Type Theory
OPLSS 2026
Eugene Oregon

H. Geuvers
Radboud University Nijmegen, NL

Exercises Lecture 3

1. Find terms of the following types (NB $\rightarrow$ binds strongest). Also write down the contexts in which these terms are typed.
   (a) $(\Pi x{:}A.P\,x \rightarrow Q\,x) \rightarrow (\Pi x{:}A.P\,x) \rightarrow \Pi x{:}A.Q\,x$
   (b) $(\Pi x{:}A.P\,x \rightarrow \Pi z.R\,z\,z) \rightarrow (\Pi x{:}A.P\,x) \rightarrow \Pi z{:}A.R\,z\,z)$.
2. In λP, give a term of type $\Pi v : D.\, R\,(f\,(f\,v))\,v \rightarrow R\,(f\,v)\,v$ in the context

$$\begin{array}{rcl} \Gamma & := & D : *, R : D \rightarrow D \rightarrow *, f : D \rightarrow D, \\ & & h \;:\; \Pi x, y : D.\, R\,(f\,x)\,y \rightarrow R\,x\,(f\,y), \\ & & r \;:\; \Pi z : D.\, R\,(f\,z)\,(f\,z) \rightarrow R\,(f\,z)\,z \end{array}$$

3. Construct a term of type $\mathsf{T}(A \Rightarrow (B \Rightarrow A))$ in the LF interpretation of minimal proposition logic. (See the slides).
4. Construct a proof-term that mirrors the (obvious) proof of $\forall x(P\,x \Rightarrow Q\,x) \Rightarrow \forall x.P\,x \Rightarrow \forall x.Q\,x$ in the LF interpretation of minimal predicate logic. (See the slides).
5. Consider the interpretation of untyped λ-calculus in λP (see the slides).
   (a) Prove (i.e. find a proof term of the associated type) $(\lambda x.x)y =_\beta y$.
   (b) Add an axiom for η-equality ($\lambda x.Px =_\eta P$ if $x \notin \mathrm{FV}(P)$) to the context and the extensionality rule $(\forall N(M\,N = P\,N \rightarrow M = N))$
   (c) Prove that η follows from extensionality.