Algebraic Effects and Handlers

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Summary

An algebraic effect is given by

- operations (i.e. *effect constructors*) with signatures
- a set of axioms

Effect handlers

- give interpretations to operations
- effects and handlers can be nested and composed

2. Effect handlers

2.2. Effect type system

Effect type system

expressions

$e := c \mid x \mid \lambda x.\ e \mid e_1\ e_2 \mid \textbf{if}\ e_1\ \textbf{then}\ e_2\ \textbf{else}\ e_3 \mid \mathit{op}\ e \mid \textbf{handle}\ h\ e$

$v := c \mid \lambda x.\ e$

$h := \{\mathit{op} \mapsto \lambda x\ k.\ e_1, \mathit{return} \mapsto \lambda x.\ e_2\}$

Effect type system

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effect labels ℓ example: state has get and put

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effects $\epsilon = \langle\rangle \mid \langle \ell \mid \epsilon \rangle$

effects as: ➤ sets (Eff) ➤ simple rows (Links) ➤ scoped rows (Koka)

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types $A := \mathit{Int} \mid \mathit{Bool} \mid A \to \epsilon\ B$

- Read $A \to \epsilon\ B$ as: a function that, when given A, may perform effects in ϵ, and returns B

Effect type system

$$\boxed{\Gamma \vdash e : A \mid \epsilon}$$

$$\frac{}{\Gamma \vdash \textbf{True} : \mathit{Bool} \mid \epsilon'}\ \mathrm{T} \qquad \frac{}{\Gamma \vdash \textbf{False} : \mathit{Bool} \mid \epsilon'}\ \mathrm{F} \qquad \frac{x : A \in \Gamma}{\Gamma \vdash x : A \mid \epsilon'}\ \mathrm{VAR}$$

$$\frac{\Gamma, x : A \vdash e : B \mid \epsilon}{\Gamma \vdash \lambda x.\ e : A \to_{\epsilon} B \mid \epsilon'}\ \mathrm{LAM} \qquad \frac{\Gamma \vdash e_1 : A \to_{\epsilon} B \mid \epsilon \qquad \Gamma \vdash e_2 : A \mid \epsilon}{\Gamma \vdash e_1\ e_2 : B \mid \epsilon}\ \mathrm{APP}$$

$$\frac{\Gamma \vdash e_1 : \mathit{Bool} \mid \epsilon \qquad \Gamma \vdash e_2 : A \mid \epsilon \qquad \Gamma \vdash e_3 : A \mid \epsilon}{\Gamma \vdash \textbf{if}\ e_1\ \textbf{then}\ e_2\ \textbf{else}\ e_3 : A \mid \epsilon}\ \mathrm{IF}$$

Effect type system

$$\boxed{\Gamma \vdash e : A \mid \epsilon}$$

$$\frac{\mathit{op} : A \rightsquigarrow B \in \Sigma(\ell) \qquad \Gamma \vdash e : A \mid \langle \ell \mid \epsilon \rangle}{\Gamma \vdash \mathit{op}\ e : B \mid \langle \ell \mid \epsilon \rangle}\ \textsc{op}$$

$$\frac{\Gamma \vdash e : C \mid \langle \ell \mid \epsilon \rangle \quad \Gamma \vdash h : C \Rightarrow \langle \ell \mid \epsilon \rangle D}{\Gamma \vdash \textbf{handle}\ h\ e : D \mid \epsilon}\ \textsc{handle}$$

Effect type system

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$$\boxed{\Gamma \vdash h : C \Rightarrow \langle \ell \mid \epsilon \rangle D}$$

$$\frac{\Gamma, x : C \vdash e : D \mid \epsilon \qquad \mathit{op}_i : A_i \rightsquigarrow B_i \in \Sigma(\ell) \qquad \Gamma, x_i : A_i, k_i : B_i \to \epsilon D \vdash e_i : D \mid \epsilon}{\Gamma \vdash \{\mathit{return} \mapsto \lambda x.\ e,\ \mathit{op}_i \mapsto \lambda x_i\ k_i.\ e_i\} : C \Rightarrow \langle \ell \mid \epsilon \rangle D}\ \text{HND}$$

Type safety

Theorem (Type Preservation)
If $\Gamma \vdash e : A \mid \epsilon$, *and* $e \longrightarrow e'$, *then* $\Gamma \vdash e' : A \mid \epsilon$.

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Question: what if e performs an operation in ϵ?

Type safety

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If $\bullet \vdash e : A \mid \epsilon$, *then either* e *is a value, or there exists* e' *such that* $e \longrightarrow e'$, *or* $e = E[op\ v]$, *where* $op \in \epsilon$ *and* $op \# E$.

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Corollary (Progress)
If $\bullet \vdash e : A \mid \langle\rangle$, *then either* e *is a value, or there exists* e' *such that* $e \longrightarrow e'$.

Algebraic effects vs monads

Haskell'19

Monad Transformers and Modular Algebraic Effects

What Binds Them Together

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Abstract

For over two decades, monad transformers have been the main *modular* approach for expressing purely functional side-effects in Haskell. Yet, in recent years algebraic effects have emerged as an alternative whose popularity is growing.

While the two approaches have been well-studied, there is still confusion about their relative merits and expressiveness, especially when it comes to their comparative modularity. This paper clarifies the connection between the two approaches—some of which is folklore—and spells out consequences that we believe should be better known.

We characterise a class of algebraic effects that is modular, and show how these correspond to a specific class of monad transformers. In particular, we show that our modular algebraic effects gives rise to monad transformers. Moreover, every monad transformer for algebraic operations gives rise to a modular effect handler.

CCS Concepts • **Software and its engineering → Functional languages**; *Control structures*; *Coroutines*; • **Theory of computation** → *Categorical semantics.*

Keywords Handlers, Effects, Monads, Transformers

ACM Reference Format:
Tom Schrijvers, Maciej Piróg, Nicolas Wu, and Mauro Jaskelioff. 2019. Monad Transformers and Modular Algebraic Effects: What

1 Introduction

For decades monads [29, 42] have dominated the scene of pure functional programming with effects, and the recent popularisation of algebraic effects & handlers [3, 7, 21, 23, 35] promises to change the landscape. However, with rapid change also comes confusion, and practitioners have been left uncertain about the advantages and limitations of the two competing approaches. This paper aims at clarifying the essential differences and similarities between them.

Working with combinations of multiple different effects demands a *modular* approach, where each effect is given semantics separately. This allows for the construction of complex custom effects from off-the-shelf building blocks.

A popular approach to achieving modularity for monads is with monad transformers [27]. Monad transformers extend an arbitrary monad with a new effect while at the same time ensuring that original effects are available. The desired combination of monads is achieved by stacking several monad transformers in a particular order.

In the algebraic effects approach modularity is conceptually achieved in two stages. First, the syntax of all operations involved in the effect are defined. Then a program is incrementally interpreted by several handlers, which in turn give the syntax of different effects a semantics.

Since each handler only knows about the part of the syntax of the effect it is handling, a modular approach to algebraic

3. Implementing effect handlers

Expensive runtime operations

$$\textbf{handle}\ h\ E[op\ v] \longrightarrow f\ v\ (\lambda x.\ \textbf{handle}\ h\ E[x]) \quad \text{where } op \mapsto f \in h, op \# E$$

Expensive runtime operations

Two potentially expensive runtime operations:

$$\textbf{handle}\ h\ E[op\ v] \longrightarrow f\ v\ \overbrace{(\lambda x.\ \textbf{handle}\ h\ E[x])}^{\textit{capturing}} \quad \text{where } op \mapsto f \in h,\ \overbrace{op \# E}^{\textit{searching}}$$

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$$\begin{aligned} &\mathit{allChoices} = \{ \\ &\quad \mathit{choose} \mapsto \lambda x\ k.\ k\ \textbf{True} +\!\!+ k\ \textbf{False}, \\ &\quad \mathit{return} \mapsto \lambda x.\ [x]\ \} \end{aligned}$$

$$\textbf{handle}\ \mathit{allChoices}\ 42 \longrightarrow [42]$$
$$\textbf{handle}\ \mathit{allChoices}\ (\mathit{toss}\ ()) \longrightarrow [\mathit{Heads}, \mathit{Tails}]$$

Compile effect handlers efficiently

- Continuation-passing style

  Closure allocation cost

Links [Hillerström et al.(2017)]

Compile effect handlers efficiently

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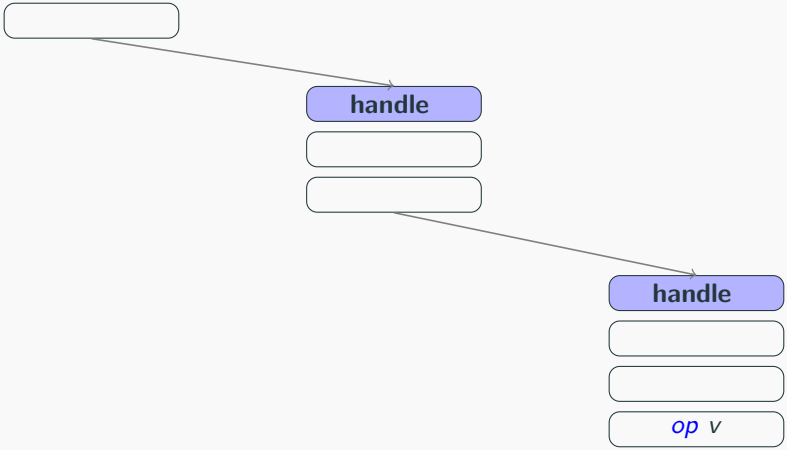
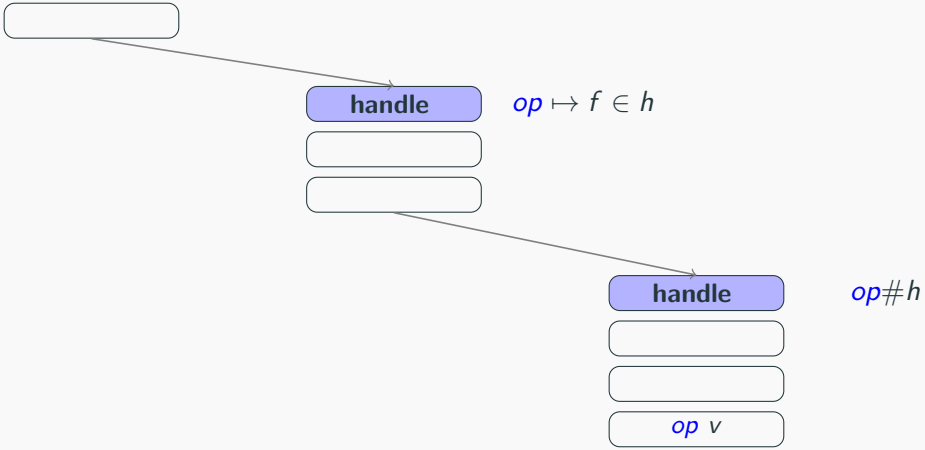
- Segmented stacks — OCaml [Sivaramakrishnan et al.(2021)]
  - Efficient one-shot resumption

Segmented stacks

Segmented stacks



Segmented stacks



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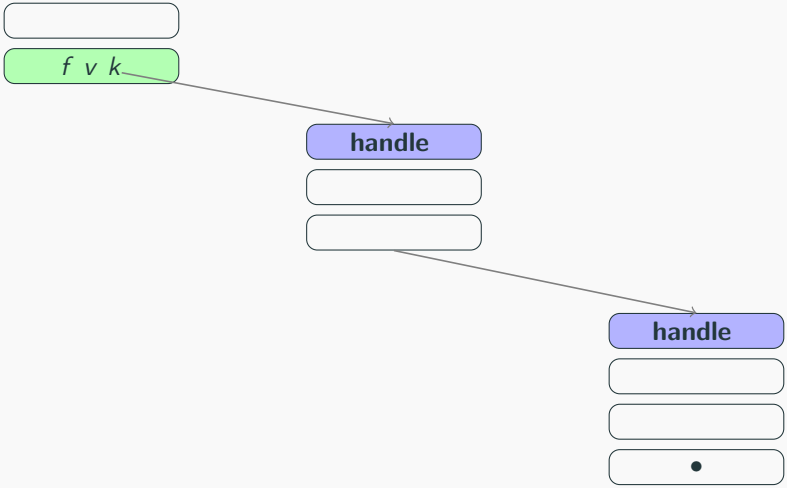

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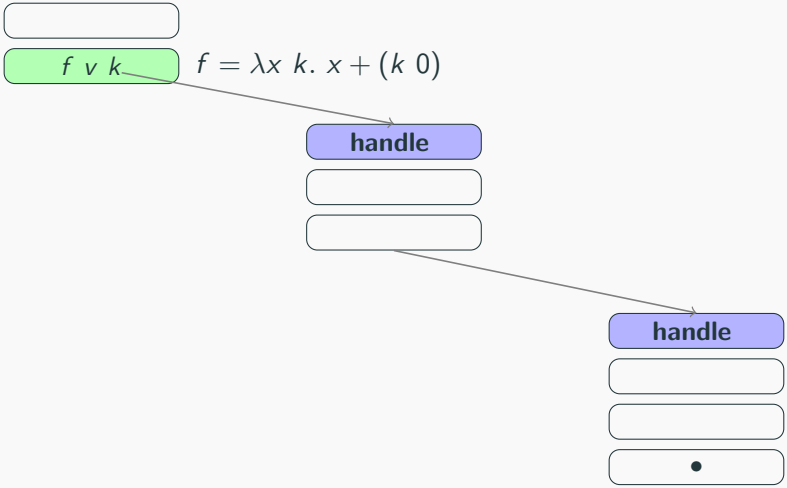

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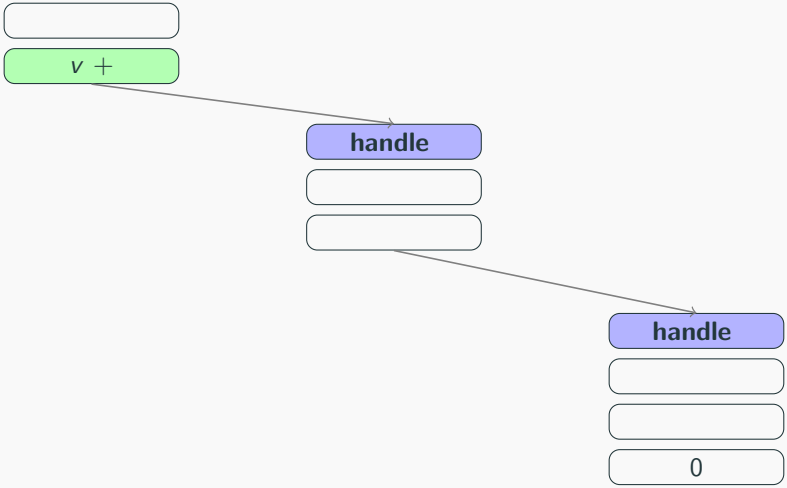

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- Evidence-passing semantics — Koka [Xie et al.(2020), Xie and Leijen(2021)]
  - Efficient tail-resumptive operations

Evidence-passing semantics

- Pass handlers down to the operation call sites
- Evaluate tail-resumptive operations in-place

ICFP'20

Effect Handlers, Evidently

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Algebraic effect handlers are a powerful way to incorporate effects in a programming language. Sometimes perhaps even *too* powerful. In this article we define a restriction of general effect handlers with *scoped resumptions*. We argue one can still express all important effects, while improving reasoning about effect handlers. Using the newly gained guarantees, we define a sound and coherent evidence translation for effect handlers, which directly passes the handlers as evidence to each operation. We prove full soundness and coherence of the translation into plain lambda calculus. The evidence in turn enables efficient implementations of effect operations; in particular, we show we can execute tail-resumptive operations *in place* (without needing to capture the evaluation context), and how we can replace the runtime search for a handler by indexing with a constant offset.

CCS Concepts: • **Software and its engineering → Control structures**; **Polymorphism**; • **Theory of computation → Type theory**.

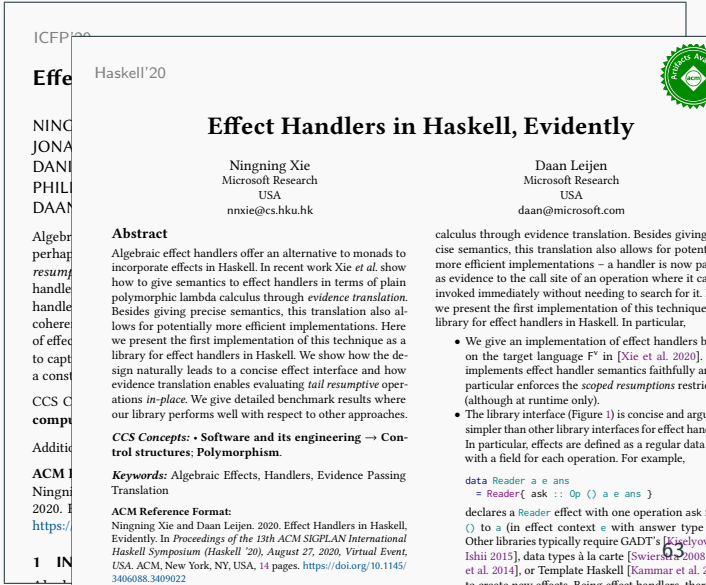
Additional Key Words and Phrases: Algebraic Effects, Handlers, Evidence Passing Translation

ACM Reference Format:
Ningning Xie, Jonathan Immanuel Brachthäuser, Daniel Hillerström, Philipp Schuster, and Daan Leijen. 2020. Effect Handlers, Evidently. *Proc. ACM Program. Lang.* 4, ICFP, Article 99 (August 2020), 29 pages. https://doi.org/10.1145/3408981

1 INTRODUCTION

Evidence-passing semantics

- Pass handlers down to the operation call sites
- Evaluate tail-resumptive operations in-place
- A Haskell library



ICFP'20

Haskell'20

Effect Handlers in Haskell, Evidently

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Abstract

Algebraic effect handlers offer an alternative to monads to incorporate effects in Haskell. In recent work Xie *et al.* show how to give semantics to effect handlers in terms of plain polymorphic lambda calculus through *evidence translation*. Besides giving precise semantics, this translation also allows for potentially more efficient implementations. Here we present the first implementation of this technique as a library for effect handlers in Haskell. We show how the design naturally leads to a concise effect interface and how evidence translation enables evaluating *tail resumptive* operations *in-place*. We give detailed benchmark results where our library performs well with respect to other approaches.

CCS Concepts: • **Software and its engineering** → **Control structures**; **Polymorphism**.

Keywords: Algebraic Effects, Handlers, Evidence Passing Translation

ACM Reference Format:
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calculus through evidence translation. Besides giving
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more efficient implementations – a handler is now pa
as evidence to the call site of an operation where it ca
invoked immediately without needing to search for it.
we present the first implementation of this technique
library for effect handlers in Haskell. In particular,

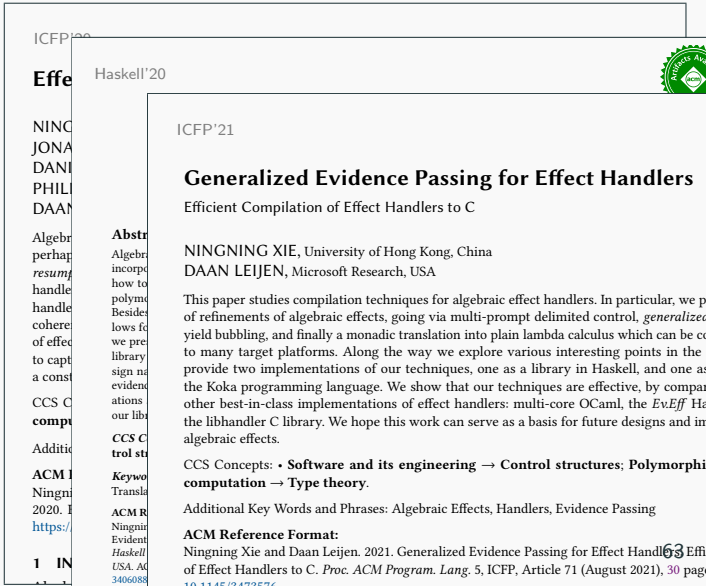
- We give an implementation of effect handlers b
on the target language F^c in [Xie et al. 2020].
implements effect handler semantics faithfully ar
particular enforces the *scoped resumptions* restri
(although at runtime only).
- The library interface (Figure 1) is concise and argu
simpler than other library interfaces for effect han
In particular, effects are defined as a regular data
with a field for each operation. For example,

```
data Reader a e ans
  = Reader{ ask :: Op () a e ans }
```

declares a Reader effect with one operation ask
() to a (in effect context e with answer type
Other libraries typically require GADT's [Kiselyov
Ishii 2015], data types à la carte [Swierstra 2008
et al. 2014], or Template Haskell [Kammar et al. 2

Evidence-passing semantics

- Pass handlers down to the operation call sites
- Evaluate tail-resumptive operations in-place
- ~~A Haskell library~~
  Two Haskell libraries



Evidence-passing semantics

- Pass handlers down to the operation call sites
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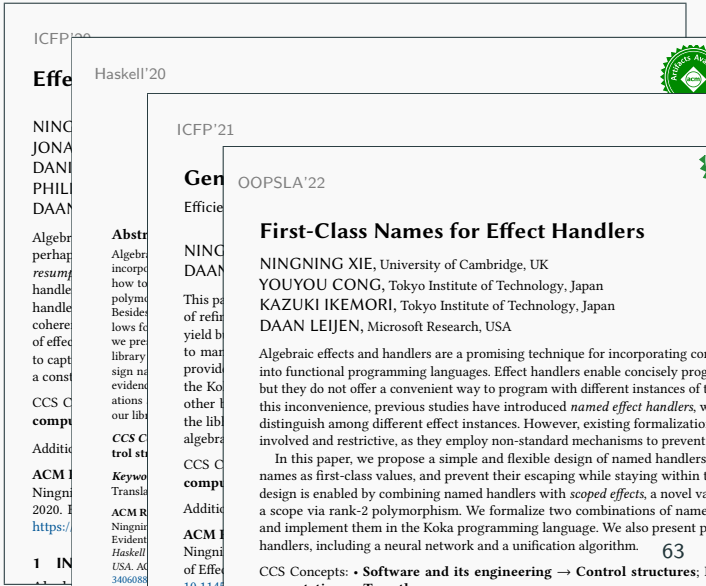
- Specify the handler when performing



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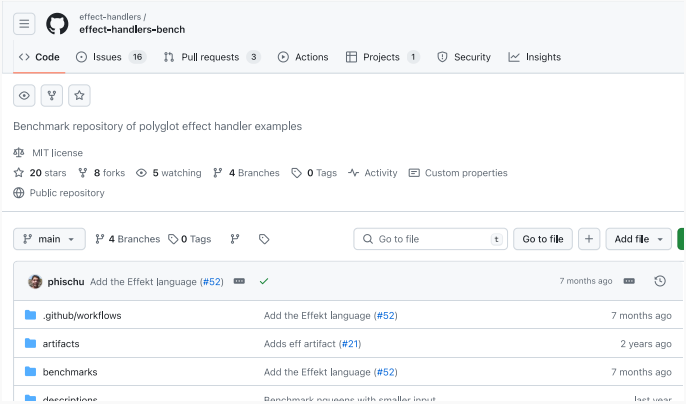
Effect handlers benchmarks suite



https://github.com/effect-handlers/effect-handlers-bench

Resources

- Effect bibliography
  `https://github.com/yallop/effects-bibliography`
- "An introduction to algebraic effects and handlers" [Pretnar(2015)]
- "Effect-Handler Oriented Programming" [Lindley, OPLSS'22]
- "Programming with Effect Handlers and FBIP in Koka" [Leijen and Xie, ICFP'21 Tutorial]
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