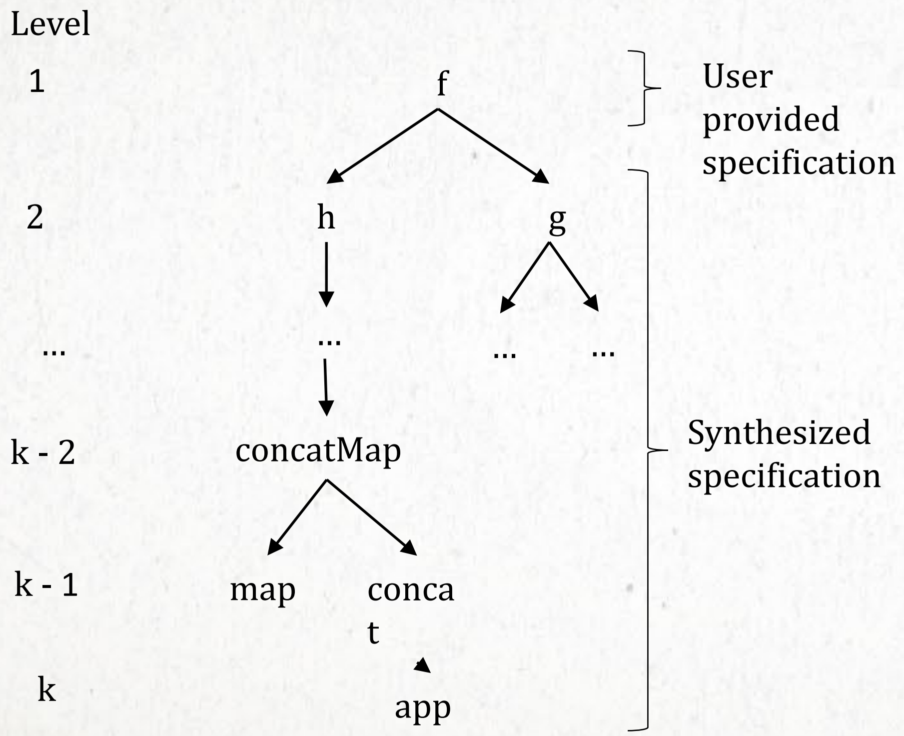
$pre_{app}([], []) \Rightarrow \neg post_{app}([], [], [0])$

$pre_{app}([], []) \Rightarrow post_{app}([], [], [])$

$pre_{app}([0], []) \Rightarrow post_{app}([0], [], [0])$

$pre_{app}([0], [0]) \Rightarrow post_{app}([0], [0], [0])$

CALL GRAPH TRAVERSAL



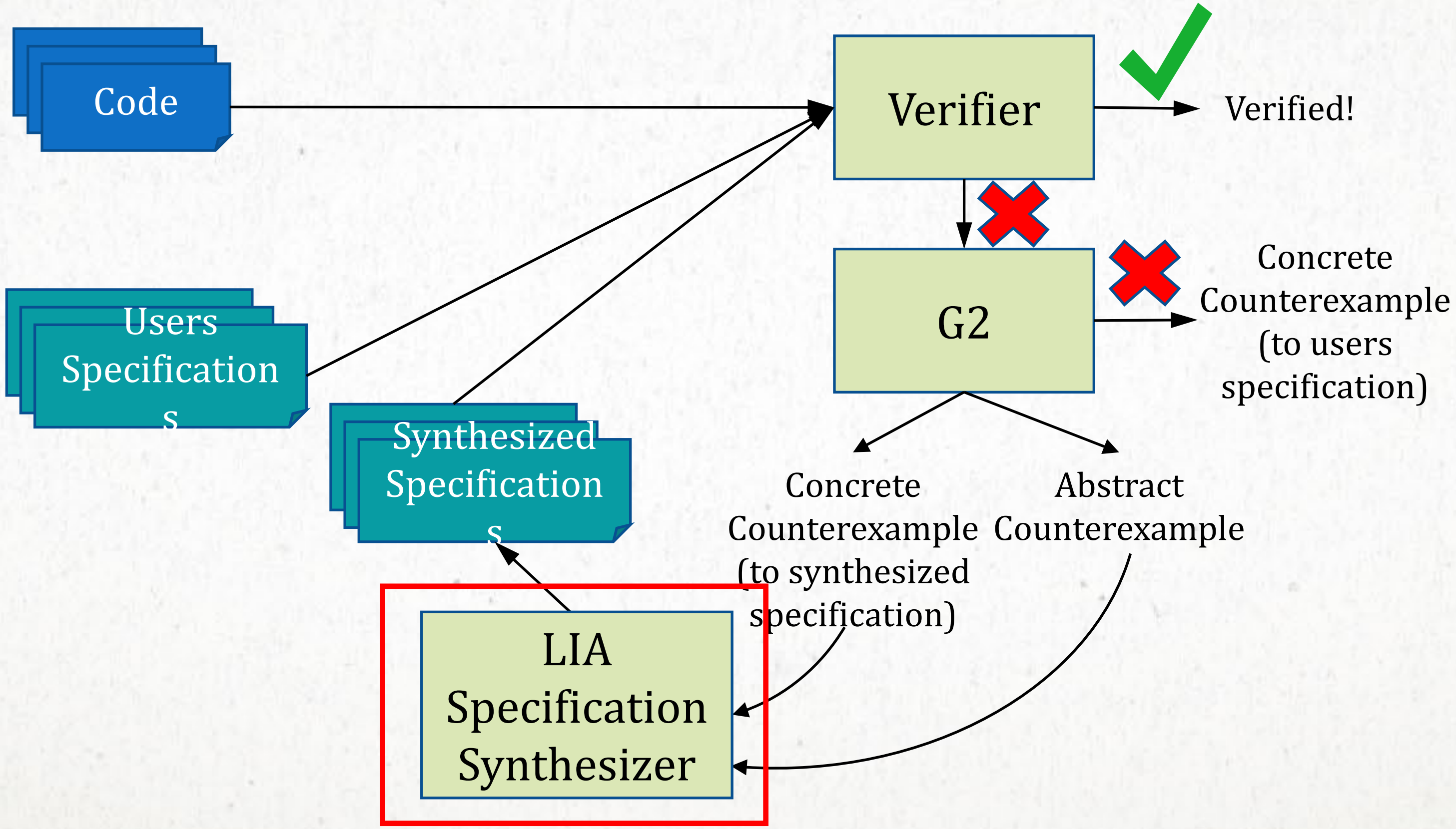
Walk **down** the call graph, from level 1 to level k.

At level i, synthesize specifications for the functions
at level i + 1 that **would** (if correct) prove specifications of functions at level i.

Backtrack if:

- a concrete counterexamples to a specification at level <= i is found
- specification synthesis problem becomes unrealizable

OVERVIEW



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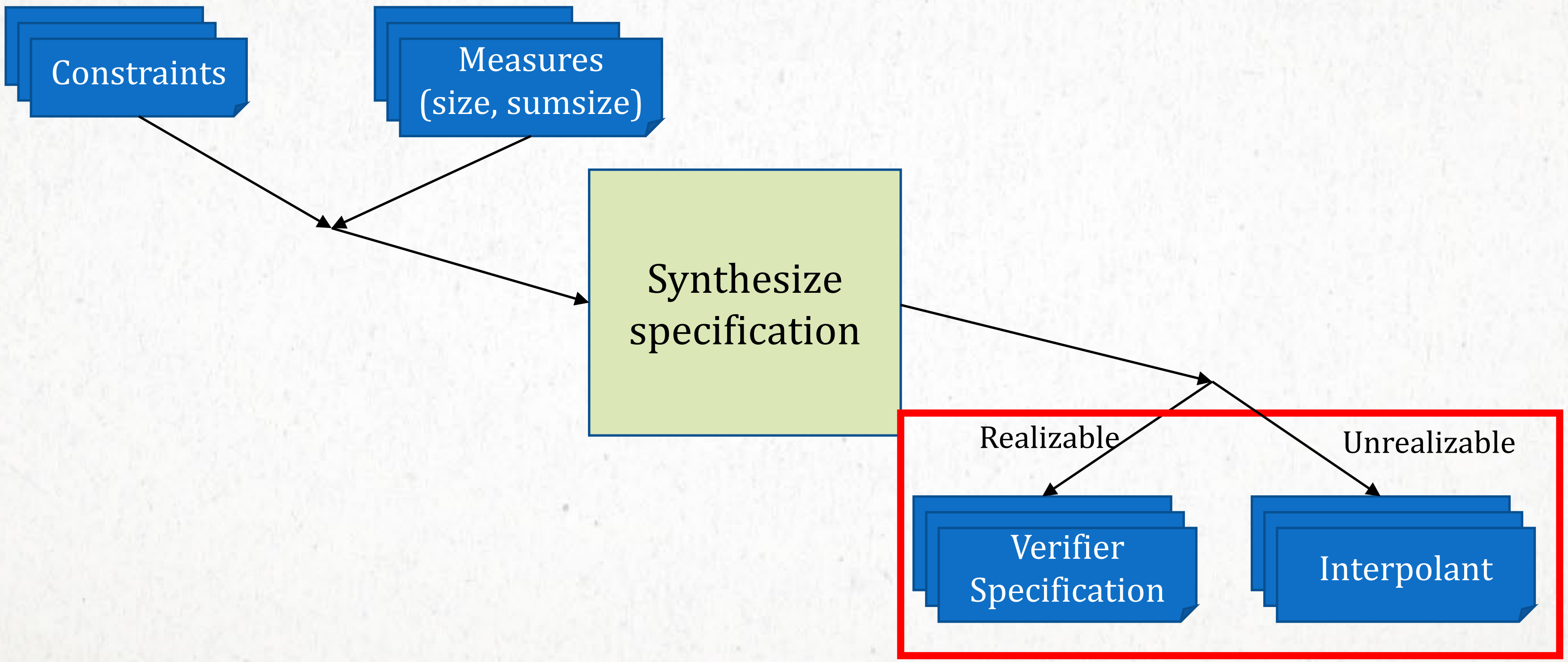
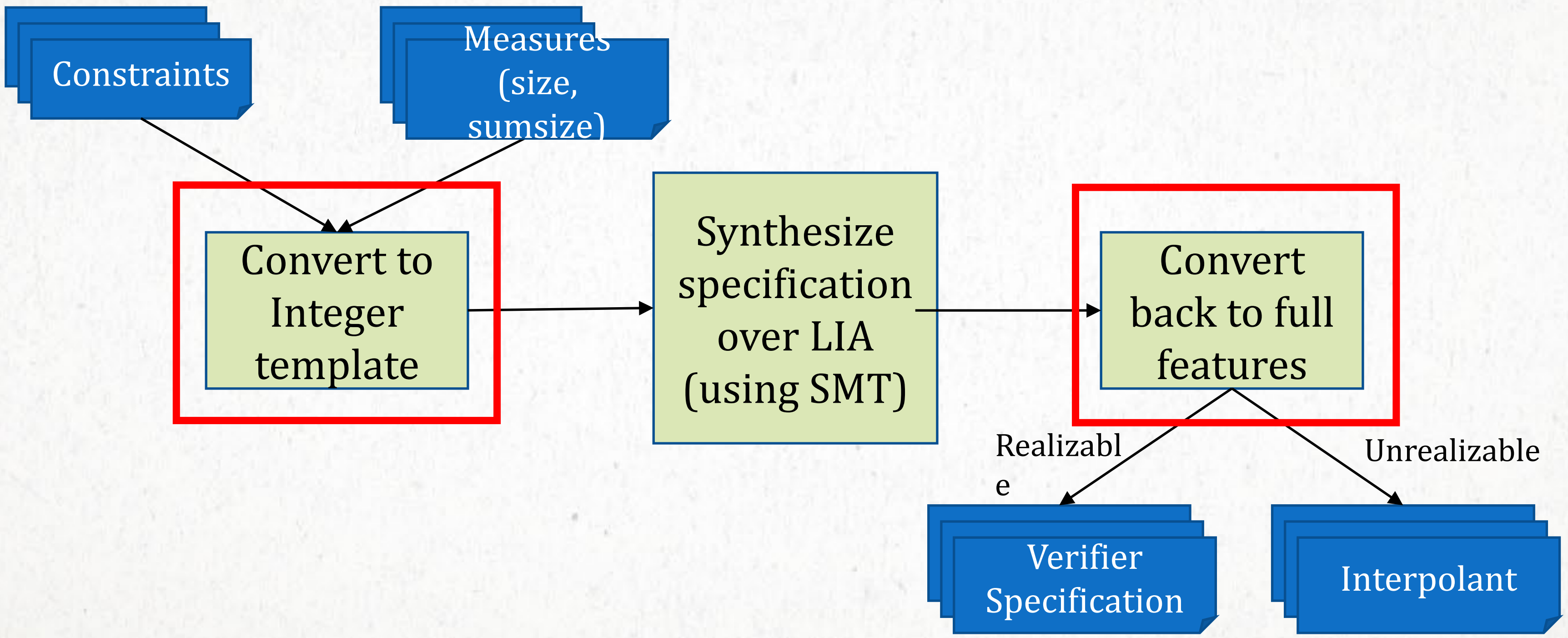
SYNTHESIZER



SYNTHESIZER



SYNTHESIZER

Synthesize LIA specifications for: f :: Int -> Int -> [Int]

f x y = [x + 4, y + 4]

Over:

Specification Type	Specification Example
Integer Literal Inputs/Outputs	f :: {x:Int | x < 0} -> { y:Int | y > 0} -> [Int]
Integer Measures	f :: Int -> Int -> { xs:[Int] | size xs > 0 } size :: [a] -> Int sumsize :: [[a]] -> Int
ADT Contents	f :: Int -> Int -> [{ x:Int | x > 0 }]

CONVERSION

Synthesize LIA specifications for: f :: Int -> Int -> [Int]

f x y = [x + 4, y + 4]

Constraint $\text{pre}_f(0, 1) \Rightarrow \text{post}_f(0, 1, [4, 5])$

Integer Measures

size [4, 5] = 2

$\text{pre}_f(0, 1) \Rightarrow \text{post}_f(0, 1, 2)$

$\text{post}_f(x, y, z) = z > 0$

$\downarrow$

$\text{post}_f(x, y, z) = \{ z:[a] \mid \text{size } z > 0 \}$

CONVERSION

Synthesize LIA specifications for: f :: Int -> Int -> [Int]

f x y = [x + 4, y + 4]

Constraint

$pre_f(0, 1) \Rightarrow post_f(0, 1, [4, 5])$

$post_{f\_cons}(x, y, r)$

ADT Contents

[4, 5]

$$pre_f(0, 1) \Rightarrow post_f(0, 1, 2) \\ \wedge\ post_{f\_cons}(0, 1, 4) \\ \wedge\ post_{f\_cons}(0, 1, 5)$$

$post_f(x, y, r) = r > 0$

$post_{f\_cons}(x, y, r) = r > 2$

$\downarrow$

{ r:[{ x:Int | x > 2 }] | size r > 0 }

SOUNDNESS AND COMPLETENESS

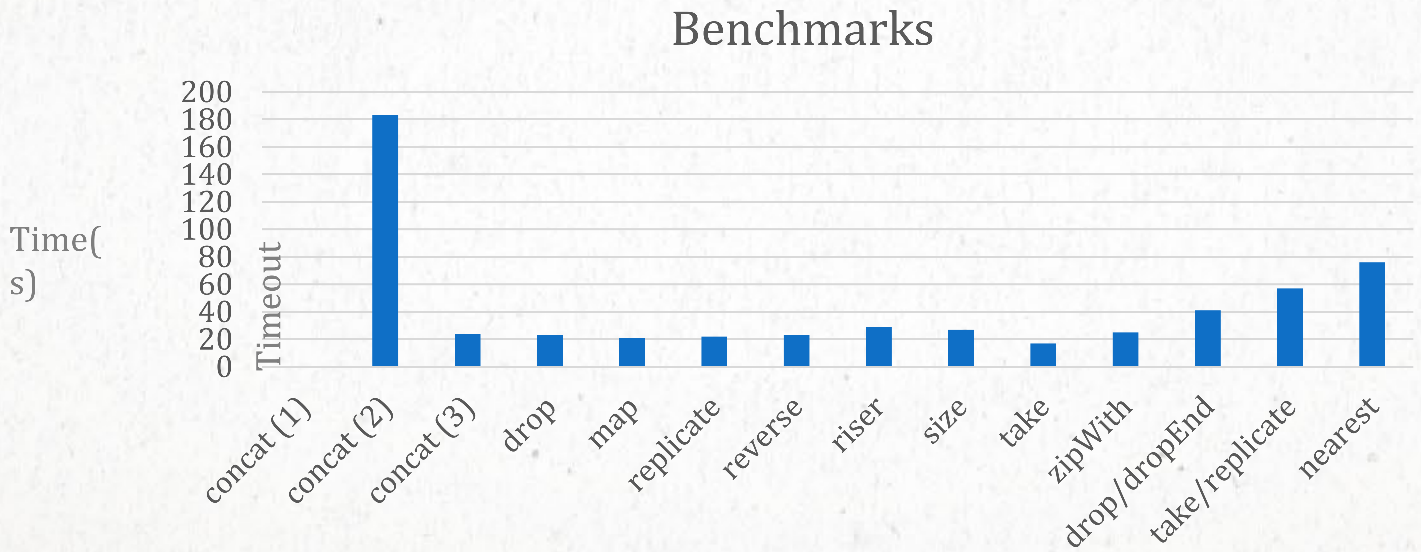
Soundness Theorem –
Assuming a sound verifier, counterexample generator, and synthesizer, the inference algorithm is sound.

Completeness Definition –
We say an inference function is complete if, whenever there exists some set of specifications that will allow verification, the algorithm succeeds in finding such a set.

Completeness Theorem –
Assuming a finite number of possible specifications, and a sound and complete verifier, counterexample generator, and synthesizer, the inference algorithm is complete.

EVALUATION

Ran the inference algorithm on 15 benchmarks, some created by us, some drawn from a graduate student level class homework assignment.



Largest benchmark is the inner loop of a kmeans implementation, involving 34 functions. We prove the codes specifications in 596 seconds (slightly under 10 minutes.)

SUMMARY

- For verification to succeed, modular verifiers require specifications to not only be correct, but be sufficiently supported by callee's specifications.
- Given specifications written by the user, our inference algorithm **automatically** finds the required set of specifications for a modular verifier to succeed.
- Using an SMT solver to synthesizer LIA specifications allows us SyGuS like synthesis, but to also prove unrealizability and get interpolants.
- Our approach is implemented to find LiquidHaskell specifications, using G2 as a counterexamples generator, and it's effectiveness is demonstrated on a variety of benchmarks.