Type Theory
OPLSS 2026
Eugene Oregon

H. Geuvers
Radboud University Nijmegen, NL

Exercises Lecture 1

1. Verify in detail (by giving a derivation in $\lambda\!\rightarrow$) that

$$\lambda x : \beta \to \alpha.\lambda y : (\beta \to \alpha) \to \alpha.y(\lambda z : \beta.x\, z) : (\beta \to \alpha) \to ((\beta \to \alpha) \to \alpha) \to \alpha$$

2. "Dress up" the λ-term $\lambda x.\lambda y.y(\lambda z.x\, z)$ with type information in such a way that it is of type $(\beta \to \gamma) \to ((\beta \to \gamma) \to \alpha) \to \alpha$

3. (a) Give the natural deduction (either in Fitch style or in tree form) that corresponds to

$$\lambda x : C \to E.\lambda y : (C \to E) \to E.y(\lambda z.y\, x) \quad : \quad (C \to E) \to ((C \to E) \to E) \to E$$

   (b) Give another term of the same type $(C \to E) \to ((C \to E) \to E) \to E$ and the natural deduction (either in Fitch style or in tree form) that it corresponds to.

4. Give a term M in $\lambda\!\rightarrow$ à la Church with type

$$((\beta{\to}\gamma){\to}\alpha){\to}((\alpha{\to}\beta){\to}\gamma){\to}\alpha.$$

   Give a typing derivation that gives the type of M.

5. Let α be a type variable and define the type $A_0 := \alpha$ and $A_{n+1} := A_n \to \alpha$.

   (a) Give a closed term of type A_3.

   (b) For which types A_n is there a closed term of type A_n? Define a term $P_n : A_n$ for those A_n.

6. (a) Give term-constructors for the type $A + B$ in $\lambda\!\rightarrow$. These are an "in-left", "in-right" and a "case" construction.

   (b) Give reduction rules for these term constructions that correspond with detour-elimination.